Coupled-channels Calculations of Heavy-ion Fusion Reactions

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• The goal is to develop a coupled-channels description that can explain phenomena observed in heavy-ion fusion reactions, e.g.,
  a) large enhancement at energies below the CB (Coulomb barrier),
  b) hindrance of fusion at extreme sub-barrier energies,
  c) suppression of fusion data far above the CB.

• The description should include couplings to
  a) low-lying $2^+$ and $3^-$ states, mutual and two-phonon exc.,
  b) excitations of rotational states (if deformed),
  c) transfer channels: $1n$, $2n$, $1p$, $2p$, $\alpha$ (if necessary.)

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• Such couplings usually explain the enhancement below the CB.

$^{64}\text{Ni} + ^{64}\text{Ni}$ by Jiang et al., PRL 93, 012701 (2004).

• In the 1970s fusion cross sections were measured at energies above the Coulomb barrier. Once you overcome a barrier you are trapped.

• Since the 1980s cross sections down to 0.1 mb were measured. Large enhancements observed. Coupled-channels calculations were developed. Once you have penetrated the barrier you are trapped.

• Since 2001 cross sections have been measured down to 10 nb. Large hindrance compared to coupled-channels calculations. Calculations are sensitive to the ion-ion potential in the interior.

• Coupled-channels calculations must be based on a realistic ion-ion potential, with a realistic pocket above the Compound Nucleus GS.

• The calculations should explain the hindrance far below the CB, and help explain the suppression far above the CB.

• EXAMPLES: $^{64}$Ni+$^{64}$Ni, $^{16}$O+$^{208}$Pb, $^{16}$O+$^{16}$O.
Proximity type Woods-Saxon (WS) potential

\[ U(r) = \frac{-16\pi \gamma a R_{aA}}{1 + \exp\left[(r - R_a - R_A)/a\right]}, \]

where \( \gamma \) is the nuclear surface tension and \( a \approx 0.6 - 0.7 \text{ fm} \).

It is realistic for large values of \( r \), where it is consistent with elastic scattering data (Rex-Winther) and with double-folding potentials (Akyüür-Winther). It provides a good description of the height of the Coulomb barrier and of fusion data with \( \sigma_f \geq 0.1 \text{ mb} \).

The force has the correct liquid drop form for touching spheres:

\[ F = -4\pi \gamma R_{aA}, \text{ where } R_{aA} = \frac{R_a R_A}{R_a + R_A}. \]

This type of potential has been very useful in the past. However, it is not realistic for overlapping nuclei.
Coupled-channels formalism.

Expand total wave function on channel-spin wave functions,

\[ \Psi_{JM} = \sum_{nIL} \frac{\psi_{nIL}(r)}{r} |n(IL)JM\rangle. \]

Channel-spin wave functions

\[ |n(IL)JM\rangle = \sum_{M_L M_I} \langle LM_L, IM_I |JM\rangle |LM_L\rangle |nIM_I\rangle. \]

\[ |L, M_L\rangle \text{ orbital angular momentum,} \]
\[ |nIM_I\rangle \text{ excited state of projectile or target,} \]
\[ |J, M\rangle \text{ total spin, which is conserved.} \]

Coupled equations:

\[ (h_L + \epsilon_{nI} - E) \psi_{nIL}(r) = \]

\[ - \sum_{n'I'L'} \langle n(IL)JM|V_{int}|n'(I'L')JM\rangle \psi_{n'I'L'}(r). \]

\( I + 1 \) channels for each state: \( L' = |L - I|, \ldots, L + I \). \text{ TOO MANY!}
Rotating frame approximation.

- Assumes that the orbital angular momentum $L$ is conserved (also known as the Iso-centrifugal approximation.)

- Then one can diagonalized the interaction matrix in such a way that there is only one channel for each excited state $(nI)$ instead of $I + 1$ channels, namely, the state $|nIM\rangle$, where $M$ is conserved.

- For fixed $L$ solve the coupled equations:

$$ (h_L + \epsilon_{nI} - E) \psi_{nI}(r) = - \sum_{n'I'} \langle nI|V_{int}|n'I'\rangle \psi_{n'I'}(r). $$

Good approximation for fusion; not so good for angular distributions of Coulomb excitation and transfer reactions at forward angles.
Example: Quadrupole excitations.

- Consider quadrupole excitations.

- The full problem has \( \sum (I + 1) = 33 \) channels.

- In the rotating frame approximation, there is only one channel \((M=0)\) for each state, i.e., we only need \( \sum 1 = 10 \) channels.

- Combine the (3) two-phonon and the (5) three-phonon states into one effective two-phonon and three-phonon state, respectively. Only 4 effective channels are needed.
Standard two-phonon calculation of fusion.

\[
\begin{array}{c|c|c|c}
2 & (2,3) & 2 \\
\hline
2^+ & 3^- & \text{Mutual} \\
\hline
\end{array}
\qquad
\begin{array}{c|c|c|c}
(3,3) & (2,3) & 2 \\
\hline
(2,2) & 3 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c}
2^+ & 3^- & 2 \\
\hline
\end{array}
\quad
\begin{array}{c|c|c|c}
2 & (2,3) & 3 \\
\hline
0^+ & \text{nucleus a} \\
\hline
0^+ & \text{nucleus b} \\
\hline
\end{array}
\]

\(1 \text{ (GS)} + 4 \text{ (1PH)} + 4 \text{ (2PH)} + 6 \text{ (Mutual)} = 15 \text{ channels} \)

(instead of the 138 channels of the full problem.)

This model works quite well for the fusion of not too heavy systems.

- It does not work for inelastic scattering at forward angles,
- in fusion reactions where transfer plays a role \((Q_{tr} > 0)\),
- for heavy, soft or strongly deformed nuclei (multiple excitations),
- in heavy systems where deep inelastic reactions may play a role.
Standard coupled-channels calculations.

- Include nuclear couplings up to second order in the dynamic surface displacement $\delta s = R \sum \alpha_{\lambda \mu} Y^*_{\lambda \mu}(\hat{r})$,

$$U(r, \delta s) = U_N(r) - \frac{dU_N}{dr}\delta s + \frac{1}{2} \frac{d^2U_N}{dr^2}(\delta s^2 - \langle \delta s^2 \rangle),$$

and Coulomb couplings up to first order in $\delta s$.

- Include one-phonon, two-phonon and mutual excitations of the low-lying $2^+$ and $3^-$ states in projectile and target.

- Use scattering boundary conditions for large $r$,

$$\psi_{nI}(r) \to \delta_{nI,0} e^{-ik_0r} + R_{nI} e^{ik_n r}, \text{ for } r \to \infty.$$
Simulate fusion by ingoing-wave boundary conditions (IWBC),

\[ \psi_n(r) \rightarrow T_n e^{-i q_n r}, \quad \text{for} \quad r \rightarrow R_{\text{pocket}}, \]

which are imposed at the minimum of the pocket.

The IWBC are sometimes supplemented with a weak, short-ranged absorption.
Double folding potentials

\[ U_N(r) = \int d\mathbf{r}_1 \, d\mathbf{r}_2 \, \rho_a(\mathbf{r}_1) \, \rho_A(\mathbf{r}_2) \, v_{NN}(\mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1). \]

The effective M3Y interaction produces a very realistic Coulomb barrier, consistent with the proximity type Akyüz-Winther potential. However, the potential is way too deep for overlapping nuclei.

Supplement the M3Y interaction with a repulsive contact term,

\[ v_{NN}^{\text{rep}} = v_{\text{rep}} \, \delta(\mathbf{r} + \mathbf{r}_2 - \mathbf{r}_1). \]

Use a smaller diffuseness of the densities, \( a_{\text{rep}} \approx 0.3–0.4 \text{ fm} \), when calculating the repulsive potential.

Adjust the strength \( v_{\text{rep}} \) so that the total nuclear interaction for overlapping nuclei is consistent with the Equation of State,

\[ U_N(r = 0) = 2A_a[\epsilon(2\rho) - \epsilon(\rho)] \approx \frac{A_a}{9} K, \]

and a nuclear incompressibility of \( K \approx 234 \text{ MeV} \).
Example: $^{64}$Ni+$^{64}$Ni.

The hindrance sets in below 89 MeV. The hindrance is an entrance channel, and not a CN effect.

The shallow M3Y+Repulsion potential has been corrected for the effect of the nuclear incompressibility.
Applied to the $^{64}$Ni$+^{64}$Ni fusion data

\[ S - \text{factor} = E_{\text{c.m.}} \sigma_f \exp(2\pi[\eta - \eta_0]), \text{ where } \eta = \frac{Z_1Z_2e^2}{\hbar\nu}. \]

The IWBC imply that $\sigma_f = 0$, for $E < V_{\text{pocket}} = 85.4 \text{ MeV}$. 
Average spin for fusion from $\gamma$-ray multiplicities. Ackerman et al., NPA 609, 91 (1996).

The WS potential predicts a constant average spin at low energy. The (CC) M3Y+repulsion calculation predicts a vanishing spin at LE. Mišicu and Esbensen, PRC 75, 034606 (2007).
The M3Y+repulsion explains qualitatively the suppression that has been observed (for some systems) at high energies.
Signs of a fusion hindrance have been observed in many systems: $^{90}\text{Zr}+^{89}\text{Y}$, $^{90,92}\text{Zr}$, $^{28}\text{Si}+^{30}\text{Si}$, $^{28}\text{Si}+^{64}\text{Ni}$, $^{58}\text{Ni}+^{58}\text{Ni}$, $^{64}\text{Ni}+^{64}\text{Ni}$, $^{32}\text{S}+^{89}\text{Y}$, $^{48}\text{Ca}+^{96}\text{Zr}$, $^{60}\text{Ni}+^{89}\text{Y}$, $^{64}\text{Ni}+^{100}\text{Mo}$, $^{16}\text{O}+^{208}\text{Pb}$.

The M3Y+rep potential has a minimum pocket energy of $V_{pocket} = 86.2$ MeV. A maximum S factor barely reached.


New data (solid points), Dasgupta et al., PRL 99, 192701 (2007), confirm the fusion hindrance. The WS potential is too deep and cannot explain the fusion hindrance.

A shallow pocket, a thicker barrier, and couplings to the ($^{16}\text{O},^{17}\text{O}$) transfer explain the data much better, HE&SM, PRC 76, 054609 (2007).
The M3Y+repulsion potential has a pocket at 65.1 MeV.

Green curve: one-neutron transfer strength was multiplied by 1.26.
This strength produces a realistic total reaction cross section.
Suppression of $^{16}\text{O}+^{208}\text{Pb}$ fusion far above the CB.

The high energy data are suppressed compared to calculations based on the WS potential. The problem can be fixed by using a large diffuseness, Newton, PLB 586, 219 (2004). Calculations based on the M3Y+repulsion potential and a weak, short-range absorption (SRAbs) reproduce the data.
$^{16}\text{O}^{16}\text{O}$ fusion data, Thomas et al., PRC 33, 1679 (1986).

**Red curve:** best fit to all data points, HE, PRC 77, 054608 (2008).

**Diamond:** Adiabatic TDHF calc. by P.G.Reinhard et al.

**Green:** Best fit to 7 lowest points; is consistent with Jiang’s extrapolation (black curve.)
Evidence for a shallow pocket in the fusion of $^{16}\text{O}+^{16}\text{O}$.

Vary $a_{\text{rep}}$, adjust $v_{\text{rep}}$ so $K=234$ MeV.
The best fit to Thomas’s data is obtained for $a_{\text{rep}} = 0.41$ fm...
$^{16}\text{O}^{+^{16}}\text{O}$ high energy fusion.

**Diamonds** data by Tserruya et al. (1978).

Kolata et al. (1977) saw similar structures.

Are also been seen in $^{12}\text{C}^{+^{12}}\text{C}$ fusion.

Blue dashed curve: based on conventional Woods-Saxon well.

Red curve: the M3Y+rep calculation that fits the Thomas data. The high energy data prefer a shallow pocket. Consistent with elastic scattering analysis by Gobbi et al. PRC 7, 30 (1973).
Conclusion

• The hindrance of fusion far below the Coulomb barrier is a general phenomenon, which has been observed in many heavy-ion systems.

• It is explained by (a posteriori) coupled-channels calculations that are based on IWBC and a shallow potential in the entrance channel.

• A shallow potential also helps resolve the problem of a suppression of high energy fusion data and explains the structures observed in the high energy $^{16}$O+$^{16}$O fusion and scattering data.

• A short-range imaginary potential is often needed at high energies to simulate the effect of the many channels that open up.

• Going beyond the Rotating Frame Approximation would be a computational challenge and require a large number of channels.
Open questions

• Expand experimental and theoretical studies to lighter systems. **WILL THE HINDRANCE PERSIST**, and how will it affect the extrapolation to astrophysical reaction rates? (Gasques et al., PRC 76, 035802, 2007).

• What is the relation to molecular resonances (Bromley et al.)?

• What is the relation to TDHF calculations (Oberacker and Umar)?

• How does the hindrance affect the production of heavy elements?

• How to model the dynamics all the way to the compound nucleus? (Ichikawa et al., PRC 75, 057603 (2007)).
Future directions

• Study more reactions of interest to astrophysics.
• Study the competition between breakup, complete and incomplete fusion of weakly bound nuclei.
• Apply CDCC calculations to deal with states in the continuum.
• A good starting point is $^9$Be. It has several advantages:
  it is weakly bound, $Q(\alpha + \alpha + n) = -1.574$ MeV, with only one (borromean) bound state. It is stable (strong beams).
  Many experiments have already been performed.
$^9\text{Be}$ is strongly deformed, $Q_0 = 26.5 \ (15) \ \text{fm}^2$. 

$$\rho(r, \theta') = C \frac{1 + \cosh(R(\theta')/a)}{\cosh(r/a) + \cosh(R(\theta')/a)}, \quad R(\theta') = R_0(1 + \beta_2 Y_{20}(\theta')).$$

$\theta'$ is the angle between $\mathbf{r}$ and the symmetry axis. Calibrate the density to give the correct RMS charge radius and quadrupole moment $Q_0$.

This is achieved for $R_0=2.08$, $a=0.375 \ \text{fm}$, and $\beta_2 = 1.18$. 
Coupled Eqs. for excitations of the Ground State rotational band of $^9\text{Be}$.

Spins: $I^\pi = 3/2^-, 5/2^-$ and $7/2^-$, exc. energies 0.0, 2.43 and 6.38 MeV.

The decay of the $7/2^-$ state, $\Gamma(7/2^-) = 1.21$ MeV, is included as an absorption. It may lead to incomplete fusion (ICF).

Coupled equations:

$$\left[ \frac{\hbar^2}{2\mu} \left( -\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} \right) + U_0(r) + E_I - i\Gamma_I/2 - E_{cm} \right] \psi_{IM}(r)$$

$$= - \sum_{\lambda>0} \sum_{I'} \langle K IM | P_\lambda(\cos(\theta')) | K I'M \rangle U_\lambda(r) \psi_{I'M}(r).$$

Esbensen, PRC 81, 034606 (2010).
K=3/2 Ground State channel potentials and the complete fusion (CF) of $^9\text{Be}$ and $^{144}\text{Sm}$.

Fusion through the tip ($m=3/2$) dominates at low energy. Fusion through the belly ($m=1/2$) is hindered at low energy.
Complete (CF) and incomplete (ICF) fusion of $^9$Be and $^{144}$Sm, Gomes et al., PRC 73, 064606 (2006).

**Figure (A)**

- $^9$Be on $^{144}$Sm
- CF and ICF graphs

**Figure (B)**

- TF, CF, ICF graphs
- $\sigma_f$ vs. $E_{c.m.}$ (MeV)

CF reproduced by IWBC. ICF reproduced by the decay.
Complete (CF) and incomplete (ICF) fusion of $^9$Be and $^{208}$Pb, Dasgupta et al., PRC 73, 024606 (2004).

CF data are suppressed by 20%. The decay explains only 1/3 of ICF.
Include a weak absorption in addition to decay,
\[ W(r) = \frac{-i \; 0.35 \text{ MeV}}{1 + \exp((r - 11.5)/0.4)}. \]

One-neutron transfer is the most likely reaction mechanism responsible for the breakup and ICF of $^9\text{Be}$, Rafiei et al., incl. Diaz-Torres, PRC 81, 024601 (20101).