Physics 554/ Astronomy 510: Nuclear Astrophysics
Problem Set 1 (Due: Monday, October 11)

1. Unless you have a good background in weak interaction and neutrino interactions, please read the two AJP articles placed on the web site. Record on your HW whether you have done this.

2. The energy density due to photons in the early universe is given (with $\hbar$ and c now properly inserted) by an integral over the photon momenta

$$\rho = \frac{2}{\hbar^3} \int \frac{d^3k}{(2\pi)^3} \frac{ck}{e^{ck/KT} - 1} \tag{1}$$

where K is Boltzmann's constant. Complete this integral. (Or work it into a form where you can look it up.) If one considers the corresponding contribution of a neutrino and an antineutrino, the integral is identical apart from the change to

$$e^{ck/KT} + 1 \tag{2}$$

in the denominator. Complete this integral and compare the resulting energy density with that for photons: what is the ratio? (These integrals are different because photons are bosons and neutrinos are fermions, which must exist in distinct quantum states.)

3. The Hubble rate was given in terms of $\sqrt{\rho(t)}$ in class, and $\rho(t)$ was expressed in terms of the temperature T and the number of degrees of freedom N, which was taken to be 43/4. The assumption that there are four neutrino species, not three, changes N to 50/4. What is the impact of this on the time/temperature of deuterium formation? (Give both the qualitative reasons for the direction of the change and provide values of $T_{deuterium}$, taking $\eta = 10^{-9}$, for N=43/4 and N=50/4.) Therefore, estimate the change in the resulting ^{4}He abundance. Compare to the Fig. 4.4 of Kolb and Turner (on reserve).

4. This problem is meant to help those who are not familiar with nuclear masses and/or have not thought about neutrinos before. The former is important to almost every nuclear reaction going on in stars. The latter control the evolution of stars ranging from red giants to core-collapse supernovae.

a) At http://www.nndc.bnl.gov/wallet/walletcontent.shtml, you will find a list of isotopes to click on. If you do so, you will find *atomic* masses listed in terms of a quantity delta, in rows corresponding to isotopes with various A and fixed Z. The atomic mass excess delta is defined as M-A, where for atomic ^{12}C, M-A=M-12=0 by definition. That is, our mass unit is 1/12th the mass of atomic ^{12}C. Now look up the mass of atomic ^{7}Li – the mass of the ^{7}Li nucleus plus the mass of the three bound electrons (you can ignore atomic binding energies, which are small in this case). Using this atomic mass table, calculate the *nuclear* mass difference between ^{7}Li and ^{7}Be. How much heavier is ^{7}Be?

b) In that same table you will find the half life of ^{7}Be listed. What is this number? This is the half life of an *atom* of ^{7}Be, the stuff we deal with on earth.

c) How do terrestrial ^{7}Be atoms decay? There are two possibilities. One is β decay, in which ^{7}Be changes to ^{7}Li by emitting a positron and a neutrino. The other is electron capture: ^{7}Be "eats" one of its atomic electrons, converting it to a neutrino. (Draw pictures of the neutrino-electron-W boson vertices if you are having trouble with this.) Show that each of the reactions conserves both charge and our other charge, lepton number, additively. Which of these two reactions is allowed energetically?

d) In the early universe ^{7}Be is produced in the big bang, but because the temperatures are high, nuclei are fully ionized: we have just bare (charged) nuclei floating about. Can these early-universe ^{7}Be nuclei decay?

e) Electrons "recombine" with atoms only after the universe cools so that the binding energy of atoms is greater than KT, T the temperature. The binding energy of the 1s state in hydrogen is about 13.6 eV. From what we did in class, make a rough estimate of this "recombination" time. This is extremely important in astrophysics: when neutral atoms form, the universe becomes transparent to photons. Thus this is the era to which we can "look back" using the primordial microwave background. Nothing subsequent to this time affects the background radiation (we think).

f) Remembering the Bohr atom, estimate when ^{7}Be decays. Thus what is your guess for the half life of primordial ^{7}Be? Is is longer/shorter than that for terrestrial ^{7}Be?

g) Note that there is a low-lying excited state in ^{7}Li. Do think there might be any signal for primordial ^{7}Be decay?