

Physics 554/ Astronomy 510  
Nuclear Astrophysics  
Assignment 4  
Due: Friday, November 23

1. In class we derived a formula for the survival probability of an electron neutrino undergoing vacuum oscillations. Following that calculation exactly, calculate the probability that it has become a muon neutrino at point  $x$ . (That is, take the overlap of the wave function with a  $|\nu_\mu\rangle$ .) Show that the sum of the two probabilities is one. Graph the electron and muon probabilities as a function of  $x$ .
  
2. Rewrite our formula for the vacuum neutrino oscillation length by using the following normalized dimensionless quantities:  $E/\text{MeV}$ ;  $\delta m^2/\text{eV}^2$ ;  $L_o/1 \text{ meter}$ . If you do things right, all that should remain is some dimensionless number relating  $L_o$  to the other quantities. Stopped pions produce neutrinos of about 35 MeV average energy. If you are searching for neutrino oscillations corresponding to  $\delta m^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$ , how far from the source should you place your detector? Do you think it might be helpful to use more than one location? What if the neutrinos had energies of 2.5 GeV? This last answer tells you how far an underground lab should be from Fermilab to test the atmospheric neutrino oscillation results of SuperKamiokande, assuming a new proton driver and neutrino superbeam.
  
3. Consider a monoenergetic electron neutrino beam of energy  $E$ . It travels from a source through vacuum a distance  $L_o$ , then through matter a distance  $L_o$ , with  $L_o$  defined as in problem 2. It exits the matter and is immediately measured. Calculate the electron neutrino survival probability as a function of the electron density in the matter. Note this problem is very different from the MSW problem considered in class. It involves propagation through constant density, except for an abrupt change in density as the neutrino enters or exits matter. The appropriate approximation to use is the sudden approximation. (See me for an explanation if you are not familiar with this.) From the result you find, describe the effects of the matter. In this exploration you can pick a  $\delta m^2$  of  $10^{-5} \text{ eV}^2$ , a mixing angle of 45 degrees, a neutrino energy of 1 GeV, and consider only two flavors (say electron and muon). Note that, if the matter is removed, the experimentalist would measure the full flux of electron neutrinos. Is it possible for the experimentalist to measure a smaller flux of electron neutrinos, because of the matter?
  
4. A supernova explodes in the Large Magellanic Cloud, 50 kpc from earth. The burst of neutrinos turns on suddenly at  $t = 0$  and then declines in time as  $e^{-t/t_0}$ , where  $t_0 \sim 2$  seconds. The distribution in energy can be taken as a Fermi-Dirac characterized by a temperature  $T \sim 5 \text{ MeV}$ . The neutrinos are observed on earth in a detector, and the spread in the arrival times of the neutrinos recorded. Estimate what neutrino mass would cause the duration of the observed events to have approximately doubled, due to spreading of the

signal during transit from the LMC to earth. Note this is about the kinematic effects of the neutrino mass, not about oscillations.

5. We mentioned in class the explosive burning of  $^{28}\text{Si}$  into iron-group nuclei. It was also noted that the peak temperature achieved when the shock wave passes through the Si shell is about  $T_9 \sim 6$ .

a) Consider the process  $^{28}\text{Si} + ^{28}\text{Si} \rightarrow ^{56}\text{Ni} + \gamma$ . Estimate the rate for this reaction in sufficient detail that you can answer the question: will this reaction make significant  $^{56}\text{Ni}$  under the conditions described above? Feel free to assume resonant production, leaving the widths as unknowns but basing your conclusions on the Coulomb physics.

b) Similarly, give a rough estimate of the rates of the reaction  $^{28}\text{Si} (\gamma, \alpha)^{24}\text{Mg}$  relative to that in a). Again, leave the widths unspecified, but otherwise handle the Coulomb physics properly.

c) Reactions such as those in b) generate free  $\alpha$ s, which can in turn do  $(\alpha, \gamma)$  reactions. Calculate the ratio

$$\frac{N(^{28}\text{Si})N(\alpha)}{N(^{32}\text{S})}$$

where N represents a number density, in terms of the mass difference Q between  $^{28}\text{Si}$  and  $\alpha + ^{32}\text{S}$ . Evaluate this expression at  $T_9=6$ . If you use the Saha equation, explain why this is ok based on b). Could I also apply the Saha equation to the reaction in a)?

d) We noted that the binding energy per nucleon increases until one reaches iron. Give a simple argument, based on physical insight or thermodynamics, about how Si burns to Fe.