Strangeness Content of the Nucleon

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Workshop on Precision ElectroWeak Interactions
College of W&M: August 16th, 2005
Outline

- The QCD Vacuum
- Quarks to Hadrons
- Measurements of Nucleon Form Factors
- Latest Results on Strangeness
- A Precise Theoretical Calculation of $G_M^s$
- What needs measuring?
Topology of QCD Vacuum
Powerful Qualitative New Insights From Lattice QCD

QCD sum rules:

\[ \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^i G_{i}^{\mu\nu} | 0 \rangle = \langle 0 | \frac{2\alpha_s}{\pi} (B^2 - E^2) | 0 \rangle = (350 \pm 30 \text{ MeV})^4, \]

- Non-trivial topological structure of vacuum linked to dynamical chiral symmetry breaking
- There are regions of positive and negative topological charge
- BUT they clearly are **NOT** spherical
- **NOR** are they weakly interacting!
Quark Condensate

\[\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle = -(225 \pm 25 \text{ MeV})^3\]

at a renormalization scale of about 1 GeV.

- commutator measures chiral symmetry breaking
- \(\frac{1}{4}\) valence + pion cloud +
- volume * (difference of condensate in & out of N)

and last term is as big as 20 MeV (or more)

i.e. presence of nucleon "cleans out" vacuum to some extent

Hence: Model independent LO term for in-medium condensate

\[
\frac{Q(\rho_B)}{Q_0} \approx 1 - \frac{\sigma_N}{f^2 \pi m^2} \rho_B
\]

BUT this has no new physics at all!
QCD and the Origin of Mass

\[ u + u + d = \text{proton} \]
\[ \text{mass: } 0.003 + 0.003 + 0.006 \neq 0.938 \]

How does the rest of the proton mass arise?
χ’al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon

FRR give same answer to <<1\% systematic error!

<table>
<thead>
<tr>
<th>Regulator</th>
<th>$a_0^A$</th>
<th>$a_2^A$</th>
<th>$a_4^A$</th>
<th>$\Lambda$</th>
<th>$c_0$</th>
<th>$c_2$</th>
<th>$c_4$</th>
<th>$m_N$</th>
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<tbody>
<tr>
<td>Monopole</td>
<td>1.74</td>
<td>1.64</td>
<td>−0.49</td>
<td>0.5</td>
<td>0.923(65)</td>
<td>2.45(33)</td>
<td>20.5(15)</td>
<td>0.960(58)</td>
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<td>Dipole</td>
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<td>1.54</td>
<td>−0.49</td>
<td>0.8</td>
<td>0.922(65)</td>
<td>2.49(33)</td>
<td>18.9(15)</td>
<td>0.959(58)</td>
</tr>
<tr>
<td>Gaussian</td>
<td>1.17</td>
<td>1.48</td>
<td>−0.50</td>
<td>0.6</td>
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<td>2.48(33)</td>
<td>18.3(15)</td>
<td>0.960(58)</td>
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<tr>
<td>Sharp cutoff</td>
<td>1.06</td>
<td>1.47</td>
<td>−0.55</td>
<td>0.4</td>
<td>0.923(65)</td>
<td>2.61(33)</td>
<td>15.3(8)</td>
<td>0.961(58)</td>
</tr>
<tr>
<td>Dim. Reg. (BP)</td>
<td>0.79</td>
<td>4.15</td>
<td>+8.92</td>
<td>0.4</td>
<td>0.875(56)</td>
<td>3.14(25)</td>
<td>7.2(8)</td>
<td>0.923(51)</td>
</tr>
</tbody>
</table>

Leinweber et al., PRL 92 (2004) 242002

Jefferson Lab
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Convergence from LNA to NLNA is Rapid – Using Finite Range Regularization

<table>
<thead>
<tr>
<th>Regulator</th>
<th>LNA</th>
<th>NLNA</th>
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</thead>
<tbody>
<tr>
<td>Sharp</td>
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<td>961</td>
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<td>960</td>
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<tr>
<td>Dim Reg</td>
<td>784</td>
<td>884</td>
</tr>
</tbody>
</table>

\( M_N \) in MeV
Comparison with $\chi$ QSM

Goeke et al., hep-lat/0505010
Analysis of pQQCD $\rho$ data from CP PACS

\[
\sqrt{(M_{V}^{\text{deg}})^2 - \Sigma_{TOT}} = (a_{0}^{\text{cont}} + X_{1}a + X_{2}a^2) + a_{2}(M_{PS}^{\text{deg}})^2 + a_{4}(M_{PS}^{\text{deg}})^4 + a_{6}(M_{PS}^{\text{deg}})^6
\]
Infinite Volume Unitary Results

All 80 data points drop onto single, well defined curve

Allton, Young et al., hep-lat/0504022

777 § 7 MeV
JLab: Unique Capabilities for Investigating QCD in the Non-Perturbative Regime

JLab is a world leader in SRF technology: SNS, 12 GeV Upgrade, FEL, RIA, and others in the Office of Science 20-Year Facilities Outlook

Superconducting rf (SRF) technology makes the circulating accelerator feasible

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High luminosity, high resolution detectors in Halls A, B, and C.

Thomas Jefferson National Accelerator Facility

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U.S. DEPARTMENT OF ENERGY
Precision Tests of Nucleon Structure

- Astonishing discovery concerning proton electric form factor

- But what about contribution from non-valence quarks
  - especially strange quarks?
Strangeness Widely Believed to Play a Major Role – Does It?

- As much as 100 to 300 MeV of proton mass:

\[ M_N = \langle N(P) \rangle - \frac{9 \alpha_s}{4 \pi} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u \bar{\psi}_u \psi_u + m_d \bar{\psi}_d \psi_d + m_s \bar{\psi}_s \psi_s |N(P)\rangle \]

\[ \Delta M^s_{N} = \frac{ym_s}{m_u + m_d} \sigma_N \]

\( y=0.2 \ § 0.2 \]

45 § 8 MeV (or 70?)

Hence 110 § 110 MeV (increasing to 180 for higher \( \sigma_N \))

- Through proton spin crisis:
  As much as 10% of the spin of the proton

- HOW MUCH OF THE MAGNETIC FORM FACTOR?
World Data @ $Q^2 = 0.1 \text{ GeV}^2$

$G_E^s = -0.013 \pm 0.028$

$G_M^s = +0.62 \pm 0.31$

$\pm 0.62 \text{ 2}\sigma$

Contours

$1\sigma, 2\sigma$

68.3, 95.5% CL

Theories

1. Leinweber, et al.
PRL 94 (05) 212001

2. Lyubovitskij, et al.
PRC 66 (02) 055204

3. Lewis, et al.
PRD 67 (03) 013003

PRD 65 (01) 014016
Simple Fits to World Hydrogen Data

- **Fit**

\[
G_E^s(Q^2) + \eta(Q^2, E_i) G_M^s(Q^2) = \\
\frac{4\pi\alpha\sqrt{2}}{G_F Q^2} \left( \frac{\varepsilon G_E^p}{1 + \rho^2} + \frac{\tau G_M^p}{\varepsilon G_E^p} \right) \left( A_{\text{phys}} - A_{\text{NVS}}(Q^2, E_i) \right)
\]

with simple forms for \( G_E^s, G_M^s \)

\[
G_E^s(Q^2) = \frac{c_2 Q^4}{1 + d_1 Q^2 + d_2 Q^4 + d_3 Q^6}
\]

à la Kelly

\[
G_M^s(Q^2) = \frac{G_M^s(Q^2 = 0)}{\left(1 + Q^2 / \Lambda_M^2\right)^2}
\]

with

\[
G_M^s(Q^2 = 0) = 0.81
\]

from \( Q^2 = 0.1 \text{ GeV}^2 \) plot, dipole ff

DHB, 17 June 2005
“Fit” to World Hydrogen Data

c_2 = -0.51 ± 0.25
d_1 = -8.5 ± 0.9
d_2 = 24 ± 6
d_3 = 1
\Lambda_{M}^{s^2} = \Lambda^2 / 1.3

\begin{align*}
G^s_E + \eta G^s_M & \quad \text{(black line)} \\
G^s_E & \quad \text{(red curve)} \\
G^s_M & \quad \text{(blue curve)}
\end{align*}

DHB, 17 June 2005
Significance & Comparison with Lattice QCD

- Size and sign of the strange magnetic moment is astonishing!
- Experimental isoscalar nucleon moment is $0.88 \mu_N$
  c.f. this result which is (Beck) $-0.54 \mu_N$: i.e. $-60\%$!!
- Also remarkable versus lattice QCD which gives
  $+0.03 \pm 0.01 \mu_N$ (Leinweber et al., PRL 94 (2005) 212001)
- Sign would require violation of universality of
  valence quark moments by $\gg 70\%$!
Magnetic Moments within QCD

\[ p = \frac{2}{3} u^p - \frac{1}{3} d^p + O_N \]
\[ n = -\frac{1}{3} u^p + \frac{2}{3} d^p + O_N \]
\[ \Sigma^+ = \frac{2}{3} u^\Sigma - \frac{1}{3} s^\Sigma + O_\Sigma \]
\[ \Xi^- = -\frac{1}{3} u^\Xi - \frac{2}{3} s^\Xi + O_\Xi \]

HENCE:

\[ O_N = \frac{1}{3} \left( 2p + n - \left( \frac{u^p}{u^p} \right) (\Sigma^+ + \Xi^-) \right) \]
\[ O_N = \frac{1}{3} \left( n + 2p - \left( \frac{u^p}{u^p} \right) (\Xi^- - \Xi^0) \right) \]

Just these ratios from Lattice QCD

OR
Constraint from Charge Symmetry

\[ O_N = \frac{2}{3} \ell G_M^u - \frac{1}{3} \ell G_M^d - \frac{1}{3} \ell G_M^s \]

\[ = \frac{1}{3} \left( \ell G_M^d - \ell G_M^s \right) , \]

\[ = \frac{\ell G_M^s}{3} \left( 1 - \frac{\ell R_d^s}{\ell R_d^s} \right) , \]

\[ G_M^s = \left( \frac{\ell R_d^s}{1 - \ell R_d^s} \right) \left[ 3.673 - \frac{u_p}{u_{\Sigma^+}} (3.618) \right] \]

\[ G_M^s = \left( \frac{\ell R_d^s}{1 - \ell R_d^s} \right) \left[ -1.033 - \frac{u_n}{u_{\Xi^0}} (-0.599) \right] \]

$u^p_{\text{valence}}$: QQCD Data Corrected for Full QCD Chiral Coeff’s

New lattice data from Zanotti et al.; Chiral analysis Leinweber et al.
$u^\Sigma_{\text{valence}}$
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