Partial Wave Analysis
results from JETSET

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representing the Jetset collaboration
with members from Bari, CERN, Erlangen, Freiburg, Genova, Illinois, Jülich, Oslo, Uppsala

• the Jetset experiment
• PWA formalism and MC tests
• results from analysis of full data set
The Jetset Experiment

Measures in-flight pbar annihilation: $\bar{p}p \rightarrow \phi \phi$

OZI-suppressed, may form glueball resonances in s-channel

Morningstar et.al., LAT991004
Total cross section $\bar{p}p \rightarrow \phi \phi$

Complete data set from Jetset

<table>
<thead>
<tr>
<th>point</th>
<th>$N(\phi\phi)$</th>
<th>$N(\text{b.g.})$</th>
<th>point</th>
<th>$N(\phi\phi)$</th>
<th>$N(\text{b.g.})$</th>
<th>point</th>
<th>$N(\phi\phi)$</th>
<th>$N(\text{b.g.})$</th>
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J values of the waves included in the partial wave analysis. All waves up to J=4, L=4 in the final state were allowed.

<table>
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<tr>
<th>wave</th>
<th>$J^P_C$</th>
<th>L initial</th>
<th>S initial</th>
<th>L final</th>
<th>S final</th>
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Getting started:

- Fit with all waves free
  - gives full freedom to the fit -> definition of “good fit”
  - errors on amplitudes are large, meaningless

- Reduce the set of allowed waves in search of a minimal set that gives a good description of the entire data set
  - gives priority to an economical description
  - adequacy judged in comparison with full fit
  - require same set of waves for all mass bins

We found 3 dominant waves
  all 2++

Method:

1. Group the data into mass bins with sufficient statistics

2. For each bin, try all waves one-by-one, keep best, repeat
   ➔ Sets agreed on 3 top waves.

3. Go back to beginning and put in waves two-by-two trying all pairs of waves together, then add one-by-one
   ➔ Sets chose same set of 3 waves as dominant.
Ambiguities

2 kinds:

1. **Essential ambiguities**
   - correspond to *invariances* in angular distributions from PWA expansion
   - continuous invariances: global phases (2)
   - discrete invariances: undetermined signs (4)
   - no others believed to exist for 2(V→2P)
   - irreducible even in limit of good acceptance and high statistics

2. **Statistical ambiguities**
   - correspond to different angular distributions which cannot be discriminated given the available data
   - discrete (different local maxima in likelihood)
   - discovered by systematic numerical search
   - reducible by good acceptance and high statistics
   - relatively few in this data set
Monte Carlo test

Ingredients:
- 1 resonant wave, two non-resonant
- experimental acceptance through simulation
- same reconstruction, analysis as for real data
Results of Monte Carlo test

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Monte Carlo test #2

- include incoherent background
- uniform angular distribution for background
- not orthogonal to waves -- check for leakage

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Results of Monte Carlo test #2

wave 2S

wave 2(D2)

wave 2(D0)

2S - 2(D2)

2(D0) - 2(D2)

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PWA Results

- 3-wave fit **identical** to Monte Carlo test #2
- simultaneous fit in mass and angular distributions
- $\phi\phi$ cross section now corrected for acceptance
  based on **measured** angular distribution

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3-wave fit

wave 2S

wave 2(D2)

wave 2(D0)

2S – 2(D2)

2(D0) – 2(D2)

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Possible Interpretation

- narrow peak seen in raw cross section
- PWA reveals 3 dominant waves in $2^{++}$
- rapid phase motion seen in two waves as expected for a Breit-Wigner resonance
Quality of the fit

- To check goodness of fit, use **likelihood ratio test**

  Define \( \chi^2 = -2 \ln \left( \frac{L}{L_0} \right) \)

  where \( L_0 \) is the likelihood maximum over the full parameter space and \( L \) is the likelihood maximum over some restricted part.

- For large \( N \), behaves like chi-square with \( n-n_0 \) d.o.f.

\[ \chi^2 = -2 \ln \left( \frac{L}{L_0} \right) \]
6-wave fit

wave 2S

wave 2(D2)

wave 2(D0)

2S − 2(D0)

2(D2) − 2(D0)

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6-wave fit

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Some strength in $2^+$ has moved to $3^+$

No obvious narrow structure is visible in $2^+$

Phase motion seen does not correspond to a simple Breit-Wigner resonance

Statistical errors do not justify a serious attempt to perform a multiple-pole fit
Conclusions

- PWA has been performed of the reaction
  \[ p \bar{p} \rightarrow \phi \phi \]

- 3 dominant waves were found, all \( 2^{++} \).

- Rapid phase motion seen in two waves consistent with a narrow \( 2^{++} \) resonance.

**BUT**

- The fit shows significant improvement if more waves are added, up to 6.

- Statistical errors do not permit a clear interpretation of 6-wave solution, but it does not favour a single narrow resonance.

**AND**

- Possible interference between the \( \phi \phi \) and an underlying \( f_0, f_0 \) background should be taken into account.
5-wave fit

wave 2S

wave 2(D2)

wave 2(D0)

2S − 2(D2)

2(D0) − 2(D2)

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5-wave fit

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