Coupled-Channels Density-Matrix Approach to Nuclear Reaction Dynamics

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Outline

- **Introduction**
  - Quantum decoherence in a broad context
  - Coherent coupled channels model
  - Quantum decoherence in nuclear collisions

- **Coupled-channels density-matrix approach**
  - Picture and formulation
  - Applications

- **Summary**
The Quantum to Classical transition - from coherent superposition to irreversibility

M. Schlosshauer, Decoherence and the quantum to classical transition, Springer (2007)

- Quantum decoherence – “dynamical dislocalization of Q.M. superpositions”
  (H.D. Zeh arXiv:quant-ph/0512078 v2)  coherence shared with (lost in) environment
Quantum Decoherence

Example: Electron entanglement with a surface

Double-slit type experiment with single electrons

P. Sonnentag et al., PRL 98 (2007) 200402

Can a quantitative model be developed for nuclear collisions?
How do these excitations affect the nuclear collision dynamics?
Low-Energy Collision Dynamics: Coherent Quantum Description

Energy

Total potential

Coulomb Barrier

Internuclear distance

Wave-packet

Relative motion

Intrinsic low-lying collective states

\(|0\rangle\)

\(|1\rangle\)
Failure of the Coherent Quantum Description

Coupling Assisted Quantum Tunnelling: Nuclear Fusion

Measure of Fusion Probability

$^{16}\text{O} + ^{208}\text{Pb}$

Above the Barrier

Below the Barrier

Quantum Decoherence in Nuclear Collisions

AD-T, Hinde, Dasgupta, Milburn & Tostevin, PRC 78 (2008) 064604

Intrinsic quantum states

Relative motion

Suppresses quantum tunnelling (sub-barrier fusion)
Quantum Decoherence in Nuclear Collisions

AD-T, Hinde, Dasgupta, Milburn & Tostevin, PRC 78 (2008) 064604

\[ \frac{\partial \hat{\rho}}{\partial t} = [\hat{L}_H + \hat{L}_D] \hat{\rho} \Rightarrow \hat{\rho}(0) = \hat{\rho}_0 \quad \text{Master equation} \]

\[ \hat{L}_H \hat{\rho} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] \quad \text{Schrödinger description} \]

\[ \hat{L}_D \hat{\rho} = \sum_k \left( \hat{C}_k \hat{\rho} \hat{C}_k^\dagger - \frac{1}{2} [\hat{C}_k^\dagger \hat{C}_k, \hat{\rho}]_+ \right) \quad \text{Decoherence} \]

\[ \hat{C}_{ij} = \sqrt{\Gamma_{ij}} |I_i \rangle \langle j| \quad \text{Absorption} \]

Example: 1-dimensional model

\[ \hat{\rho}(t) = \sum_{ij,rs} |i \rangle \langle i| \rho_{ij}^{rs}(t) \langle j| \langle s| \Rightarrow \rho_{ij}^{rs}(0) = \rho_{00}^{rs}(0) = g_0(r)g_0^*(s) \]

|\langle i|, i = 1, \ldots N \rangle \quad \text{Intrinsic (energy) basis} |

|\langle r |, r = 1, \ldots M \rangle \quad \text{Coordinate (grid) basis} |
Quantum Decoherence in Nuclear Collisions

Absence of Decoherence in the Optical Potential Model

Decoherence significantly affects quantum tunnelling, and thus scattering as well.
\[ |\chi\rangle = \sum_{LJ M} \psi_{k_0}(r) |0 L; J M\rangle \Rightarrow \hat{\rho}_0 = |\chi\rangle \langle \chi| \]

\[ \hat{\rho}_0 = \sum_{\alpha, \alpha', rs} |r\rangle |\alpha\rangle \rho_{\alpha\alpha'}^{rs}(t = 0) \langle \alpha'| (s|, \]

where \( \alpha \equiv (IL; JM) \), \( |\alpha\rangle \) and \( |r\rangle \) are the coupled angular momentum basis and the discrete grid-basis describing the internuclear separations, respectively.

\[ \rho_{\alpha\alpha'}^{rs}(t = 0) = N^2 \exp \left[ - \frac{(r - r_0)^2}{2\sigma^2} \right] e^{ik_0 r} \]

\[ \times \exp \left[ - \frac{(s - r_0)^2}{2\sigma^2} \right] e^{-ik_0 s} \delta_{I 0} \delta_{I' 0}, \]

where \( N \) is determined from the normalization condition \( \sum_{r\alpha} \rho_{\alpha\alpha}^{rr} = 1. \)
Coupled-Channels Density-Matrix Approach

Equations of Motion

\[ i\hbar \dot{\rho}_{\alpha\alpha'}^{rs} = \sum_t (T^{rt} \rho_{\alpha\alpha'}^{ts} - \rho_{\alpha\alpha'}^{rt} T^{ts}) \]
\[ + [U_\alpha(r) - U_\alpha'(s)] \rho_{\alpha\alpha'}^{rs} \]
\[ + \sum_{\beta} [V_{\alpha\beta}(r) \rho_{\beta\alpha'}^{rs} - \rho_{\alpha\beta}^{rs} V_{\beta\alpha'}(s)] \]
\[ + (\varepsilon_\alpha - \varepsilon_{\alpha'}) \rho_{\alpha\alpha'}^{rs} \]
\[ + i\hbar \{ \delta_{\alpha\alpha'} \sum_\mu \sqrt{\Gamma_{r\mu}} \rho_{\mu\mu}^{rs} \sqrt{\Gamma_{s\mu}} \}
\[ - \frac{1}{2} \sum_\mu (\Gamma_{r\mu} + \Gamma_{s\mu}^{rs}) \rho_{\alpha\alpha'}^{rs} \} \]

\[ \dot{\rho}_{\bar{\alpha}\bar{\alpha}'}^{rs} = \delta_{\bar{\alpha}\bar{\alpha}'} \sum_\mu \sqrt{\Gamma_{\bar{\alpha}\mu}} \rho_{\mu\mu}^{rs} \sqrt{\Gamma_{\bar{\alpha}\mu}} \]
\[ - \frac{1}{2} \sum_\mu (\Gamma_{\mu\bar{\alpha}} + \Gamma_{\mu\bar{\alpha}'}) \rho_{\bar{\alpha}\bar{\alpha}'}^{rs} \]

Expectation value of an observable: \[ \langle \hat{O}(t) \rangle = \text{Tr}[\hat{O} \hat{\rho}(t)] \]
The probability for producing the target in state \((I, M_I)\) with the relative coordinate in the direction \(\hat{r}'\):

\[
\frac{dW}{d\Omega}(I, M_I) = \sum_q C_{LmIM_I}^{JM} Y_{Lm}(\hat{r}') S_{\gamma\lambda}(t_f) 
\times C_{L'm'IM_I}^{J'M'} Y_{L'm'}^*(\hat{r}'),
\]

where \(q \equiv (L, m, J, M, L', m', J', M')\), \(\gamma \equiv (IL; JM)\), \(\lambda \equiv (IL'; J'M')\), and \(S_{\gamma\lambda}(t_f) = \sum_{r'} \rho_{\gamma\lambda}^{rr'}(t_f)\).

Integrating over all directions \(\hat{r}'\) of solid angles, and summing over all \(M_I\), the total probability for producing the target in state \(I\) (population) is obtained:

\[
W(I) = \sum_{M_I} \sum_{LmJM} (C_{LmIM_I}^{JM})^2 S_{\gamma\gamma}(t_f)
\]
Track decoherence and absorption through different mechanisms
Environments are specific to particular degrees of freedom

\[ (Z_T + Z_P) \]

Nucleonic d.o.f.

Nuclei with different charge
\[ (Z_T + 2)(Z_P - 2) \]

Reduced system

GDR

Nuclear Molecular d.o.f.

Transfer

Interacting individual nuclei
\[ (Z_T)(Z_P) \]
Application: Understanding fusion of astrophysically-important collisions at low energies

AD-T, Gasques & Wiescher, PLB 652 (2007) 255

E.F. Aguilera et al., PRC 73 (2006) 064601

$^{12}\text{C} + ^{12}\text{C}$

Complex excitation modes in dinuclear system

Fusion probability at astrophysical energies?

Origin of the resonances?

AD-T, PRL 101 (2008) 122501
Neutron molecular shell structure of two interacting deformed $^{12}$C

$$V = \sum_{s=1}^{2} e^{-iR_s \hat{k}} \hat{U}(\Omega_s) V_s \hat{U}^{-1}(\Omega_s) e^{iR_s \hat{k}}$$

$$V_s \approx \sum_{\nu\mu}^{N} |s\nu\rangle V_{\nu\mu}^s \langle s\mu|$$

AD-T, PRL 101 (2008) 122501
Application: Unified quantum description of reaction processes of neutron-rich, weakly-bound nuclei
Classical dynamical model

Quantitative calculations of CF, ICF and NCBU yields above the barrier

Main Ingredient:

\[ P_{BU}^L(R) dR \] probability of breakup on the interval \( R + dR \)

\[ P_{BU}(R_{min}) = 2 \int_{R_{min}}^{\infty} P_{BU}^L(R) dR = A \exp(-\alpha R_{min}) \]
Fusion excitation functions: \( {^8}\text{Be} \) + 208Pb

Breakup function

CF & ICF excitation functions
Direct alpha-production yields: “$^8\text{Be}$” + $^{208}\text{Pb}$

$E_{\text{cm}}/B_0 = 1.57$

$E_{\text{cm}}/B_0 = 1.08$
Spin distribution of fusion products: $^{216}\text{Rn} \& ^{212}\text{Po}$

$E_{\text{cm}}/B_0 = 1.57$

$E_{\text{cm}}/B_0 = 1.08$
Excitation energy distribution of ICF product $^{212}$Po

$E_{cm}/B_0 = 1.57$

$E_{cm}/B_0 = 1.08$
Summary

- **Innovative quantum dynamical approach**, which will quantify the **importance of quantum decoherence** in various areas of reaction theory of stable and exotic nuclei.

- **Quantum decoherence should always be explicitly included** when modelling low-energy nuclear collision dynamics within a truncated model space of reaction channels.

- **Links with decoherence in other quantum systems**

**Workshop on Decoherence in Quantum Dynamical Systems**

To be held at ECT* Trento, IT, April 26\textsuperscript{th} – 30\textsuperscript{th}, 2010

Registration: until April 9

http://www.nucleartheory.net/Decoherence/
Let us define two probabilities: (i) the probability of breakup between \( R \) and \( R + dR \), \( \rho(R)dR \) [being \( \rho(R) \) a density of probability], and (ii) the probability of the weakly-bound projectile's survival from \( \infty \) to \( R \), \( S(R) \). The survival probability at \( R + dR \), \( S(R + dR) \), can be written as follows

\[
S(R + dR) = S(R) [1 - \rho(R)dR]. \tag{A.1}
\]

Expression (A.1) suggests the following differential equation for the survival probability \( S(R) \),

\[
\frac{dS(R)}{dR} = -S(R) \rho(R), \tag{A.2}
\]

whose solution is \([S(\infty) = 1]\):

\[
S(R) = \exp(- \int_{\infty}^{R} \rho(R)dR). \tag{A.3}
\]

From (A.3), the breakup probability at \( R \), \( B(R) = 1 - S(R) \). If \( \int_{\infty}^{R} \rho(R)dR \ll 1 \), \( B(R) \) can be written as

\[
B(R) \approx \int_{\infty}^{R} \rho(R)dR. \tag{A.4}
\]

From (A.4), identifying \( \rho(R) \) with \( \mathcal{P}_{BU}^{L}(R) \), we obtain expression (1) for the breakup probability integrated along a given classical orbit.
Measure of Coherence

For a pure state described by the state vector $|\chi\rangle$:

$\hat{\rho} = |\chi\rangle\langle \chi|$, and $\text{Tr}(\hat{\rho}) = \langle \chi|\chi\rangle$.

$\hat{\rho}^2 = |\chi\rangle\langle \chi|\langle \chi|\chi\rangle$, and $\text{Tr}(\hat{\rho}^2) = \langle \chi|\chi\rangle \langle \chi|\chi\rangle = [\text{Tr}(\hat{\rho})]^2$.

Hence, $\text{Tr}(\hat{\rho}^2)/[\text{Tr}(\hat{\rho})]^2 = 1$, for nonzero values of $\text{Tr}(\hat{\rho})$.

For a mixed state, there is no single state vector describing the system:

$\text{Tr}(\hat{\rho}^2)/[\text{Tr}(\hat{\rho})]^2 < 1$.

The transition from a pure state to a mixed state is caused by decoherence.