Light Meson Radiative Decays

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- Radiative transitions have proved their value in the baryon sector, successfully reproducing the magnitudes and relative phases of over 100 helicity amplitudes for photoexcitation of the proton and neutron.

- Calculations of light meson radiative decays have concentrated mainly on ground-state to ground-state decays.

- New and proposed facilities (ISR at BABAR & BELLE, CLEO-C, JLab, Novosibirsk) promise greatly increased statistics and open up the possibility of studying radiative decays of excited light mesons.

Several key radiative widths are found to be large, \( \gtrsim 500 \text{ keV} \), and offer strong discriminatory power. We shall concentrate on three aspects:

- discrimination between the radial \( (2^3S_1) \) and orbital \( (1^3D_1) \) excitations of the \( \rho \)

- discrimination between these and the \( J^{PC} = 1^{--} \) hybrid

- discrimination among different \( q\bar{q} \) and glueball mixing scenarios in the scalars
The Model

Wave functions are taken as Gaussian, that is of the form \(\exp(-p^2/\beta^2)\) multiplied by the appropriate polynomial, and the parameter \(\beta\) found for each of the \(1S, 1P, 2S, 1D\) states by treating it as the variational parameter in the Hamiltonian

\[
H = \frac{p^2}{m_q} + \sigma r - \frac{4 \alpha_s}{3 r} + C
\]

with standard quark-model parameters:

\(m_{u,d} = 0.33\ \text{GeV}, \ m_s = 0.45\ \text{GeV}, \ \sigma = 0.18\ \text{GeV}^2, \ \alpha_s = 0.5\)

The decay at rest of the meson \(A\) to the meson \(B\) and a photon with three-momentum \(p\) has the form

\[
M_{A \to B} = M_{A \to B}^q + M_{A \to B}^{\bar{q}}.
\]

with

\[
M_{A \to B}^q = \frac{I_q}{2m_q} \int d^3k \left[ \text{Tr} \left\{ \phi_B^\dagger(k - \frac{1}{2}p)\phi_A(k) \right\} (2k - p) - i\text{Tr} \left\{ \phi_B^\dagger(k - \frac{1}{2}p)\sigma\phi_A(k) \right\} \times p \right]
\]

and

\[
M_{A \to B}^{\bar{q}} = \frac{I_{\bar{q}}}{2m_q} \int d^3k \left[ \text{Tr} \left\{ \phi_A(k)\phi_B^\dagger(k + \frac{1}{2}p) \right\} (2k + p) - i\text{Tr} \left\{ \phi_A(k)\sigma\phi_B^\dagger(k + \frac{1}{2}p) \right\} \times p \right]
\]

where \(I_q\) and \(I_{\bar{q}}\) are isospin factors and \(m_q\) is the quark mass.
The differential decay rate is then given by
\[
\frac{d\Gamma}{d \cos \theta} = 4p \frac{E_B}{m_A} \alpha I \sum |M_{A \rightarrow B}|^2
\]
where the sum is over final-state polarisations and \( I = I_q^2 = I_{\bar{q}}^2 \) is the isospin factor.

The pure electric-dipole (E1) transition is well-defined for heavy quarks, but is certainly a bad approximation for light quarks so we include the magnetic quadrupole (M2) transition as well.

This approach has a long history of success in the baryon sector even though the M2 terms are the same order in \( p^2 \) as E1 corrections such as:
- anomalous magnetic moments of the constituents
- spin-orbit terms
- Thomas precession
- binding effects

The success in the baryon sector suggests that the collective effect of these corrections is small.
Within this “leading multipole” hypothesis there are checks on our procedures.

• Ground-state to ground-state transitions, e.g. $\rho \to \eta\gamma$, $\omega \to \eta\gamma$, given correctly by the model.

• The predicted width for $\Gamma(f_1(1285) \to \gamma\rho)$ is in good accord with experiment.

• The prediction that $\Gamma(f_2(1270) \to \gamma\rho) \lesssim 0.5\Gamma(f_1(1285) \to \gamma\rho)$ is in qualitative accord with experiment as there is no evidence for the radiative decay of $f_2(1270)$ in either the MARK III or WA102 experiments and both have strong $f_2$ signals.

• From general considerations we can form a positivity constraint among a combination of widths which is satisfied by our explicit model and enables us to draw a more general conclusion, namely that $\Gamma(f_0 \to \gamma\rho) \sim \Gamma(f_1 \to \gamma\rho)$ in agreement with our calculation.
A Hidden Hybrid?

- The data on $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$, excluding $\omega\pi$, are completely consistent with $e^+e^- \rightarrow a_1\pi$ up to 1.65 GeV and the cross section is large (CLEO-$\tau$, Novosibirsk).

- The data on $e^+e^- \rightarrow \pi^+\pi^-$ require a $\rho'$ state ($2\,^3S_1$) at about 1.45 GeV and there is additional structure at higher mass (CLEO-$\tau$, Novosibirsk, $e^+e^-$).

- The data on $e^+e^- \rightarrow \omega\pi$ are consistent with the tail of the $\rho$ plus the ($2\,^3S_1$) (CLEO-$\tau$, CLEO-B, Novosibirsk, $e^+e^-$).

- There is no strong $\omega\pi$ signal above the $\rho'(1450)$ (CLEO-B, $e^+e^-$).

- The $^3P_0$ model cannot explain the large $4\pi$ cross section, excluding $\omega\pi$:

$$\Gamma(\rho_{2S} \rightarrow a_1\pi \rightarrow 4\pi) \sim 3 \text{ MeV}$$
$$\Gamma(\rho_{2S} \rightarrow h_1\pi \rightarrow 4\pi) \sim 1 \text{ MeV}$$
$$\Gamma(\rho_{2S} \rightarrow \omega\pi) \sim 115 \text{ MeV}$$
$$\Gamma(\rho_{2S} \rightarrow \omega\pi) \sim 68 \text{ MeV}$$

- The cross section for 'direct' production of $a_1\pi$ via $\rho$ dominance is also very small.

- One solution is to invoke a vector hybrid as its dominant decay mode is $a_1\pi$. 
But

• The hybrid has no direct $e^+e^-$ coupling. It must be induced by mixing. To get a large $4\pi$ cross section the mixing must be maximal with the $\rho'(1450)$. This then implies a large cross section for $e^+e^- \rightarrow \omega\pi$ above the $\rho'(1450)$ which is not observed!

However

• Pham, Roiesnel & Truong (1978) and Penso & Truong (1980) argued that in the special case of $a_1\pi$ it is incorrect to use naive $\rho$ dominance and that the axial current matrix element (which is dominated by the $a_1$) should be used. This gives a large (non-resonant) cross section for $e^+e^- \rightarrow a_1\pi$.

• The $1^3D_1$ state (the $\rho'(1700)$) has a large $4\pi$ decay width:
  $\Gamma(\rho_{1D} \rightarrow a_1\pi \rightarrow 4\pi) \sim 104$ MeV
  $\Gamma(\rho_{1D} \rightarrow h_1\pi \rightarrow 4\pi) \sim 105$ MeV

• The (rather strong) direct $a_1\pi$ can interfere with the (comparatively weak) $1^3D_1$ state boosting it significantly.

So goodbye hybrid?

• Not necessarily. There will be some mixing between the hybrid and the $1^3D_1$ and the direct $a_1\pi$ can boost the mixed states.
Vector Meson Radiative Decays

- Radiative decays can distinguish between the $2^3S_1$ and the $1^3D_1$. For example:
  - $\Gamma(\rho(1450) \rightarrow f_2(1270)\gamma) \sim 700$ keV
  - $\Gamma(\rho(1700) \rightarrow f_2(1270)\gamma) \sim 140$ keV
  - $\Gamma(\rho(1450) \rightarrow f_1(1285)\gamma) \sim 350$ keV
  - $\Gamma(\rho(1700) \rightarrow f_1(1285)\gamma) \sim 1100$ keV

- Radiative decays can separate cleanly the $\phi(1690)$. For example the $f_2(1525)\gamma$ decay of the $\phi(1690)$ provides a unique signature for the $s\bar{s}$ state, albeit with a smallish width:
  - $\Gamma(\phi(1690) \rightarrow f_2(1525)\gamma) \sim 200$ keV

- Radiative decays can resolve the issue of the $J^{PC} = 1^{--}$ hybrid, $\rho_H$. As the $q\bar{q}$ pair in the hybrid is in a spin-singlet state, radiative decays to the spin-triplet $f_2(1270)$ and $f_1(1285)$ will be suppressed. The dominant radiative decay should be to the spin-singlet $b_1(1235)$, which is suppressed for the spin-triplet $\rho(1450)$ and $\rho(1700)$. Specific calculation (Close and Dudek) gives
  - $\Gamma(\rho_H(1700) \rightarrow b_1(1235)\gamma) \sim 700$ keV
• Vector meson radiative decays can also tell us something about the scalar glueball.

• There are three scalar states when, if we only have $q \bar{q}$ states, there should be two.

• If there is no mixing among the scalars so that the $f_0(1370)$ is pure $n\bar{n}$ and the $f_0(1710)$ is pure $s\bar{s}$ then

\[
\Gamma(\rho(1700) \rightarrow f_0(1370)\gamma) \sim 900 \text{ keV}
\]

\[
\Gamma(\rho(1450) \rightarrow f_0(1370)\gamma) \sim 65 \text{ keV}
\]

\[
\Gamma(\phi(1900) \rightarrow f_0(1710)\gamma) \sim 190 \text{ keV}
\]

• The result of mixing is that the bare $n\bar{n}$ and $s\bar{s}$ states contribute in varying degrees to each of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$
• Three different mixing scenarios have been proposed:
  the bare glueball is lighter than the bare $n\bar{n}$ state
  the bare glueball lies between the bare $n\bar{n}$ state and the
  bare $s\bar{s}$ state
  the bare glueball is heavier than the bare $s\bar{s}$ state

• Each one affects the radiative decays in a unique way:

<table>
<thead>
<tr>
<th></th>
<th>$\rho(1700)$</th>
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<th>$\phi(1900)$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>M</td>
<td>H</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>174</td>
<td>440</td>
<td>603</td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>520</td>
<td>301</td>
<td>98</td>
</tr>
<tr>
<td>$f_0(1710)$</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

• So the relative rates of the radiative decays of the $\rho(1700)$
  to $f_0(1370)$ and $f_0(1500)$, and of the $\phi(1900)$ to $f_0(1500)$ and
  $f_0(1710)$ change radically according to the particular model for
  $q\bar{q}$-glueball mixing.

• An important check on this phenomenology is provided by
  the decay $\omega(1650) \to a_0(1450)\gamma$, predicted width $\sim 610$ keV.
Scalar Meson Radiative Decays

- A complementary approach to flavour-filtering among the scalars is provided by the radiative decays of the scalars to the ground-state vectors $\rho$, $\omega$ and $\phi$.

<table>
<thead>
<tr>
<th></th>
<th>$\rho(770)$</th>
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<th>$\phi(1020)$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>M</td>
<td>H</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>443</td>
<td>1121</td>
<td>1540</td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>2519</td>
<td>1458</td>
<td>476</td>
</tr>
<tr>
<td>$f_0(1710)$</td>
<td>42</td>
<td>94</td>
<td>705</td>
</tr>
</tbody>
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- The width of the decay $f_1(1285) \rightarrow \rho \gamma$ is measured and provides a good check on the model: $1320 \pm 312$ keV compared to a predicted value of $\sim 1400$ keV.

- The predicted width for the decay $f_2(1270) \rightarrow \rho \gamma$ is $\sim 640$ keV. Experimentally this width is small as neither MARK III nor WA102 see it although both have a large $f_2(1270)$ signal.

- The branching fractions for radiative decay of $J/\psi$ to $f_1(1285)$ and $f_2(1270)$ are comparable at $(6.1 \pm 0.9) \times 10^{-4}$ and $(1.30 \pm 0.14) \times 10^{-3}$, so the non-observation of any $f_2(1270)$ signal in the decay $J/\psi \rightarrow \gamma(\gamma \rho)$ is meaningful.

- A similar situation holds in central production in high-energy proton-proton interactions and one can deduce an upper limit on $\Gamma(f_2(1270) \rightarrow \rho \gamma)$ of 500 keV at 95% confidence level.

- A further check on the phenomenology would be provided by the decay $a_0(1450) \rightarrow \omega \gamma$, predicted width $\sim 2100$ keV.
Single quark transitions

• Independent of details of binding dynamics it is possible to obtain relations among helicity amplitudes, and hence widths, that depend only on the assumption that the mesons are $q\bar{q}$ $P$ and $S$ states. The width for the decays $V \to \gamma f_J$ is

$$\Gamma(V \to \gamma f_J) = 2p\alpha \frac{E_B}{m_A} I \frac{1}{3} \sum |A_\lambda|^2$$

and for the decays $f_J \to \gamma V$ is

$$\Gamma(f_J \to \gamma V) = 2p\alpha \frac{E_B}{m_A} I \frac{1}{2J + 1} \sum |A_\lambda|^2$$

where $A_\lambda$ are the helicity amplitudes.

• In terms of electric dipole and magnetic quadrupole transitions

$$A_0 = \sqrt{2}(E_1 + 2E_R)$$

for $^3S_1 \to ^3P_0$ transitions,

$$A_0 = \sqrt{3}(E_1 + E_R + M) \quad A_1 = \sqrt{3}(E_1 + E_R - M)$$

for $^3S_1 \to ^3P_1$ transitions and

$$A_0 = (E_1 - E_R + 3M) \quad A_1 = \sqrt{3}(E_1 - E_R + M)$$

$$A_2 = \sqrt{6}(E_1 - E_R - M)$$

for $^3S_1 \to ^3P_1$ transitions.
• Consider the decays $V \rightarrow \gamma f_J$. For equal phase space and equal form factors the $|M|^2$ term and the cross terms between $E_1$ and $E_R$ can be eliminated and a combination formed proportional to $|E_0|^2 + |E_R|^2 \geq 0$. The resulting inequality is

$$\Gamma(\rho(2S) \rightarrow \gamma f_2) + 7\Gamma(\rho(2S') \rightarrow \gamma f_0) \geq 3\Gamma(\rho(2S) \rightarrow \gamma f_1)$$

Similarly for the decays of pure $n\bar{n}$ states

$$5\Gamma(f_2 \rightarrow \gamma \rho) + 7\Gamma(f_0 \rightarrow \gamma \rho) \geq 9\Gamma(f_1 \rightarrow \gamma \rho)$$

• As $\Gamma(f_1 \rightarrow \gamma \rho) \sim 1300$ keV, this latter equation requires that one or other of $f_0$, $f_2$ must have a radiative width $\sim 1000$ keV. As there is no evidence for the $f_2$ decay it follows that $f_0 \rightarrow \gamma \rho$ should be large, in line with our specific calculations.
Summary

- In general, radiative decays are a better probe of meson structure than hadronic decays as the coupling to the charges and spins of constituents gives detailed information on wave functions.

- Specifically, light-quark radiative decays provide a strong discriminatory mechanism and act as a good flavour filter.

  Discrimination: $\rho(1450), \rho(1700), \rho_H(1700)$.

  Flavour filter: glueball mixing in the scalars $f_0(1370), f_0(1500), f_0(1710)$.

- Specific and general checks of the model are in good agreement with experiment.

- New and proposed facilities promise greatly increased statistics and will allow these decays to be measured.

- Some of the radiative decays may be detectable in present experiments:

  E852 sees $\omega(1650) \rightarrow \omega\eta$

  - $^3P_0$ model predicts $\Gamma(\omega(1650) \rightarrow \omega\eta) \sim 13 \text{ MeV}$
  - $\Gamma(\omega(1650) \rightarrow a_1(1260)\gamma) \sim 1000 \text{ keV} \sim 8\%$ of $\omega\eta$ width

  VES sees $\rho(1450) \rightarrow \rho\eta$ and $\rho(1700) \rightarrow \rho\eta$

  - $^3P_0$ model $\Gamma(\rho(1450) \rightarrow \rho\eta) \sim \Gamma(\rho(1700) \rightarrow \rho\eta) \sim 25 \text{ MeV}$
  - $\Gamma(\rho(1450) \rightarrow f_2(1270)\gamma) \sim 700 \text{ keV}$
  - $\Gamma(\rho(1700) \rightarrow f_1(1285)\gamma) \sim 1100 \text{ keV}$