

Physics 554/Astronomy 510
Problem Set #2
Due Wednesday, October 24

1. For a Fermi-Dirac distribution, calculate the total energy E and particle number N , treating $(kT/\epsilon_F)^2$ as small (keeping corrections to this order). Calculate dE/dN , the energy required to add a particle. Generalize the $T=0$ formula given in class for the Fermi energy.
2. Starting with the Maxwell distribution as given on page 2 of the notes, taking the chemical potential as zero, derive the Maxwell velocity distribution.
3. Among particles in thermal equilibrium with photons in the universe, the entropy $g(RT)^3$ remains constant, where g is the number of degrees of freedom. So consider g when the photons, electrons, and positrons are in equilibrium, then the new g later, when the electrons and positrons have annihilated. Use the constancy of the entropy to determine the ratio of RT before and after annihilation. Use this result to explain trends in R and T seen in the table from Weinberg on page 9 of the notes for Chapter 1. What is T in that table? Expand the table so that there are two temperature columns, one corresponding to photons and one to neutrinos.
4. Calculate the number of degrees of freedom relevant to the very early high-temperature universe, assuming the relativistic particles are the muons, electrons, photons, and three types of neutrinos, in units where the photon yields a weight of 2.
5. Use the Saha equation to calculate the temperature of recombination, which you can define as the time when there is an equal amount of neutral hydrogen and free protons/electrons. You can simplify the hydrogen atom to be a single state at -13.6 eV.
6. Generalize the Saha equation for the case of two protons combining with two neutrons directly – not through deuterium – to form 4He . Taking $\eta = 10^{-9}$, find the temperature where half of the neutrons are bound in 4He . How does this temperature compare to the deuteron bottleneck temperature? Comment on this result, in comparison with the BBN calculation done in class.
7. The textbooks state that nucleosynthesis terminates (almost) at 4He because there are no stable isotopes at $A=5$ and 8 . Consider the reaction $4\text{He} + p \rightarrow 5\text{Li} + \gamma$. Assume 5Li is bound relative to the initial channel by some amount E_b . Use the Saha equation to evaluate the abundance of 5Li at the time of deuteron formation, taking $\eta = 10^{-9}$. What value of the binding gives a Li/He ratio of 5% and 0.5% ? Critique the statement that the absence of 5Li at the time of deuteron formation is due to the lack of a stable nucleus at $A=5$.