Physics 554/ Astronomy 510: Nuclear Astrophysics
Problem Set 1 (Due: Monday, October 11)

1. There are two AJP articles on the web site that review basic neutrino and weak interaction physics. If you are a bit uncertain about weak interactions, you might review these. Equation (4) of the first paper gives an approximate formula for beta decay. Use it to derive a total beta decay rate for the neutron. By looking up Q (the neutron-proton mass difference) and the neutron lifetime, deduce the size of the squared matrix element in that formula (which you can assume to be a constant). What do you think is the most important approximation that has been made in Eq. (4), in terms of numerical impact? That is, can you think of any physics, relevant at low energies, that has been omitted?

2. We estimated the temperature T_d for deuterium formation. Consider the analogous two-state problem of d+d vs. ^{4}He in equilibrium. Looking up masses, find the temperature where half of the deuterons would have combined to form He. Is this temperature lower or higher than T_d? Why, in the big bang, don't two neutrons and two protons combine directly to form He, before the time of deuterium formation?

3. The energy density due to photons in the early universe is given (with $\hbar$ and c now properly inserted) by an integral over the photon momenta

$$\rho = \frac{2}{\hbar^3} \int \frac{d^3k}{(2\pi)^3} \frac{ck}{e^{ck/KT} - 1} \tag{1}$$

where K is Boltzmann's constant. If one considers the corresponding contribution of a neutrino and an antineutrino, the integral is identical apart from the change to

$$e^{ck/KT} + 1 \tag{2}$$

in the denominator. (These integrals are different because photons are bosons and neutrinos are fermions, which must exist in distinct quantum states.) Suppose we lived in a world where there was no parity violation – neutrinos and antineutrinos (assumed to be distinct particles) would each have a left-handed state and a right-handed state. Furthermore assume three families of neutrinos, electron, muon, and tauon. What is the ratio of the energy density in neutrinos to the energy density in photons?

4. The Hubble rate was given in terms of $\sqrt{\rho(t)}$ in class, and $\rho(t)$ was expressed in terms of the temperature T and the number of degrees of freedom N, which was taken to be 43/4. The assumption that there are four neutrino species, not three, changes N to 50/4. What is

the effect on the estimated neutron-proton decoupling temperature and on the n/p ratio at that time?

5. Using the web or any other resources you can find, determine the *nuclear* mass difference between ^{7}Li and ^{7}Be. How much heavier is ^{7}Be? (Note that tabulated masses are often atomic (with the electrons) not those of the bare nuclei. So careful.)
b) Look up the half life of ^{7}Be. What is this number? This is the half life of an *atom* of ^{7}Be, the stuff we deal with on earth.
c) How do terrestrial ^{7}Be atoms decay? There are two possibilities. One is β decay, in which ^{7}Be changes to ^{7}Li by emitting a positron and a neutrino. The other is electron capture: ^{7}Be "eats" one of its atomic electrons, converting it to a neutrino. (Draw pictures of the neutrino-electron-W boson vertices if you are having trouble with this.) Show that each of the reactions conserves both charge and our other charge, lepton number, additively. Which of these two reactions is allowed energetically?
d) In the early universe ^{7}Be is produced in the big bang, but because the temperatures are high, nuclei are fully ionized: we have just bare (charged) nuclei floating about. Can these early-universe ^{7}Be nuclei decay?
e) Electrons "recombine" with atoms only after the universe cools so that the binding energy of atoms is greater than KT, T the temperature. The binding energy of the 1*s* state in hydrogen is about 13.6 eV. From what we did in class, make an estimate of this "recombination" time. This is extremely important in astrophysics: when neutral atoms form, the universe becomes transparent to photons. Thus this is the era to which we can "look back" using the primordial microwave background. Nothing subsequent to this time affects the background radiation (we think).
f) Remembering the Bohr atom, estimate when ^{7}Be decays. Thus what is your guess for the half life of primordial ^{7}Be? Is is longer/shorter than that for terrestrial ^{7}Be?
g) Note that there is a low-lying excited state in ^{7}Li. Do think there might be any signal for primordial ^{7}Be decay?