

Physics 554/ Astronomy 510: Nuclear Astrophysics  
Problem Set 3 (Due: Monday, November 7)

1. In class we estimated the electron capture rate on  ${}^7\text{Be}$  as capture of an electron from the plasma. This assumes that  ${}^7\text{Be}$  is fully ionized under conditions characteristic of the solar center.

a) The binding energies of the s and p orbital electrons in  ${}^7\text{Be}$  are about 112 and 3 eV respectively. Compare this with the Bohr atom prediction for  $Z=4$ . Now derive the "effective  $Z$ " for a Bohr atom that would yield these energies. Comment on the result – its physical origin.

b) Consider the two-state problem where an electron is either in the s-state or the continuum. Note this is the Bohr atom, by assumption. Calculate the probability the s-state is occupied by this electron for a temperature of  $T_7 = 1.5$ .

2. Calculate carefully the energy deposited in the sun by each of the ppI, ppII, and ppIII cycles, assuming that all of the energy is deposited except for that carried off in neutrinos. For neutrinos, neglect the  $p+p+e^-$  reaction and assume, in beta decays, that the neutrino carries, on average, half of the kinetic energy available to the outgoing neutrino and positron. Look up the solar constant. Use this to derive a model-independent constraint on the sum of the  $pp + {}^7\text{Be} + {}^8\text{B}$  neutrino fluxes, assuming only that the sun burns in a steady state (so that today's neutrino fluxes and photon luminosity are coupled).

3. Calculate the relative abundance of  ${}^{12}\text{C}$ ,  ${}^{13}\text{C}$ ,  ${}^{14}\text{N}$ , and  ${}^{15}\text{N}$  in the CN cycle at equilibrium for a temperature of  $T_7 = 1.583$ , the value predicted by the standard solar model of Bahcall and Pinsonneault at the solar center. According to this model, the mass fractions of  ${}^{12}\text{C}$  and  ${}^{14}\text{N}$  are  $2.42 \times 10^{-5}$  and  $5.93 \times 10^{-3}$ . Do these numbers seem consistent with estimates you made?

4. Bethe originally thought the sun might be powered by the CNO cycle. If it were so powered, and if neutrino oscillations are ignored, what would be the terrestrial fluxes of electron neutrinos? (Assume the CNO-I cycle is burning in equilibrium, and use arguments similar to problem 2.) Compare this to the  ${}^7\text{Be}$  neutrino flux given in class. Estimate the relative rates of CNO-cycle neutrino and  ${}^7\text{Be}$  neutrino absorption in the Davis Cl detector, considering just the transition to the  ${}^{37}\text{Ar}$  ground state, for a CNO-powered sun vs. a pp-cycle sun. In estimating the neutrino absorption relative cross sections, feel free to approximate Coulomb effects for the outgoing electron by some average value, in the case of continuous neutrino spectra from beta decay.

5. Calculate the temperature dependence of the CNO cycle. Remember that  ${}^{14}\text{N}(p,\gamma)$  is the controlling reaction at the energies of interest to us. You can either do this numerically, by calculating at  $T_7 = 2$  and  $T_7 = 2.1$ , for example, or by expanding in a Taylor series around  $T_7 = 2$ . But  $T_7 = 2$  is a good choice for the evaluation.

6. Consider a two-nucleon wave function, which you will form as a product of the single-particle wave functions (or as a sum of such products). Each of the single-particle wave functions has quantum numbers  $n, \ell, m, s = 1/2, m_s, \tau = 1/2, m_\tau$ . Assume both nucleons are in the  $n = 1, \ell = 0$  state. Form an antisymmetric two-proton state and an antisymmetric proton-neutron state, with  $m_s(1) + m_s(2) = 0$ . Calculate the Fermi and Gamow-Teller matrix elements between these states.