Radiative Corrections to Parity-Violating Electron Scattering Experiments

Andrei Afanasev
Jefferson Lab
JLab/INT Precision ElectroWeak Workshop
August 15, 2005
Plan of talk

Overview of radiative corrections for parity-violating experiments

Role of Two-Photon Exchange effects in \( e+p \rightarrow e+p \)
  - Factorizable and non-factorizable corrections
  - GPD-based calculation
Atomic Parity Violation

- Review by M. Kuchiev, V. Flambaum, hep-ph/0305053
  - Atomic PNC experiments reached accuracy of ~0.3% (Boulder group)
  - Including full QED correction is crucial. Outstanding problems with self-energy correction (~0.7%) were resolved, bringing suggested 2σ-deviation into agreement with SM
Leptonic Process: Moller Scattering

- PV in Moller scattering, SLAC E158, hep-ph/0504049
  \[ A_{PV} = -131\pm14\pm10 \text{ ppb} \]
- Radiative Corrections are be calculated within Standard Model:
- QED correction for asymmetry is \(-1\%\), with \(2\gamma\) and \(\gamma Z\)-boxes included
Semi-Leptonic Processes involving nucleons

- Neutrino-nucleon scattering
  Per cent level reached by NuTeV. Radiative corrections for DIS calculated at a partonic level (D. Bardin et al.)

- Neutron beta-decay: Important for $V_{ud}$ measurements; axial-vector coupling $g_A$


- Parity-violating elastic ep (strange quark effects, weak mixing angle)
Elastic Nucleon Form Factors

- Based on one-photon exchange approximation

\[ M_{fi} = M_{fi}^{1\gamma} \]

\[ M_{fi}^{1\gamma} = e^2 \bar{u}_e \gamma_\mu u_e \bar{u}_p (F_1(t) \gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u_p \]

- Two techniques to measure

\[ \sigma = \sigma_0 (G_M^2 \tau + \varepsilon \cdot G_E^2) \quad \text{: Rosenbluth technique} \]

\[ \frac{P_x}{P_z} = - \frac{A_x}{A_z} = - \frac{G_E \sqrt{\tau} \sqrt{2\varepsilon(1-\varepsilon)}}{G_M \tau \sqrt{1-\varepsilon^2}} \quad \text{: Polarization technique} \]

\[ G_E = F_1 - \varepsilon F_2, \quad G_M = F_1 + F_2 \]

\( (P_y = 0) \)

Latter due to: Akhiezer, Rekalo; Arnold, Carlson, Gross
Bethe-Heitler corrections to polarization transfer and cross sections


Figure 2: Radiative corrections to the unpolarized cross section (left plot) and polarization asymmetries (right plot) defined in (41). Solid and dashed lines corresponds to longitudinal and transverse cases. $S=8$ GeV$^2$.

In kinematics of elastic ep-scattering measurements, cross sections are more sensitive to RC
Full Calculation of Bethe-Heitler Contribution

Additional work by AA et al., using MASCARAD (Phys. Rev. D64: 113009, 2001)
Full calculation including soft and hard bremsstrahlung

Radiative leptonic tensor in full form
AA et al, PLB 514, 269 (2001)

\[ L_{\mu \nu} = -\frac{1}{2} Tr (\hat{k}_2 + m) \Gamma_{\mu \alpha} (1 + \gamma_5 \hat{s}_e) (\hat{k}_1 + m) \Gamma_{\alpha \nu} \]

\[ \Gamma_{\mu \alpha} = \left( \frac{k_{1 \alpha}}{k \cdot k_1} - \frac{k_{2 \alpha}}{k \cdot k_2} \right) \gamma_\mu - \frac{\gamma_\mu \hat{k}_\gamma \gamma_\alpha}{2k \cdot k_1} - \frac{\gamma_\alpha \hat{k}_\gamma \mu}{2k \cdot k_2} \]

\[ \Gamma_{\alpha \nu} = \left( \frac{k_{1 \alpha}}{k \cdot k_1} - \frac{k_{2 \alpha}}{k \cdot k_2} \right) \gamma_\nu - \frac{\gamma_\alpha \hat{k}_\gamma \nu}{2k \cdot k_1} - \frac{\gamma_\nu \hat{k}_\gamma \alpha}{2k \cdot k_2} \]

Additional effect of full soft+hard brem → +1.2% correction to \( \varepsilon \)-slope
Resolves additional ~25% of Rosenbluth/polarization discrepancy!
Do the techniques agree?

Both early SLAC and recent JLab experiments on (super)Rosenbluth separations followed Ge/Gm~const

JLab measurements using polarization transfer technique give different results (Jones'00, Gayou'02)

Radiative corrections, in particular, a short-range part of 2-photon exchange is a likely origin of the discrepancy
Electron Scattering: LO and NLO in $\alpha_{em}$

Radiative Corrections:
- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c, d)
- Two-photon exchange (e, f)
- Proton vertex and VCS (g, h)
- Corrections (e-h) depend on the nucleon structure

"Guichon & Vanderhaeghen'03:
Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3% x 2 ...

Main issue: Corrections dependent on nucleon structure
Model calculations:

Jefferson Lab
Office of Science
U.S. Department of Energy

Operated by the Southeastern Universities Research Association for the U.S. Dept. of Energy  Andrei Afanasev, Radiative Corrections to Parity-Violating Experiments, PEW05, 8/16/05
Separating soft photon exchange

- Tsai, Maximon & Tjon
- We used Grammer & Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to the cross section
- NB: Corresponding effect to polarization transfer and/or spin asymmetry is zero

\[ \delta_{\text{Soft}} \]

\[ Q^2 = 6 \text{ GeV}^2 \]
Lorentz Structure of 2-γ amplitude

Generalized form factors are functions of two Mandelstam invariants; specific dependence is determined by nucleon structure

\[ M_{fi} = M_{fi}^{1\gamma} + M_{fi}^{2\gamma} \]

\[ M_{fi}^{1\gamma} = \bar{u} e \gamma_\mu u e \bar{u} p (F_1(t) \gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u p \]

\[ M_{fi}^{2\gamma} = V_e \otimes V_p + A_e \otimes A_p \]

\[ V_e = \bar{u} e \gamma_\mu u e, \quad V_p = \bar{u} p (F_1'(s,u) \gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2'(s,u)) u p \]

\[ A_e = \bar{u} e \gamma_\mu \gamma_5 u e, \quad A_p = \bar{u} p G_A(s,u) \gamma_\mu \gamma_5 u p \]
New Expressions for Observables

We can formally define ep-scattering observables in terms of the new form factors:

\[
\sigma = \sigma_0 \left( |G_M|^2 \tau + \epsilon \cdot |G_E|^2 + 2\sqrt{\tau(1+\tau)(1-\epsilon^2)} \Re G_M^* G_A \right)
\]

\[
\frac{P_x}{P_z} = -\frac{\Re(G_E^* G_M)\sqrt{\tau} \sqrt{2\epsilon(1-\epsilon)} + \Re(G_E^* G_A)\sqrt{1+\tau} \sqrt{2\epsilon(1+\epsilon)}}{|G_M|^2 \tau \sqrt{1-\epsilon^2} + \Re(G_M^* G_A)2\sqrt{\tau(1+\tau)}}
\]

\[
P_y = \frac{\Im(G_E^* G_M)\sqrt{\tau} \sqrt{2\epsilon(1+\epsilon)} + \Im(G_E^* G_A)\sqrt{1+\tau} \sqrt{2\epsilon(1-\epsilon)}}{|G_M|^2 \tau + \epsilon \cdot |G_E|^2 + 2\sqrt{\tau(1+\tau)(1-\epsilon^2)} \Re G_M^* G_A}
\]

For the target asymmetries: \(A_x = P_x\), \(A_y = P_y\), \(A_z = -P_z\)
Calculations using Generalized Parton Distributions

Model schematics:
- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
  - Use Grammer-Yennie prescription

Hard interaction with a quark

AA, Brodsky, Carlson, Chen, Vanderhaeghen,
Short-range effects; on-mass-shell quark
(AA, Brodsky, Carlson, Chen, Vanderhaeghen)

Two-photon probe directly interacts with a (massless) quark
Emission/reabsorption of the quark is described by GPDs

\[ A_{e_q \rightarrow e_q}^{2\gamma} = \frac{e_q^2}{t} \frac{\alpha_{em}}{2\pi} (V_{\mu}^e \otimes V_{\mu}^q \times f_{\gamma} + A_{\mu}^e \otimes A_{\mu}^q \times f_A), \]

\[ V_{\mu}^{e,q} = \overline{u}_{e,q} \gamma_{\mu} u_{e,q}, \quad A_{\mu}^{e,q} = \overline{u}_{e,q} \gamma_{\mu} 5 u_{e,q} \]

\[ f_{\gamma} = -2[\log\left(\frac{-u}{s}\right) + i\pi]\log\left(-\frac{t}{\Lambda^2}\right) - \frac{t}{2s} \left[ 2 \left( \log\left(\frac{-t}{u}\right) + i\pi \right) \right] - \frac{1}{u} \log\left(-\frac{s}{t}\right) \]

\[ + \frac{(u^2 - s^2)}{4} \left( \frac{1}{s^2} \left( \log^2\left(\frac{u}{t}\right) + \pi^2 \right) + \frac{1}{u^2} \log\left(-\frac{s}{t}\right)\left( \log\left(-\frac{s}{t}\right) + i2\pi \right) \right) + i\pi \frac{u^2 - s^2}{2su} \]

\[ f_A = -\frac{t}{2} \left[ \frac{1}{s} \left( \log\left(\frac{u}{t}\right) + i\pi \right) + \frac{1}{u} \log\left(-\frac{s}{t}\right) \right] + \]

\[ + \frac{(u^2 - s^2)}{4} \left( \frac{1}{s^2} \left( \log^2\left(\frac{u}{t}\right) + \pi^2 \right) - \frac{1}{u^2} \log\left(-\frac{s}{t}\right)\left( \log\left(-\frac{s}{t}\right) + i2\pi \right) \right) + i\pi \frac{t^2}{2su} \]

Note the additional effective (axial-vector)^2 interaction; absence of mass terms
`Hard' contributions to generalized form factors

GPD integrals

\[ A \equiv \int_{-1}^{1} \frac{dx}{x} \frac{(s - \hat{u}) \bar{f}_1^{hard} - \hat{s} \hat{u} \bar{f}_3}{(s - u)} \sum_q e_q^2 \left( H^q + E^q \right), \]

\[ B \equiv \int_{-1}^{1} \frac{dx}{x} \frac{(s - \hat{u}) \bar{f}_1^{hard} - \hat{s} \hat{u} \bar{f}_3}{(s - u)} \sum_q e_q^2 \left( H^q - \tau E^q \right), \]

\[ C \equiv \int_{-1}^{1} \frac{dx}{x} \bar{f}_1^{hard} \text{sgn}(x) \sum_q e_q^2 \tilde{H}^q \]

Two-photon-exchange form factors from GPDs

\[ \delta \tilde{G}_M^{hard} = C \]

\[ \delta \tilde{G}_E^{hard} = -\left(\frac{1 + \varepsilon}{2\varepsilon}\right)(A - C) + \sqrt{\frac{1 + \varepsilon}{2\varepsilon}} B \]

\[ \tilde{F}_3 = \frac{M^2}{\nu} \left(\frac{1 + \varepsilon}{2\varepsilon}\right)(A - C) \]
Two-Photon Effect for Rosenbluth Cross Sections

- Data shown are from Andivahis et al., PRD 50, 5491 (1994)
- Included GPD calculation of two-photon-exchange effect
- Qualitative agreement with data:
- Discrepancy likely reconciled

\[ \sigma_R / (\mu_p G_D)^2 \]

\[ Q^2 = 3.25 \text{ GeV}^2 \]

\[ Q^2 = 4 \text{ GeV}^2 \]

\[ Q^2 = 5 \text{ GeV}^2 \]

\[ Q^2 = 6 \text{ GeV}^2 \]
Updated Ge/Gm plot


Rosenbluth w/2-γ corrections vs. Polarization data

G_E / G_M vs. Q^2 (GeV^2)

Pol.: Jones et al.
Pol.: Gayou et al.
Pol.: Gayou et al. fit
Rosenbluth, Mo-Tsai corr. only
Rosenbluth, incl. 2γ corr. w/ gaussian GPD

Operated by the Southeastern Universities Research Association for the U.S. Dept. of Energy
Andrei Afanasev, Radiative Corrections to Parity-Violating Experiments, PEW05, 8/16/05
Polarization transfer

- Also corrected by two-photon exchange, but with little impact on Gep/Gmp extracted ratio.
Charge asymmetry

. Cross sections of electron-proton scattering and positron-proton scattering are equal in one-photon exchange approximation
  . Different for two- or more photon exchange

To be measured in JLab Experiment 04-116, Spokepersons W. Brooks et al.
Parity Violating elastic e-N scattering

Longitudinally polarized electrons, unpolarized target

\[
A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[ -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \right] \frac{A_E + A_M + A_A}{2\sigma_{\text{unpol}}}
\]

\[
A_E = \varepsilon(\theta) G_E Z G_E^\gamma
\]

\[
A_M = \tau G_M Z G_M^\gamma
\]

\[
A_A = -(1 - 4\sin^2\theta_W)\varepsilon' G_A G_M^\gamma
\]

Neutral weak form factors contain explicit contributions from strange sea

\[
G_{E,M}^Z(Q^2) = (1 - 4\sin^2\theta_W)(1 + R_A^p)G_{E,M}^p - (1 + R_A^n)G_{E,M}^n - G_{E,M}^s
\]

\[
G_A^e(Q^2) = -G_A^Z + (\eta F_A^\gamma + R^e) + \Delta s
\]

\[
\eta = \frac{8\pi\alpha\sqrt{2}}{1 - 4\sin^2\theta_W} = 3.45
\]

\[
G_A^Z(0) = 1.2673 \pm 0.0035 \text{ (from } \beta \text{ decay)}
\]
Born and Box diagrams for elastic ep-scattering

(b) Presumed small, e.g., M. Ramsey-Musolf, Phys. Rev. C60 (1999) 015501
New Expressions for PV asymmetry

PV-asymmetry, Born Approximation

\[ A_{PV}^{\text{Born}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \left[ A_E^{\text{Born}} + A_M^{\text{Born}} + A_A^{\text{Born}} \right] \]

\[ A_E^{\text{Born}} = -2g_A^e g_{Ep}^Z G_{Ep}^\nu, \quad A_M^{\text{Born}} = -2g_A^e \tau G_{Mp}^Z G_{Ep}^\nu, \]

\[ A_A^{\text{Born}} = 2g_V^e \epsilon \sqrt{\tau(1+\tau)(1-\epsilon^2)} G_A^Z G_{Mp}^\nu, \]

\[ g_V^e = -(1-4\sin^2 \theta_W)/2, \quad g_A^e = -1/2 \]

PV asymmetry in terms of generalized form factors including multi-photon exchange

\[ A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \tau \left| G_{Mp}^\nu \right|^2 + \epsilon \left| G_{Ep}^\nu \right|^2 + 2\sqrt{\tau(1+\tau)} G_{Ap}^\nu \]
2\(\gamma\)-exchange Correction to Parity-Violating Electron Scattering

\[
\begin{align*}
\gamma & \quad \gamma & \quad \times & \quad Z^0 \\
\text{Electromagnetic} & \quad & \text{Neutral Weak}
\end{align*}
\]

**New** parity violating terms due to \((2\gamma)x(Z^0)\) interference should be added:

\[
A'_M = -2g^e_A \sqrt{\tau(1 + \tau)(1 - \varepsilon^2)} G^Z_M \text{Re}(G'_A)
\]

\[
A'_A = 2g^e_V (1 + \tau) G^Z_A \text{Re}(G'_A)
\]
GPD Calculation of $2\gamma \times Z$-interference

- Can be used at higher $Q^2$, but points at a problem of additional systematic corrections for parity-violating electron scattering. The effect evaluated in GPD formalism is the largest for backward angles:

$$A_{PV}(1\gamma+2\gamma)/A_{PV}(1\gamma)$$

- $Q^2=2 \text{ GeV}^2$
- $Q^2=5 \text{ GeV}^2$
- $Q^2=9 \text{ GeV}^2$


Important note: (nonsoft) $2\gamma$-exchange amplitude has no $1/Q^2$ singularity; $1\gamma$-exchange is $1/Q^2$ singular $\Rightarrow$ At low $Q^2$, $2\gamma$-corrections is suppressed as $Q^2$. P. Blunden used this formalism and evaluated correction of 0.16% for $Q_{\text{weak}}$. 

Operated by the Southeastern Universities Research Association for the U.S. Dept. of Energy

Andrei Afanasev, Radiative Corrections to Parity-Violating Experiments, PEW05, 8/16/05
Two-photon exchange for electron-proton scattering

- Quark-level short-range contributions are substantial (3-4%)
- Structure-dependent radiative corrections calculated using GPDs bring into agreement the results of polarization transfer and Rosenbluth techniques for Gep measurements
- $2\gamma$-correction to parity-violating asymmetry does not cancel. May reach a few per cent for GeV momentum transfers
- Corrections are angular-dependent, not reducible to re-definition of coupling constants
  - Revision of $\gamma Z$-box contribution and extension of model calculations to lower $Q^2$ is necessary
- Experimentally measurable directly by comparing electrons vs positrons on a spin-0 target- it is difficult $\Rightarrow$ in the meantime need to rely on the studies of $2\gamma$-effect for parity-conserving observables