Nuclei, Neutron Matter and Cold Atoms
J. Carlson - LANL

Light Nuclei

Neutron Star

Cold Atom Fermi Condensates

U. Arizona

C. Regal et al. PRL 2004
A Nuclear Hamiltonian:

Pion Exchange

Shorter-range interactions
spin-orbit, etc.

Also need a three-body force
Monte Carlo Methods

\[ \Psi = \exp[-H\tau] \Psi_0 \]

Initial Guess or `Source'
Shell Model plus Correlations

Two-Body solutions plus 3-nucleon interaction

- Monte Carlo for spatial integrals
- Explicit Sums for spin/isospin
- In general approximate solutions

Alpha Particle,... Trivial

\(^{12}\)C uses 100K cores (or more) for Ground/Hoyle states
Light Nuclear Spectra

Argonne v$_{18}$
With Illinois-2
GFMC Calculations

$^{4}\text{He}$  $^{6}\text{He}$  $^{6}\text{Li}$  $^{7}\text{Li}$  $^{8}\text{He}$

$^{8}\text{Be}$  $^{9}\text{Be}$  $^{10}\text{Be}$  $^{10}\text{B}$

$^{12}\text{C}$

$^{12}\text{C}$ IL2 result is preliminary.
Recent Calculations

\[ {}^{12}\text{C}(\text{gs}) - \text{AV18+IL7 & Modified SCC} v'_{\phi} - \rho_p - 20 \text{ Jun 2009} \]

\begin{align*}
\text{12C Form Factor} \\
\text{Mostly ‘Postdictions’}
\end{align*}

\begin{align*}
\text{n-alpha scattering} \\
\text{Nollett, et al, PRL 2007}
\end{align*}
Higher Momentum States in the Nucleus
Back-to-Back np vs. pp momentum distribution

\[ n_{N,N'}(\mathbf{k}, P = 0) = \langle 0 | a_{N}^{\dagger}(\mathbf{k})a_{N'}^{\dagger}(-\mathbf{k}) a_{N}(\mathbf{k})a_{N'}(-\mathbf{k}) | 0 \rangle \]

Piasetsky et al PRL 2006; Shneor et al 2007

see Alvioli, et al PRL 2008
Neutron Rich Matter: Helium Charge Radii

Norterhauser, et al, PRL 2009
Neutron/Neutron-Star Matter

**Low Density:** Exterior of Nuclei
Neutron Star Crust

Simple well-understood interaction
Intriguing physics: strongly paired fermions
Many experimental tests

**High Density:** Bulk of Neutron Star

Interaction Poorly Known
Required to understand cold, dense matter
Observational Tests
Unitary Regime and Low-Density Neutron Matter

\[ H = \sum_i T_i + \sum_{i<j} V_0 \, \delta(r_{ij}) \]

One Parameter: \( V_0 \)
Cold Atom Experiments:

Diagram from Innsbruck

Fermions: $^6$Li, $^{40}$K
Density $\sim 1/ \mu m^3$
Temperature $\sim 200$ nK $\sim 0.1$ Ef

First Fermi Condensates 2004
Experimental Results in Cold Atoms

- (nearly) Free Fermions
- (nearly) Free Bosons
- ‘Universality’ and the BCS-BEC transition
- Polaron
- Efimov States
- Superfluid Fermions (s-, p-, d-wave,... pairing)
- Exotic Polarized Superfluids (FFLO, breached pair,...)
- PseudoGap States
- Itinerant Ferromagnetism
- ‘Perfect’ Fluids
- Reduced Dimensionality
- More than pairing (3-,4-body condensates, ...)
- Bose, Fermi Hubbard Models,
Unitarity

Unitarity = limit of 0 pair binding
\[ a_0 = \infty \]

All quantities multiples of Fermi Gas at same \( \rho \)
At zero polarization, expect strong pairing

\[ E = \xi \epsilon_{FG} = \xi \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} \]
\[ \Delta = \delta \frac{\hbar^2 k_F^2}{2m} \]

Values of \( \xi, \delta, t \) are independent of \( \rho \)

\[ T_c = t \frac{\hbar^2 k_F^2}{2m} \]
\[ E = \xi E_{FG} = \xi \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} \]
Calculations at Unitarity: \( \# \quad \uparrow \quad \Rightarrow \quad \#
\)

\[ \xi = 0.40(01) \]

Experiments at Unitarity: \( \# \bullet \bigcirc = \# \bigcirc \)

**Cloud Size and Sound Velocity**

**Cloud Size vs E (B)**

\[ \xi = 0.39(02) \]

**Energy vs. Entropy**

\[ \xi = 0.41(02) \]

\[ \frac{c_0}{v_f} = \frac{\xi^{1/4}}{\sqrt{5}} \]

scaling verified as \( \rho \) varied by 30!

\[ \xi = 0.435(15) \]
Relation to Neutron Matter: Equation of State

Neutrons vs. Cold Atoms    adding p-wave interactions

Gezerlis and JC, PRC 2008, 2010
Pairing Gap at Unitarity - Cold Atom Calculations

Add one to fully-paired system
Energy cost for an unpaired particle: $\mu + \Delta$

Computational Cost Large:
$E(N+1) - 1/2( E(N)+E(N+2)$

Quasiparticle Dispersion

$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$

$\delta = 0.50 (03)$

$(k_{min}/k_f)^2 = 0.80(10)$

JC and Reddy, PRL 2005
Pairing Gap at Unitarity - Experiment

Spin up, down densities in a trap

Polarization

radius

Polarization
Pairing Gap at Unitarity - Experiment

Polarization in a trap

\[ \Delta = \delta \frac{\hbar^2 k_F^2}{2m} \]
\[ \delta = 0.45(05) \]

JC and Reddy, PRL 2007 analyzing MIT data
Pairing Gap at Unitarity - Experiment

RF response

Shift of response of paired vs. unpaired atoms

Shin, Ketterle, ... 2008
**Pairing Gap: Cold Atoms and Neutron Matter**

At small $|k_F a|$ consistent with Gorkov polarization suppression of BCS

\[ \frac{\Delta}{\Delta_{BCS}} = \frac{1}{(4e)^{1/3}} \approx 0.45 \]

At large densities consistent with unitary results
Neutron Matter EOS strongly constrained at low-moderate densities
Beyond Bulk Matter

Neutron Drops in an External Well (HO)

\[ \omega = 10 \text{ MeV} \]

\[ \omega = 5 \text{ MeV} \]

Preliminary

Implies significantly more repulsive isovector gradient terms  
Carlson, Pieper, Gandolfi, preliminary
Neutron Drop Densities

\[ \frac{1}{2} \sum_{s = 1}^{2s + 1} \rho_n(r) = \frac{1}{2} \sum_{s = 1}^{2s + 1} \rho_n(r) \]

\[ \hbar \omega = 5, 10 \text{ MeV} \]

\[ 8^\text{n}, 5 \text{ MeV} \]
\[ 8^\text{n}, 10 \text{ MeV} \]
\[ 14^\text{n}, 5 \text{ MeV} \]
\[ 14^\text{n}, 10 \text{ MeV} \]

8,14\text{n} – (5 & 10 MeV) H.O. Well+AV8'+UIX - Ratio extrp $\rho_n$ - 12 May 2009
Neutron Matter at Intermediate Densities

Equation of State

Schwenk, 2010
Low-Momentum 2 Nucleon Forces

2N interaction yields identical results in truncated space

Chiral 3-N forces

2 Pion Exchange similar to Urbana, Illinois models
Others typically fit to $A=3,4......$
Chiral 3-N forces

Only 2-Pion Exchange contributes in neutron matter
Neutron Matter at Intermediate Densities

Hebeler and Schwenk, 2010

GFMC- AV8': JC, Morales, Pandharipande
AFDMC-AV8': Gandolfi, et al, PRC 2009
AFDMC-AV8'+UIX: Gandolfi, et al
Also introduced density-dependent TNI

\[ S = S_0 + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \ldots \]

Gives reasonable symmetry energy: \( S_0 = 31.3 \text{ MeV}; L \sim 70 \)

Compare with Tsang, et al (PRL 2009) and references therein: 30-34
Astrophysical Constraints on Neutron Star Matter

Apparent surface area during cooling phase of burst:

\[ A = \frac{R^2}{D^2 \frac{f^4}{c^2}} \left(1 - \frac{2GM}{Rc^2}\right)^{-1} \]

Eddington Luminosity

\[ F_{Edd} = \frac{GMc}{k_{es} D^2} \left(1 - \frac{2GM}{Rc^2}\right)^{1/2} \]

Ozel, Baym, Guyver arXiv:1002.3153

FIG. 2: The pressure of cold matter at (top) 7.4 and 3.7 \( \text{ns} \) and (bottom) 1.85 and 3.7 \( \text{ns} \).
Really Need:

3-neutron interactions
Hyperon-Nucleon interactions
Hyperon-Hyperon interactions

from Lattice QCD?

Mass/Radius for range of neutron stars

Could know neutron star matter EOS better than nuclear matter
Microscopic Constraints from Observations

Observations: Ozel, Baym, Guyver

Calculations
Gandolfi, Illarionov, Fantoni, Miller, Pederivak, Schmidt: arxiv 0909.3487
Future Challenges in Theory/Computation

Neutron Matter:
- Few Protons (Neutron Star Matter)
- Data from Expt (PREX, FRIB, ...)
- Generalized Static Response
- Drops in Various External Fields
- Matter in the Crust

Nuclei / Nuclear Matter:
- Low-Energy Reactions w/ GFMC
- AFDMC with `realistic’ interactions
- Pairing in Finite Nuclei
- Larger Nuclei, Matter, ...
- Neutrino Response
- Finite Temperature