Low Energy Theory of the Neutron Star Crust and its Observable Implications.

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Collaborators:
Aguilera, Cirigliano, Chamel, Cumming, Page, Pethick, Pons & Sharma
Neutron Star Crust:

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New Phenomena in Neutron Stars
- a window into the thermal and mechanical properties of the crust.

- Crustal heating and subsequent thermal relaxation in accreting neutron stars.
- Possible excitation of shear modes in the solid crusts of magnetars during giant flares.
Transient Accretion

• Nuclear reactions heat the crust during accretion.

• Crust relaxes during quiescence.
Transient Accretion

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Transient Accretion

- Nuclear reactions heat the crust during accretion.
- Crust relaxes during quiescence.
More than one source!

Cackett et al. 2006

MXB 1659-29

6.6 yr

$\tau_{\text{Cool}} = 465 \pm 25$ days

Cackett et al. 2008

KS 1731-260

4.4 yr

$\tau_{\text{Cool}} = 305 \pm 50$ days
Connecting to Crust Microphysics

\[ \tau_{\text{Cool}} \simeq \frac{C_V}{\kappa} \left( \Delta R \right)^2 \]
Connecting to Crust Microphysics

Crustal Specific Heat

$$\tau_{\text{Cool}} \sim \frac{C_V}{\kappa} (\Delta R)^2$$
Connecting to Crust Microphysics

\[ \tau_{\text{Cool}} \approx \frac{C_V}{\kappa} (\Delta R)^2 \]

Crustal Specific Heat

Thermal Conductivity
Connecting to Crust Microphysics

\[ \tau_{\text{Cool}} \approx \frac{C_V}{\kappa} (\Delta R)^2 \]

Crustal Specific Heat

Crust Thickness

Thermal Conductivity
Explosions on Magnetars: Giant Flares

Anomalous X-Ray Pulsars (10)
Soft Gamma Repeaters (8)

Inferred to have surface fields of the order of $10^{15}$ Gauss.


SGRs exhibit powerful outburst $\sim 10^{46}$ ergs/s

SGR 0525-66 : (1979)
SGR 1627-41  (1998)

Hurley et al. (2005)
QPOs are likely to be shear modes in the solid crust


\[ \omega_{n=0} \approx \frac{2}{c_t R} \]
\[ \omega_{n=1} \approx \frac{\pi}{R} \frac{c_t}{R} \frac{\Delta R}{R} \]
\[ \omega_{n=0, l=2} \approx \frac{2}{c_t} \frac{R}{R} \]

Similar frequencies observed in 2 sources.

SGR 1806
2004 Giant Flare
QPOs are likely to be shear modes in the solid crust


\[ \omega_{n=0, l=2} \approx \frac{\pi c_t}{R} \frac{\Delta R}{R} \]

\[ \omega_{n=1} \sim \frac{\pi c_t}{R} \frac{\Delta R}{R} \]

Similar frequencies observed in 2 sources.

SGR 1806
2004 Giant Flare

Shear mode velocity
Microscopic Structure of the Crust

Baym Pethick & Sutherland (1971)  Negele & Vautherin (1973)
Microscopic Structure of the Crust

Baym Pethick & Sutherland (1971)  Negele & Vautherin (1973)
Separation of Scales

- Protons cluster (pairing + shell gaps)
- Proton clusters form a Coulomb lattice.
- Neutrons pair to form a superfluid.

\[ \omega_{\text{plasma}} = \sqrt{\frac{4\pi\alpha Z^2 n_I}{A m_n}} \]

\[ \omega_{\text{Debye}} \sim \frac{c}{a} \approx 0.45 \, \omega_{\text{plasma}} \]

\[ \Delta \propto E_{F_n} \exp\left(\frac{-1}{N(0) V_{nn}}\right) \]

Longitudinal and Transverse Lattice Phonons

Superfluid Phonons

Nuclei (protons)

Neutrons
Superfluid Critical Temperature

\[ T_c \simeq 0.57 \Delta \ (?) \]

Pairing gap is difficult to calculate in strong coupling. No small expansion parameter \( k_F a > 1 \).

Cold atom experiments at \( k_F a = \infty \) validate QMC.

Polarization measurements in imbalanced systems are exponentially sensitive to the gap.

At \( k_F a = \infty \): \( \delta = \frac{\Delta}{E_F} = 0.45 \pm 0.05 \)
Superfluid Critical Temperature

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Gezerlis & Carlson (2009)

Carlson & Reddy (2010)
Relevant Temperature Scales in the Crust

- Electrons $T_F$
- Ion $T_{\text{Melt}} (\Gamma=200)$
- Ion $T_{\text{Plasma}}$
- Ion $T_{\text{DeBroglie}}$
- Umklapp

Graph showing temperature scales versus density ($\rho$) in the crust.
Relevant Temperature Scales in the Crust

Temperature scales include:

- Electrons $T_F$
- Ion $T_{\text{Melt}} (\Gamma=200)$
- Ion $T_{\text{Plasma}}$
- Ion $T_{\text{DeBroglie}}$
- Ion $T_{\text{Umklapp}}$

The graph shows the variation of temperature $T$ with density $\rho$ in the crust, distinguishing between outer and inner crust regions.

Accreting and Magnetized Neutron Stars

Tuesday, July 6, 2010
Low Energy Excitations

\[ \omega_{lPh}(q) = c_l \, q \quad \omega_{sPh}(q) = \nu_s \, q \]
\[ \omega_{tPh}(q) = c_t \, q \quad \omega_{\text{electron}} = q \]

How are these low energy modes coupled?
Low Energy Effective Field Theory

Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.
Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.
Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

\[
\langle \psi^\uparrow(r)\psi^\downarrow(r) \rangle = |\Delta| \exp(-2i\theta)
\]

"coarse-grain"

Collective coordinates:
Vector Field: \(\xi_i(r, t)\)
Scalar Field: \(\phi(r, t)\)
Inner Crust EFT

**Protons:**

\[
\mathcal{L}_p = \frac{1}{2} n_p m \left( \partial_t \xi_i \partial_t \xi_i - \frac{1}{2} K \partial_i \xi_i \partial_i \xi_i - \frac{1}{4} \mu_s \xi_{ij} \xi_{ij} + \cdots \right)
\]

\[\xi_{ij} = \partial_i \xi_j + \partial_j \xi_i - \frac{2}{3} \delta_{ij} \partial_k \xi_k\]

**Compressibility**

\[K \propto \frac{\partial^2 E}{\partial n_p \partial n_p}\]

**Shear Modulus**

\[\mu_s \propto \frac{Z^2 e^2}{a^4}\]

**Neutrons:**

\[
\langle \psi_\uparrow(r) \psi_\downarrow(r) \rangle = |\Delta| \exp(-2i \theta)
\]

\[\theta = \mu_n t\]

**Ground-state**

\[\mathcal{L}_n = P(\mu_n)\]

Son & Wingate (2006)
Inner Crust EFT

**Protons:**

\[ \mathcal{L}_p = \frac{1}{2} n_p m \, \partial_t \xi_i \, \partial_t \xi_i - \frac{1}{2} K \, \partial_i \xi_i \partial_i \xi_i - \frac{1}{4} \mu_s \, \xi_{ij} \, \xi_{ij} + \cdots \]

\[ \xi_{ij} = \partial_i \xi_j + \partial_j \xi_i - \frac{2}{3} \delta_{ij} \, \partial_k \xi_k \]

**Compressibility**

\[ K \propto \frac{\partial^2 E}{\partial n_p \partial n_p} \]

**Shear Modulus**

\[ \mu_s \propto \frac{Z^2 \, e^2}{a^4} \]

**Neutrons:**

\[ \langle \psi_\uparrow(r) \psi_\downarrow(r) \rangle = |\Delta| \exp(-2i \, \theta) \]

\[ \theta = \mu_n \, t - \phi \]

**Ground-state**

**Fluctuations (Superfluid Phonons)**

\[ \mathcal{L}_n = P(\mu_n) \]

Son & Wingate (2006)
Coupling Neutrons and Protons.
(or the superfluid and the lattice)

$$\mathcal{L}_n = P(\mu_n) + \frac{\partial P}{\partial \mu_n} \, \delta \mu_n + \frac{1}{2} \frac{\partial^2 P}{\partial \mu_n \partial \mu_n} \, \delta \mu_n^2 + \cdots$$

Gibbs-Duhem Relation:

\[
\delta \mu_n = E_{nn} \, \delta n_n + E_{np} \delta n_p
\]
\[
\delta \mu_n = -\partial_t \phi - E_{np} n_p \partial_i \xi_i
\]

Velocities and current-current coupling:

\[
\vec{v}_n = \frac{\partial_i \phi}{m}
\]
\[
\vec{v}_p = m \, \partial_t \xi_i
\]

\[
\delta \mu_n = -\frac{(\partial_i \phi)^2}{2m} + \frac{1}{2} \gamma \, m \, (\vec{v}_n - \vec{v}_p)^2
\]
Coupling Neutrons and Protons.
(or the superfluid and the lattice)

\[ \mathcal{L}_n = P(\mu_n) + n_n \delta \mu_n + \frac{1}{2} \chi n \delta \mu_n^2 + \cdots \]

Gibbs-Duhem Relation:

\[ \delta \mu_n = E_{nn} \delta n_n + E_{np} \delta n_p \]

\[ \delta \mu_n = -\partial_t \phi - E_{np} n_p \partial_i \xi_i \]

\[ \uparrow \]

density-density interaction

Velocities and current-current coupling:

\[ \ddot{v}_n = \frac{\partial_i \phi}{m} \]

\[ \ddot{v}_p = m \partial_t \xi_i \]

\[ \delta \mu_n = -\frac{(\partial_i \phi)^2}{2m} + \frac{1}{2} \gamma m (\ddot{v}_n - \ddot{v}_p)^2 \]

\[ \uparrow \]

current-current interaction
The Coupled System

\[ \mathcal{L}_{n+p} = \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} v_s^2 (\partial_i \phi)^2 + \frac{1}{2} (\partial_t \xi_i)^2 - \frac{1}{2} (c_l^2 - g^2) (\partial_i \xi_i)^2 \]

Velocities:

\[ v_s^2 = \frac{n_f}{m \chi \eta} \quad c_l^2 = \frac{K + 4 \mu_s / 3}{m (n_p + n_b)} \]

Entrainment: protons drag neutrons.

\[ \begin{cases} \text{Bound neutrons:} & n_b = \gamma n_n \\ \text{Free neutrons:} & n_f = n_n (1 - \gamma) \end{cases} \]
The Coupled System

\( \mathcal{L}_{n+p} = \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} v_s^2 (\partial_i \phi)^2 + \frac{1}{2} (\partial_t \xi_i)^2 - \frac{1}{2} (c_l^2 - g^2) (\partial_i \xi_i)^2 \)

\[ + g \, \partial_t \phi \, \partial_i \xi_i + \tilde{\gamma} \, \partial_i \phi \, \partial_t \xi_i \]

Velocities:
\( v_s^2 = \frac{n_f}{m \chi_n} \quad c_l^2 = \frac{K + 4 \mu_s / 3}{m (n_p + n_b)} \)

Entrainment: protons drag neutrons.

\{ Bound neutrons: \( n_b = \gamma \, n_n \)

Free neutrons: \( n_f = n_n \, (1 - \gamma) \)

Longitudinal lattice phonons and superfluid phonons are coupled:

\[ g = n_p \, E_{np} \sqrt{\frac{\chi_n}{m(n_p + n_b)}} \quad \tilde{\gamma} = \frac{-n_b \, v_s}{\sqrt{(n_p + n_b)n_f}} \]
The Coupled System

\[ \mathcal{L}_{n+p} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}v_s^2 (\partial_i \phi)^2 + \frac{1}{2}(\partial_t \xi_i)^2 - \frac{1}{2}(c_l^2 - g^2) (\partial_i \xi_i)^2 \]

\[ + g \partial_t \phi \partial_i \xi_i + \tilde{\gamma} \partial_i \phi \partial_t \xi_i \]

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Transverse lattice phonons:

\[ \mathcal{L}_t = \frac{1}{2}(\partial_t \xi_i)^2 - \frac{1}{2}c_t^2 (\partial_i \xi_j + \partial_j \xi_i)^2 \quad \Rightarrow \quad c_t^2 = \frac{\mu_s}{m(n_p + n_b)} \]
List of Low Energy Constants

<table>
<thead>
<tr>
<th>$C_l$</th>
<th>$C_t$</th>
<th>$\nu_s$</th>
<th>$g$</th>
<th>$\tilde{\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$\mu_s$</td>
<td>$E_{nn}$</td>
<td>$E_{np}$</td>
<td>$n_b$</td>
</tr>
</tbody>
</table>

Thermodynamic Derivatives:

- $E_{nn} = \frac{\partial^2 E}{\partial n_n \partial n_n}$
- $E_{pp} = \frac{\partial^2 E}{\partial n_p \partial n_p}$
- $E_{np} = \frac{\partial^2 E}{\partial n_n \partial n_p}$

Coupling between superfluid and lattice:

- $g \approx 10^{-3} - 10^{-2}$
- $\tilde{\gamma} \approx 10^{-3} - 10^{-2}$

Tuesday, July 6, 2010
Two Applications

- Superfluid heat conduction.
- Effects of mixing and entrainment on the sound speeds.
Mixing and Dissipation

Mixing of sound modes

Longitudinal Lattice Phonon
Superfluid Phonon
Transverse Lattice Phonons

ρ (g/cm³)

Speed (in units of c=3x10⁻¹⁰ cm/s)

10¹¹ 10¹² 10¹³ 10¹⁴

0.01 0.1
Mixing and Dissipation

Mixing of sound modes

Dissipation of IPH leads to dissipation of sPH

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Mixing and Dissipation

Mixing of sound modes

Dissipation of $\text{lPh}$ leads to dissipation of $\text{sPh}$
Mixing and Dissipation

Mixing of sound modes

Dissipation of lPh leads to dissipation of sPh

\[
\lambda_{\text{abs}}(\omega) = \frac{\nu_s^2}{g_{\text{mix}}^2} \frac{1 + (1 - \alpha^2)^2 (\omega \tau_{\text{lPh}})^2}{\alpha (\omega \tau_{\text{lPh}})^2} \lambda_{\text{lPh}}(\omega)
\]

sPh mean free path

lPh mean free path
Mixing and Dissipation

Mixing of sound modes

Dissipation of lPh leads to dissipation of sPh

\[
\lambda_{\text{abs}}(\omega) = \frac{v_s^2}{g_{\text{mix}}^2} \frac{1 + (1 - \alpha^2)^2 \left( \frac{\omega}{\tau_{\text{lPh}}} \right)^2}{\alpha \left( \frac{\omega}{\tau_{\text{lPh}}} \right)^2} \quad \lambda_{\text{lPh}}(\omega)
\]

\[
\tilde{g}_{\text{mix}} = g + \tilde{\gamma} \quad \alpha = \frac{c_l}{v_s}
\]

\[
\lambda_{\text{lPh}} = c_l \tau_{\text{lPh}}
\]

sPh mean free path

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Mixing and Dissipation

Mixing of sound modes

Dissipation of lPh leads to dissipation of sPh

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\[ \alpha = \frac{c_l}{v_s} \]

\[ \lambda_{\text{lPh}}(\omega) \]

\[ \tilde{g}_{\text{mix}} = g + \tilde{\gamma} \]

Away from resonance

\[ \lambda_{\text{sPh}} \simeq 10^5 \lambda_{\text{lPh}} \]
Thermal Conductivity

\[ \kappa = \frac{1}{3} C_v \times v \times \lambda \]

Typically electrons dominate heat conduction.

Processes:
- Electron-phonon
- Electron-impurity

Flowers & Itoh (1976)
Uripin & Yakovlev (1980)
Thermal Conductivity

\[ \kappa = \frac{1}{3} C_v \times u \times \lambda \]

For \( T > 10^8 \) K, superfluid phonons play a role.

Typically electrons dominate heat conduction.

Processes:
- Electron-phonon
- Electron-impurity

Flowers & Itoh (1976)
Uripin & Yakovlev (1980)
Conductivity in Magnetized Neutron Stars

Magnetic field suppresses transverse conduction

\[ \kappa_\perp = \frac{\kappa_\parallel}{1 + (\omega_g \tau_e)^2} \]

\[ \kappa_\parallel = \kappa_{el}(B = 0) \]

\[ \omega_g = \frac{eB}{\mu_e} \quad \text{=Gyrofrequency} \]

\[ \tau_e = \text{Collision time} \]

Canuto and Ventura (1977)
Urpin & Yakovlev (1980)
Conductivity in Magnetized Neutron Stars

Magnetic field suppresses transverse conduction

Aguilera et al. (2009)
Specific Heat

Lowest energy modes dominate the specific heat.

Shear Mode Velocity:

\[ C_V \approx \frac{4\pi^2}{15} \frac{T^3}{c_t^3} \quad (T < T_{\text{Debye}}) \]

\[ c_t^2 = \frac{\mu_s}{m(n_p + n_b)} \]

\[ \mu_s \propto \frac{Z^2 e^2}{a_i^4} \]

\[ n_b = \gamma n_n \]

Entrainment effects

Chamel, Pethick, Reddy (in prep)
Specific Heat at $10^{12}$ g/cm$^3$

- Electrons
- Shear mode
- Superfluid mode
- Longitudinal mode

$C_v$ (in units of $n_i$)

$T$ (K)

$10^7$ $10^8$ $10^9$
Specific Heat at $10^{12}$ g/cm$^3$

$$C_V \sim C_V^{\text{normal}} \exp \left( -\frac{\Delta}{T} \right)$$

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$C_V$ (in units of $n_i$)</th>
</tr>
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<tbody>
<tr>
<td>$0.0001$</td>
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</tbody>
</table>

- Normal Neutrons!
- Shear mode
- Electrons
- Superfluid mode
- Longitudinal mode

Tuesday, July 6, 2010
Shear Mode Velocity & Magnetar QPOs

\[ \omega_{n=0, l=2} \approx \frac{2}{c_t} \frac{\Delta R}{R} \]

\[ \omega_{n=1} \approx \frac{\pi}{R} \frac{c_t}{R} \frac{\Delta R}{R} \]

Duncan (1998)
Watts & Strohmayer (2006)
Conclusions

Astronomy (Observations)

Astrophysics (Theory)

Nuclear Physics & Many-body Theory

\[ \tau_{\text{Cool}} \approx \frac{C_V}{\kappa} (\Delta R)^2 \]

Insights about the nature of matter at extreme density
Back up slides
Mixing

Mixing leads to oscillations

\[
\rho (\text{g/cm}^3)
\]

Longitudinal Lattice Phonon
Superfluid Phonon
Transverse Lattice Phonons

Speed (in units of \(c=3\times10^{10}\) cm/s)

\(10^{11}\)  
\(10^{12}\)  
\(10^{13}\)  
\(10^{14}\)
Dissipative Processes

Electrons

$E(p) = p$

$\omega(p) = c \ p$

IPhs

$\omega(p) = v \ p$

sPhs

Electron-phonon processes

Impurity (Rayleigh) scattering

Multi electron and phonon processes
Dissipative Processes

\[ E(p) = p \]
\[ \omega(p) = c \, p \]
\[ \omega(p) = v \, p \]

Electrons

Electron-phonon processes

Impurity (Rayleigh) scattering

Multi electron and phonon processes
Coupling to Electrons - Landau Damping

Electron particle-hole excitations damp collective modes and vice-versa.

\[ \mathcal{L}_{el-ph} = \frac{1}{f_{el-ph}} \partial_i \xi \psi_e^\dagger \psi_e \]

Induced coupling between superfluid phonons and electrons:

\[ \mathcal{L}_{el-sPh} \sim \frac{\tilde{g}_{mix}}{v_s^2 (1 - \alpha^2)} \frac{1}{f_{el-ph}} \partial_t \phi \psi_e^\dagger \psi_e \]

where \[ \tilde{g}_{mix} = g + \tilde{\gamma} \]

\[ \alpha = \frac{c_l}{v_s} \]
Heating in an Accreting Crust

Non-equilibrium reactions:

Electron capture: \( e^- + [A, Z] \rightarrow [A, Z - 1] + \nu_e \text{ + Heat} \)

Neutron transfers: \( n + [A, Z] \rightarrow [A + 1, Z] + \text{Heat} \)

Pycno-nuclear fusion: