Partial Wave Analysis

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• What is it
• Relation to physical properties
• Properties of the S-matrix
• Limitation and perspectives
• Examples: peripheral production of 2- and 3-particle final states
\[
\frac{d^n \sigma}{dx_1 \cdots dx_n} = |Ampl|^2 = \left| \sum_{\alpha} a_{\alpha}(x_i) Y_{\alpha}(x_i) \right|^2
\]

- Kinematical variables \( (s_{cd}, \theta_{cd}, \phi_{cd}, \cdots) \)
- Production amplitudes \( a_{\alpha}(x_i) \) depend on fit parameters (output)
- Production amplitudes \( Y_{\alpha}(x_i) \) (input)

\[
\frac{d^n \sigma}{dx_1 \cdots dx_n} = \frac{\Delta N_{ev}}{\Delta x_1 \cdots \Delta x_n}
\]

- \( \Delta x_i \) (mass, t, \cdots bins)
… depending on how much we know about the amplitude:

- $Ampl = Y(x_i)$  Know everything!
  
  This is best case scenario
  
  $Y(x_i)$ includes it all: kinematics and dynamics,
  There is nothing to fit!

- $Ampl = a(x_i)$  Know nothing
  
  Worst case scenario

  Usually somewhere in between

- $Ampl = \sum_\alpha a_\alpha(x_i)Y_\alpha(x_i)$
Example: $\pi^- p \rightarrow \eta \pi^0 n$

\[ \frac{d\sigma}{dtdM_{\eta\pi}d\Omega} = |\sum_l m a_{lm}(t, M_{\eta\pi}) Y_{lm}(\Omega)|^2 \]

\[ \frac{d\sigma}{dtdM_{\eta\pi}d\Omega} = \left| \sum_l m a_{lm}(t) \frac{Y_{lm}(\Omega)}{D_l(M_{\eta\pi})} \right|^2 \]
… the less you know the more ambiguous the answer …

0 physics input
“maximal’ ambiguity

\[
\frac{d^n \sigma}{dx_1 \cdots dx_n} = |a(x_i)|^2
\]
\[
= |a(x_i)e^{if(x_i)}|^2
\]

some physics input
“moderate” ambiguities

\[
\frac{d^n \sigma}{dx_1 \cdots dx_n} = |\sum_\alpha a_\alpha x^\alpha|^2
\]
\[
= |\Pi_\alpha (x - x_\alpha(a))|^2
\]

know everything
no ambiguities

You do it in all possible way
to study systematics
\[ \pi^- p \rightarrow \pi^0 \pi^0 n \]

\[ \pi^0 \pi^0 \text{ spectrum} \]

\[ \frac{d^n\sigma}{dx_1 \cdots x_n} = |\sum_\alpha a_\alpha x^\alpha|^2 = |\prod_\alpha (x - x_\alpha(a))|^2 \]
Assume $a_0$ and $a_2$ resonances

(\textit{i.e. a dynamical assumption})
.. so, how much we know about the S-matrix …

- kinematical constraints
- dynamics is much harder …

- QCD (ca. 1970)
- Jlab upgrade (ca. Now !)
HM: reconstruct $S$ given Mandelstam representation and the unitarity condition

- $S$ – matrix has specific analytic properties (causality)

\[
O(t) = \int dt_0 g(t - t_0) I(t_0)
\]

\[
g(\tau) = 0 \text{ for } \tau < 0 \quad G(E) = \int_0^\infty d\tau g(\tau) e^{iE\tau}
\]

- Unitarity (conservation of energy)

\[
2i\text{Im} T = T^\dagger \rho T
\]
Mandelstam hypothesis:

1. There is an analytical representation for $S$ (which includes poles and cuts of physical origin) (so far unknown)

2. Given a representation a unique solution can be found (not true)
Complete representation: complete dynamics

Non-relativistic example: need the potential

\[ f(s, t) = f_B(t) + \text{integrals} \]

Relativistic example:
... a “given” representation can have multiple solutions! (incomplete knowledge of dynamics)

Non-relativistic example: Blaschke product

\[ S(k) = e^{-2ik\alpha} \prod_n B_n(k, k_n) \]

Relativistic example: (Castillejo, Daliz, Dyson (CDD) poles)

\[ f_i = \begin{cases} 1 - & \text{Have the same representation} \\ \end{cases} \]
Good news:

- **Low energy:**
  - Effective range expansion (low energy)
  - Two body unitarity
  - Small number of (renormalized) parameters
  - QCD input

- **High energy:**
  - Regge behavior
  - Asymptotic freedom
Illustration: $\pi\pi$ (S=I=0)

$\pi\pi$ only
(no KK, no resonances)

2 Resonances @ ~1.3, 1.5 GeV

$\pi\pi+KK$
Regge poles

\[ f(E, \cos \theta) \rightarrow \frac{1}{E-E_0+i\Gamma} \rightarrow e^{-\Gamma t} \]

\[ \sum_{l} \frac{1}{l^\alpha(E)} P_l(\cos(\theta)) \rightarrow e^{-\theta \text{Im} \alpha} \]

\[ f(s, t) \sim s^\alpha(t) \]
... combine low (chiral) and high energy information
\[ \pi^- (18\text{GeV}) p \rightarrow X p \rightarrow \eta \pi^- p \rightarrow \eta' \pi^- p \], \quad \sim 30\,000\, \text{events}

\[ N_{\text{events}} = N(s, t, M_{\eta\pi}, \Omega) \]
… PWA determined by maximizing likelihood function over an even sample …

\[ \pi^- p \ \eta \pi^0 p \]

DATA (from E852)

```
[adam@mantrid00 data]$ ls -l
total 835636
-rw-r--r-- 1 adam adam 87351564 May 10 10:40 ACC
-rw-rw-r-- 1 adam adam 4882894 May 10 10:39 DAT
-rw-r--r-- 1 adam adam 762596488 May 10 10:42 RAW
[adam@mantrid00 data]$ 
```

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<tbody>
<tr>
<td><strong>E</strong></td>
<td><strong>px</strong></td>
<td><strong>py</strong></td>
<td><strong>pz</strong></td>
<td>( (E^2 - p^2 ) )(^{1/2} )</td>
<td></td>
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<tr>
<td>Event 1</td>
<td>5.673920</td>
<td>-0.269088</td>
<td>-0.463492</td>
<td>5.646950</td>
<td>0.1345 (m_\pi)</td>
</tr>
<tr>
<td></td>
<td>12.561666</td>
<td>0.458374</td>
<td>0.228348</td>
<td>12.539296</td>
<td>0.5471 (m_\eta)</td>
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<tr>
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<td>1.001964</td>
<td>-0.260447</td>
<td>0.202660</td>
<td>0.112333</td>
<td>0.9393 (m_N) (recoil)</td>
</tr>
<tr>
<td></td>
<td>18.299278</td>
<td>-0.071161</td>
<td>-0.032484</td>
<td>18.298578</td>
<td>0.1396 (m_\pi) (beam)</td>
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<tr>
<td>Event 2</td>
<td>7.348978</td>
<td>-0.405326</td>
<td>0.602844</td>
<td>7.311741</td>
<td>0.9393 (m_N) (recoil)</td>
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<td>11.217298</td>
<td>-0.360824</td>
<td>-0.456940</td>
<td>11.188789</td>
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<td>1.255382</td>
<td>0.718019</td>
<td>-0.177694</td>
<td>0.382252</td>
<td>0.1396 (m_\pi) (beam)</td>
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<tr>
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<td>18.883385</td>
<td>-0.048131</td>
<td>-0.031789</td>
<td>18.882782</td>
<td>0.1396 (m_\pi) (beam)</td>
</tr>
</tbody>
</table>
Results of $\pi^- p ! \eta\pi^0 n$ analysis (part 1)

Assume BW resonance in all, $M=\xi 1,0$, P-waves

$\pi_1(900 - 5\text{GeV})$ emerges

Intensity in the weak P-waves is strongly affected by the $a_2(1320)$, strong wave due to acceptance corrections
E852 $\eta'\pi^-$ analysis

1 BW resonance in $P_+$

2 BW resonances in $D_+$

$a_2(1320)$

$a_2(1800) = ?$
Results of coupled channel analysis of \( \pi^- p ! \eta \pi^- p \) and \( \pi^- p ! \eta' \pi^- p \)
\[ \phi(P^+) - \phi(D^+) \]
Peripheral production of hybrid mesons

Quarks

- $S = 0$
- $L = 0$
- $J^{PC} = 0^{-+}$
  - like $\pi$, $K$
- $S = 1$
- $L = 0$
- $J^{PC} = 1^{--}$
  - like $\gamma$, $\rho$

Excited Flux Tube

$J^{PC} = \begin{cases} 1^{+-} \\ 1^{--} \end{cases}$

Hybrid Meson

- $J^{PC} = \begin{cases} 1^{--} \\ 1^{++} \end{cases}$

- Exotic
  - $J^{PC} = \begin{cases} 0^{-+} \\ 1^{--} \\ 2^{++} \end{cases}$

So only parallel quark spins lead to exotic $J^{PC}$
Photo production enhances exotic mesons

Non-exotic mesons, $\pi, K, \eta$

$\gamma$ : quark with spins aligned

$\pi$ : quark with spins anti-aligned

QCD ! Exotic :

$\gamma \rightarrow \rho(J^{PC}=1^-) \rightarrow \pi_1(J^{PC}=1^+)$

VMD "pluck" the string
Implications for exotic meson searches

- Possible narrow QCD exotic (M=1.6 GeV) (E852 $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$)

exotic/non-exotic $\gg 0.1$

exotic/non-exotic $\gg 1$
Photo production enhances exotic mesons

$\gamma \rightarrow \rho(J^{PC}=1--) \rightarrow \pi_1(J^{PC}=1++)$

VMD "pluck" the string ($S=1, L_{QQ}=0 \rightarrow L_g=1$)

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>$\rho \pi$ decay mode</th>
<th>Mass (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>$\Gamma_{3\pi}/\Gamma$</th>
<th>$\sigma_Y (\mu b)$</th>
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<tr>
<td>$\rho_1$</td>
<td>$1^{++}$ S</td>
<td>1260</td>
<td>400</td>
<td>99% 1%</td>
<td>0.03</td>
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<td>$\rho_2$</td>
<td>$2^{++}$ D</td>
<td>1320</td>
<td>110</td>
<td>70% 1%</td>
<td>0.50</td>
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<td>$\rho_3$</td>
<td>$2^{--}$ P</td>
<td>1670</td>
<td>260</td>
<td>30% 1%</td>
<td>0.02</td>
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<tr>
<td>$\rho_4$</td>
<td>$1^{--}$ P</td>
<td>1600</td>
<td>160</td>
<td>50%</td>
<td></td>
</tr>
</tbody>
</table>
CORRECTIONS TO THE ISOBAR MODEL FOR THREE HADRON FINAL

by

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1. WHY THE ISOBAR MODEL NEEDS TO BE CORRECTED (1):

UNITARITY IN FINAL STATE TWO-PARTICLE
(SUB-ENERGY) CHANNELS,
Problems with the isobar (sequential decay) model

\[ A(s_1, s_2, M) = C^J_{LS}(M) / D_1(s_1) \]

1 \leftrightarrow 2 \quad F(s_1, s_2, M) = f_1(M)/D_1(s_1) + f_2(M)/D_2(s_2)

Independent on 2-particle sub-channel energy: violates unitarity!
\[ f_2(s_{2+}) - f_2(s_{2-}) = 2i \rho(s_2) \text{ P.V.} s \frac{f_1(s_1)}{D_1(s_1)} \]

B-W: unitary in sub-channel energy

\[ f_i = f_i(s_i, M) \]

Has to depend on 2-particle energy
\[ F_1(s_1, M)_i = H(s_1, M)_{ij} C_j(M) \quad i, j = \sigma, \rho \]
$3\pi$ sample

Analyzed

Available!
Summary

• Need theory input to minimize (mathematical) ambiguities and to understand systematic errors

• Need more theory input to determine physical states (coherent background vs resonances)

• There is lots of data to work with and there will be more especially needed for establishing gluonic excitations

• $\pi_1(1400)$ and $\pi_1(1600)$ in $\eta'\pi$ are most likely due to Residual interactions much like the $\sigma$ meson in the $\pi\pi$ S-wave