Jefferson Lab in the 12 GeV Era

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Rolf Ent (Jefferson Lab)
Outline

• Cool Facts about QCD and Nuclei
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• Introduction to Jefferson Lab
• The 6-GeV Science Program – what did we learn?

• Gluons and QCD – The Need for 3D Atomic Structure
• JLab @ 12 GeV – Towards a New Paradigm for Structure
  — Femtography of valence quarks in nucleons and nuclei
  — Role of gluonic excitations in the spectroscopy of light mesons
  — Search for new physics Beyond the Standard Model
• The US-Based Electron-Ion Collider (EIC) – The Role of Gluons
• JLab @ 12 GeV (& EIC) – A Portal to a New Frontier
Did you know that …?

• If an atom was the size of a football field, the (atomic) nucleus would be about the size of the umpire.
• Despite its tiny size, the nucleus accounts for 99.9% of an atom’s mass.
• Protons and neutrons swirl in a heavy atomic nucleus with speeds of up to some \( \frac{3}{4} \) of \( c \). More commonly, their speed is some \( \frac{1}{4} \) the speed of light. The reason is because they are “strong-forced” to reside in a small space.
• Quarks (and gluons) are “confined” to the even smaller space inside protons and neutrons. Because of this, they swirl around with the speed of light.
Cool Facts about QCD and Nuclei

- The strong force is so strong, that you can never find one quark alone (this is called “confinement”).
- When pried even a little apart, quarks experience ten tons of force pulling them together again.
- Quarks and gluons jiggle around at nearly light-speed, and extra gluons and quark/anti-quark pairs pop into existence one moment to disappear the next.
- This flurry of activity, fueled by the energy of the gluons, generates nearly all the mass of protons and neutrons, and thus ultimately of all the matter we see.
- Even the QCD “vacuum” is not truly empty. Long-distance gluonic fluctuations are an integral part. Quarks have small mass themselves, but attain an effective larger mass due to the fact that they attract these gluonic fluctuations around them.
- Nuclear physicists are trying to answer how basic properties like mass, shape, and spin come about from the flood of gluons, quark/anti-quark pairs (the “sea”), and a few ever-present quarks.
A small fraction of the force between quarks and gluons “leaks out” of protons and neutrons, and binds them together to form tiny nuclei. The long-range part of this process can be well described as if protons and neutrons exchange pions.

Nuclear physicists are only now starting to understand how this “leakage” occurs, and how it results in the impressive variety of nuclei found in nature.

A nucleus consisting of some 100 protons and 150 neutrons can be the same size as one with 3 protons and 8 neutrons.

Despite the variety of nuclei found in nature, we believe we miss quite some more. These are necessary to explain the origin of nuclei and the abundance of elements found in the cosmos.
Elementary Particles

• Protons, neutrons and electrons ($p, n, e$) build all the atoms.
• Proton and neutrons make up 99.9% of the visible mass in the universe.
• Dozens of new particles were discovered in the past century.
• Strong interaction: strength can be 100 times the electromagnetic one

**leptons** ($e, \mu, \nu, \ldots$): not involved in strong interaction

**hadrons** [mesons($\pi, K, \ldots$) and baryons($p, n, \ldots$)]: involved in strong interaction
QUANTUM ELECTRODYNAMICS (QED)

\[ \mathcal{L} = \bar{\psi} \left( i \partial^\mu \gamma_\mu - m \right) \psi + e \bar{\psi} \gamma^\mu \psi A_\mu - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} \]

where \[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

• \( \mathcal{L} \) unchanged under \( U(1) \) local gauge transformation

\[ \psi(x) \rightarrow e^{i\alpha(x)} \psi(x) \quad A_\mu(x) \rightarrow A_\mu(x) + \frac{1}{e} \partial_\mu \alpha(x) \]

\[ e \bar{\psi} \gamma^\mu \psi A_\mu \]

is interaction between electron and photon
Gauge theories are plagued by infinities

These can be absorbed in a well-defined way into the “renormalized” (measurable) couplings and masses

- renormalization -> “running couplings”

\[
\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log \left( \frac{Q^2}{\mu^2} \right)}
\]
ELECTROWEAK THEORY

- one of the biggest achievements in the 20th century: electromagnetic and weak interactions unified based on gauge group \( SU(2) \times U(1) \) (Glashow, Salam, Weinberg)
- cornerstone of the Standard Model

\[
\mathcal{L}_{\text{int}} = -\frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i \gamma^\mu (1 - \gamma^5) (T^+ W^\mu_\mu + T^- W^-_\mu) \psi_i
\]

charged currents

\[-e \sum_i q_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu\]

electromagnetic interaction

\[-\frac{g}{2 \cos \theta_W} \sum_i \bar{\psi}_i \gamma^\mu (g^i_V - g^i_A \gamma^5) \psi_i Z_\mu\]

weak neutral current
A FAMOUS EXAMPLE OF A RUNNING COUPLING

\[ \alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi} (33 - 2n_f) \log \left( \frac{Q^2}{\mu^2} \right)} \]

“Confinement”

“Asymptotic Freedom”

Gross, Wilczek; Politzer

QCD
GLUONS AND QCD

• QCD is the fundamental theory that describes structure and interactions in nuclear matter.
• Without gluons there are no protons, no neutrons, and no atomic nuclei.
• Gluons dominate the structure of the QCD vacuum.

\[
L_{QCD} = \sum_{j=u,d,s,...} \bar{q}_j [i \gamma^\mu D_\mu - m_j] q_j - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}
\]

\[
D_\mu = \partial_\mu + ig^{1/2} \lambda^a A_\mu^a, \quad G^{a\mu\nu} = \partial_\mu A_\nu + \partial_\nu A_\mu + igf^{abc} A_\mu^b A_\nu^c
\]

• Facts:
  – Unique aspect of QCD is the self interaction of the gluons.
  – The essential features of QCD - asymptotic freedom and (the emergent features) dynamical chiral symmetry breaking and color confinement - are all driven by gluons!
  – Mass from massless gluons and nearly massless quarks
    • Most of the mass of the visible universe emerges from quark-gluon interactions
    • The Higgs mechanism has almost no role here

Emergent mass of the visible universe
QUANTUM CHROMODYNAMICS (QCD)

Gluons mediate the strong (color) force, just like photons mediate the electromagnetic force, but … gluons interact with themselves … which gives QCD unique properties

\[
L_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu - m_j) q_j
\]

\[
F_{\mu\nu}^a = \delta_\mu A_\nu^a - \delta_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c
\]

\[
D_\mu = \delta_\mu + ig A_\mu^a \frac{\lambda_a}{2}
\]

**QCD Lagrangian: quarks and gluons**
Nuclear Physics Model is an effective (but highly successful!) model using free nucleons and mesons as degrees of freedom.
QCD – impressive tool at high energies

Z production at the LHC
At small distance scales we have a Coulomb-like asymptotically free theory.

At larger distances we have a linear confining potential ~ 1GeV/fm.

Color Field: the field lines are compressed to vortex lines like the magnetic field in a superconductor.
The Quest to Understand the Fundamental Structure of Matter

- **1808**: Atom
- **1874**: Electron
- **1911**: Nucleus
- **1913**: Proton
- **1932**: Neutron
- **1968**: Quarks
  - Up-type: $u$, $u$
  - Down-type: $d$, $d$
- **1978**: Gluon

Diagram showing the evolution of understanding from molecules to atoms, electrons, and eventually to quarks and gluons.
The Quest to Understand the Fundamental Structure of Matter

Discovery

Application

Understanding
New Recipes for Stopping Neutrons

Neutrons are a common byproduct of particle accelerator operations. They can cause radiation damage and single-event upsets causing sensitive data acquisition systems to fail mid-experiment.

Preventing this was a chief goal of nuclear scientists in the design of a shield house for an apparatus at Jefferson Lab, and guided an optimal shielding strategy, with light-weight concrete followed by a boron-rich sheet and finally a layer of lead.

Applications:
- Nuclear power
- Experimental particle physics shielding
- Shielding for special materials
- Shielding around neutron sources (SNS, ESS)
- Medical proton therapy facility shielding
- Shielding for electronics to prevent “single event failure” (ESA)

Neutron Radiation Sources:
- Particle accelerators
- Nuclear reactors
- Cosmic rays

Applications of Nuclear Physics
Detector Spin-Off Advances Patient Care

Nuclear physics detector technology developed to explore the structure of matter at Jefferson Lab leads to new and advanced tools for better patient care.

**Tools for nuclear physics research:**
photomultiplier tubes, silicon photo multipliers, scintillator and detector electronics

**Tools for better patient care:**

- Compact gamma camera for breast cancer detection
- Hand held gamma camera to guide surgeons
Radiotherapy with a proton accelerator (up to 250 MeV) allows oncologists to design fine-tuned three-dimensional cancer treatment plans.

Fundamental nuclear physics: Bragg peak determines proton energy loss = dose to patient
- Enables higher precision localized treatment
- Has fewer side effects due to reduced stray radiation outside the tumor region
- Nuclear physics technology to enable this includes simulation, beam transport, acceleration, dose monitoring, and more
- Jefferson Lab scientists have been instrumental in the design, construction and treatment plans at nearby Hampton University Proton Therapy Center, and other facilities.
Nuclear physics detector technology incorporates real time dosimetry monitoring into radiotherapy cancer treatment procedures.

- Radiation treatment dose uncertainties can affect tumor control and may increase complications to surrounding normal tissues.
- The current standard of pre-treatment quality assurance measurement does not provide information on actual delivered dose to the tumor and can not predict all clinically relevant patient dose uncertainties.
- **Nuclear scientists at Hampton University and Jefferson Lab developed real-time, in vivo, dosimetry technology.**
- They use plastic scintillating detector (PSD) fiber technology with balloon-type patient organ immobilizers to measure real time dose delivery.

The OARtrac® system, built with detector technologies used in nuclear physics, has been cited as a 2018 R&D 100 Award Winner by R&D Magazine.
The Quest to Understand the Fundamental Structure of Matter

Discovery

Application

Understanding
From 3D atomic structure to the quantum world

- **Atomic structure**: dating back to Rutherford’s experiment:
  \[ \alpha + Au \rightarrow \alpha + X \]

  Over 100 years ago

  Atom:
  - J.J. Thomson’s plum-pudding model
  - Rutherford’s Experiment - Data
  - Quantum orbitals

  Discovery:
  - Tiny nucleus - *less than 1 trillionth in volume of an atom*
  - Quantum probability - *the Quantum World!*

- **Localized mass and charge centers** – vast “open” space:

  Molecule:
  - “Water”

  Crystal:
  - Rare-Earth metal

  Nanomaterial:
  - Carbon-based

- Not so in proton structure!
From 3D hadron structure to QCD

- A modern “Rutherford” experiment (about 50 years ago):
  - Nucleon: *The building unit of all atomic nuclei*
  - Prediction
    - If proton “charge cloud”:
    - If proton contains point charges, some of time see:
  - Discovery of quarks!

- Discovery of Quantum Chromodynamics (QCD):
  - Nanometer
  - Femtometer
  - Gluons
  - No still picture!
  - No fixed structure!

- Discovery of Quantum Chromodynamics (QCD):
  - Nucleon: *The building unit of all atomic nuclei*
  - Prediction
    - If proton “charge cloud”:
    - If proton contains point charges, some of time see:
  - Discovery of quarks!
ELECTRON SCATTERING

Electrons as probe of nuclear structure have some distinct advantages over other probes like hadrons or γ-rays:

• The interaction between the electron and the nucleus is known; it is the electromagnetic interaction with the charge $ρ$ and the current $\mathbf{J}$ of the nucleus: $V_{\text{int}} = ρ\phi + \mathbf{J}\cdot\mathbf{A}$, where $\phi$ and $\mathbf{A}$ are the scalar and vector potentials generated by the electron.

• The interaction is weak, so that in almost all cases it can be treated in the “one-photon exchange approximation” (OPEA), i.e., two-step processes (two-photon exchange) are small. One exception is charge elastic scattering of the Coulomb field of a heavy-Z nucleus.

• The energy ($ω$) and linear momentum ($\mathbf{q}$) transferred to the nucleus in the scattering process can be varied independently from each other. This is very important, as for a certain $|\mathbf{q}|$ one effectively measures a Fourier component of $ρ$ or $\mathbf{J}$. By varying $|\mathbf{q}|$ all Fourier components can be determined and from these the radial dependence of $ρ$ and $\mathbf{J}$ can be reconstructed.

• Because the photon has no charge, only the $\mathbf{J}\cdot\mathbf{A}$ interaction plays a role, leading to magnetic $M_\lambda$ and electric $E_\lambda$ transitions. In electron scattering, one can also have charge $C_\lambda$ transitions.
Electron scattering has also some disadvantages:

- The interaction is weak, so cross sections are small, but one can use high electron beam currents and thick targets. *At Jefferson Lab, we can measure neutrino-like cross sections in a day or so.*

- Neutrons are less accessible than protons, since they do not have a net electric charge. *Weak interaction can come to the rescue through parity-violating electron scattering.*

- Because electrons are very light particles, they easily emit radiation (so-called Bremsstrahlung). This gives rise to radiative tails, with often large corrections for these processes. *This can pose limits for kinematics, if the radiative correction factor would become too large.*
Electron Scattering Kinematics

**Virtual photon → off-mass shell**

\[ q \mu q^\mu = v^2 - q^2 \neq 0 \]

Define **two invariants**:

1) \[ Q^2 = -q_\mu q^\mu = -(k_\mu - k'_\mu)(k^\mu - k'^\mu) \]
   \[ = -2m_e^2 + 2k_\mu k'^\mu \]
   \[ (m_e \sim 0) \quad = 2k_\mu k'^\mu \]
   \[ (LAB) \quad = 2(EE' - k.k') \]
   \[ = 2EE'(1 - \cos(\Theta)) \]
   \[ = 4EE'\sin^2(\Theta/2) \]
   only assumption: neglecting \( m_e^2 \)!!

2) \[ 2Mv = 2p_\mu q^\mu = Q^2 + W^2 - M^2 \]

**Elastic scattering**

\[ \rightarrow W^2 = M^2 \rightarrow Q^2 = 2M(E-E') \]
Use electron scattering as high-resolution microscope to peer into matter

Extracting the \((e,e')\) cross section

\[ N_N(\text{cm}^{-2}) \]

Scattering probability or cross section

\[
\left\langle \frac{d^3\sigma}{d\Omega_e dp_e} \right\rangle = \frac{\text{Counts}}{N_e N_N \Delta \Omega_e \Delta p_e}
\]
Electron Scattering at Fixed $Q^2$

\[ \frac{d^2\sigma}{d\nu d\Omega} \]

Quark (fictitious)

Elastic

$\frac{Q^2}{2m} + 300\text{MeV}$

Proton

Deep Inelastic

Jefferson Lab
Electron Scattering at Fixed $Q^2$

\[ \frac{d^2\sigma}{d\nu d\Omega} \]

![Graph showing elastic, quasielastic, and deep inelastic scattering](image)

Nucleus

Proton

$\frac{Q^2}{2M}$ $\frac{Q^2}{2m}$ $\frac{Q^2}{2m} + 300\text{MeV}$ $\nu$

$\frac{Q^2}{2m}$ $\frac{Q^2}{2m} + 300\text{MeV}$ $\nu$
ELECTRON SCATTERING OFF POINT PARTICLES

\[ k = (E,k) \quad k' = (E',k') \]

\[ p = (M,0) \quad p' \]

\[ \frac{1}{\rho_2 \rho_1} \left| v_{1}^{\text{lab}} \right| |M|^2 \frac{d^3k'}{(2\pi^2 2E')^3} \frac{d^3p'}{(2\pi^2 2E_p')^3} (2\pi)^4 \delta^4(k + p - k' - p') \]

Working out will render the so-called Mott cross section.
ELECTRON SCATTERING OFF COMPOSITE TARGET

$k = (E, k)$

$k' = (E', k')$

Recall: matrix element $= \langle \Psi^* | O | \Psi \rangle$

Define transition matrix element $T_{fi}$:

$$T_{fi} = \langle f | T | i \rangle$$

$$= \int d^3r' d^3r \, \Psi_{k'}^*(r) \, \Psi_k(r) \frac{\alpha}{|r - r'|} \, \Psi^*(r') \, \Psi(r')$$

$$= \int d^3r' \, d^3R \, e^{iqr} \frac{\alpha}{|R|} \, e^{iqr'} \, |\Psi(r')|^2$$

which separates into two integrals and gives the well-known result:

$$\langle f | T | i \rangle = \frac{4\pi\alpha}{q^2} F(q)$$

where $F(q)$ is the form factor:

$$F(q) = \int d^3r \, e^{iqr} \, |\Psi(r)|^2$$

$$= \int d^3r \, e^{iqr} \, \rho(r) = (4\pi/q) \int dr \, r \, \rho(r) \sin(qr)$$
Amplitude at q:  \[ F(q) = \int d\vec{r} A(\vec{r}) e^{i\vec{q} \cdot \vec{r}} \]

Phase difference:  \[ \Delta \varphi = (\vec{k} - \vec{k}') \cdot \vec{r} = \vec{q} \cdot \vec{r} \]

\[ \Delta \varphi_1 = \vec{k} \cdot \vec{r} \]

\[ \Delta \varphi_2 = -\vec{k}' \cdot \vec{r} \]
Electron-Charge Scattering

Form Factors characterize internal structure of particles

- Elastic cross section:
  \[
  \left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left| F(q^2) \right|^2
  \]

- Form factor:
  \[
  F(q^2) = \int e^{iqx/\hbar} \rho(x) \, d^3x
  \]

The form factor as a Fourier transformation of the charge distribution is a non-relativistic concept.
FORM FACTORS OF NUCLEI AT LOW ENERGY

Elastic scattering \( \rightarrow W^2 = M^2 \rightarrow Q^2 = 2M(E-E') \rightarrow Q^2 \) and \( \nu \) are correlated

\[
\frac{d\sigma}{d\Omega} \quad \text{(and not } \frac{d\sigma}{d\Omega} dE) = \sigma_M F_0^2(q) \]

- For a point charge with charge \( Z \) one has \( F_0(q) = Z \).
- For a charge with a finite size \( F_0(q) \) will be smaller than \( Z \), because different parts of \( \rho(r) \) will give destructive contributions in the integral that constitutes \( F_0(q) \).
- Often one includes the factor \( Z \) in \( \sigma_M \) and not in \( F_0 \), such that \( F_0(0) = 1 \).

\[
F(q) = \frac{4\pi}{Zq} \int \rho(r) \sin(qr) rdr
\]

Scatter from uniform sphere with radius \( R \) at low \( q \): \( \sin(qr) = qr - (1/6)(qr)^3 \)

1\textsuperscript{st} term disappears (charge normalization)

2\textsuperscript{nd} term gives direct \( R_{\text{RMS}} \) measurement (for \( q \) low enough)

At higher \( q \) pattern looks like slit scattering with radius \( R \)
In ‘70s large data set was acquired on elastic electron scattering (mainly from Saclay) over large $Q^2$-range and for variety of nuclei. “Model-independent” analysis provided accurate results on charge distribution well described by mean-field Density-Dependent Hartree-Fock calculations.
ELASTIC SCATTERING FROM A PROTON AT REST

Before

\[ \left( \omega, q \right) \rightarrow \left( m, 0 \right) \]

After

Proton is on-shell \( \Rightarrow \)

\[ (\omega + m)^2 - q^2 = m^2 \]

\[ \omega^2 + 2m\omega + m^2 - q^2 = m^2 \]

\[ \omega = Q^2 / 2m \]
\[ \langle p, s_f | J^\mu | p - q, s_i \rangle = \bar{U}_f \Gamma^\mu U_i \]

Vertex fcn

\[ \Gamma^\mu = \gamma^\mu \]

point proton

structure/anomalous moment
Vertex fcn: \[ \Gamma^\mu = \gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_v}{2m} \kappa F_2(Q^2) \]

Sachs FF’s

\[ \begin{align*}
G_E(Q^2) &= F_1(Q^2) - \tau \kappa F_2(Q^2) \\
G_M(Q^2) &= F_1(Q^2) + \kappa F_2(Q^2)
\end{align*} \]

with \( \tau = \frac{Q^2}{4m^2} \)

\( G_E \) and \( G_M \) are the Fourier transforms of the charge and magnetization densities in the Breit frame.
CROSS SECTION FOR \( EP \) ELASTIC

Recoil factor for scattering from proton with mass

\[
\frac{d\sigma}{d\Omega} = f_{\text{rec}} \sigma_{\text{M}} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau \tan^2 \frac{\theta}{2} G_M^2 \right]
\]

Mott Cross Section for scattering from pointlike particle

\[
\frac{d\sigma}{d\Omega} = f_{\text{rec}} \sigma_{\text{M}} \left[ A + B \tan^2 \frac{\theta}{2} \right]
\]
ELASTIC SCATTERING FROM A MOVING PROTON

Before

\( (\omega, q) \) \( \rightarrow \) \( p \)

After

\( p \) \( \rightarrow \) \( (\omega + E, q + p) \)

\[
(\omega + E)^2 - (q+p)^2 = m^2
\]

\[
\omega^2 + 2E\omega + E^2 - q^2 - 2p \cdot q - p^2 = m^2
\]

\[
Q^2 = 2E\omega - 2p \cdot q
\]

\[
\omega \left(\frac{E}{m}\right) = (Q^2 / 2m) + p \cdot q / m
\]
QUASIELASTIC SCATTERING

For $E \approx m$:

$$\omega \approx \left(\frac{Q^2}{2m}\right) + \frac{\mathbf{p} \cdot \mathbf{q}}{m}$$

If we “quasielastically” scatter from nucleons within nucleus:

Expect peak at:

$$\omega \approx \left(\frac{Q^2}{2m}\right)$$

Broadened by Fermi motion:

$$\mathbf{p} \cdot \mathbf{q} / m$$
QUASIELASTIC ELECTRON SCATTERING

History: Knock a Proton out of a Nucleus

\[ A(e,e'p)B \]

scattering plane

reaction plane

“out-of-plane” angle

Neglecting \( m_e \):
\[ Q^2 \equiv - q_\mu q^\mu = q^2 - \omega^2 = 4ee' \sin^2 \theta/2 \]

Known: \( e \) and \( A \) + Detect: \( e' \) and \( p \) → Infer:

Missing momentum:
\[ p_m = q - p = p_{A-1} \]

Missing mass:
\[ \varepsilon_m = \omega - T_p - T_{A-1} \]
A(E,E’P)B (OR IN GENERAL ANY (E,E’H) REACTION)

Extracting the (e,e’p) cross section

\[
\left\langle \frac{d^6 \sigma}{d\Omega_e d\Omega_p dp_e dp_p} \right\rangle = \frac{\text{Counts}}{N_e N_N \Delta \Omega_e \Delta \Omega_p \Delta p_e \Delta p_p}
\]

\( N_N \) (cm\(^{-2}\))

\((\Delta \Omega_e, \Delta p_e)\)

\((\Delta \Omega_p, \Delta p_p)\)
CROSS SECTION FOR A(E,E'P)B

In One-Photon Exchange Approximation

\[ d\sigma_{\text{lab}} = \frac{1}{\beta} \frac{m_e}{e} \sum_{i f} |M_{fi}|^2 \left[ \frac{m_e}{e'} \frac{d^3 k'}{(2\pi)^3} \right] \left[ \frac{m}{E} \frac{d^3 p}{(2\pi)^3} \right] \times (2\pi)^4 \delta^4 (P + P_{A-1} - Q - P_A) \]

where

\[ M_{fi} = \frac{4\pi\alpha}{Q^2} \left\langle k' \lambda' | j_\mu | k \lambda \right\rangle \left\langle Bp | J^\mu | A \right\rangle \]

Current-Current Interaction
\[
\sum_{if} \left| M_{fi} \right|^2 = \left( \frac{4\pi\alpha}{Q^2} \right)^2 \sum_{if} \left\{ \langle k' \lambda' | j_\mu | k \lambda \rangle \langle k' \lambda' | j_\nu | k \lambda \rangle \right\}^* \sum_{if} \langle Bp | J^\mu | A \rangle^* \langle Bp | J^\nu | A \rangle
\]

\[\eta_{\mu\nu}\]

\[W_{\mu\nu}\]
CROSS SECTION IN TERMS OF TENSORS

\[ \frac{d^6\sigma}{d\Omega_e d\Omega_p dp d\omega} = \sigma_M \eta_{\mu\nu} W^{\mu\nu} \]

- Mott cross section
- Leptonic tensor
- Hadronic tensor
Consider unpolarized case

Lorentz Vectors/Scalars

3 indep. momenta: \( Q, P_i, P \ (P_{A-1} = Q + P_i - P) \)

6 indep. scalars: \( P_i^2, P^2, Q^2, Q \cdot P_i, Q \cdot P, P \cdot P_i \)

\[ = M_A^2 = m^2 \]
\[ W^{\mu\nu} = X_1 g_{\mu\nu} + X_2 q^\mu q^\nu + X_3 p_i^\mu p_i^\nu \]
\[ \quad + X_4 p^\mu p^\nu + X_5 q^\mu p_i^\nu + X_6 p_i^\mu q^\nu \]
\[ \quad + X_7 q^\mu p^\nu + X_8 p^\mu q^\nu + X_9 p^\mu p_i^\nu \]
\[ \quad + X_{10} p_i^\mu p^\nu \]
\[ \quad + (PV \text{ terms like } \varepsilon_{\mu\nu\rho\sigma} q_\rho p_\sigma) \]

\(X_i\) are the response functions
IMPOSE CURRENT CONSERVATION

\[ S^\nu \equiv q_\mu W^{\mu\nu} = 0 \]
\[ T^\mu \equiv q_\nu W^{\mu\nu} = 0 \]

Then \( q_\nu S^\nu = 0, \quad p_\nu S^\nu = 0, \quad p_{i\nu} S^\nu = 0 \)
\[ q_\mu T^\mu = 0, \quad p_\mu T^\mu = 0, \quad p_{i\mu} T^\mu = 0 \]

Get 6 equations in 10 unknowns

4 independent response functions
PUTTING IT ALL TOGETHER …

\[
\left( \frac{d^6 \sigma}{d \Omega_e d \Omega_p d p d \omega} \right)_{LAB} = \frac{pE}{(2\pi)^3} \sigma_M [v_L R_L + v_T R_T \\
+ v_{LT} R_{LT} \cos \varphi_x + v_{TT} R_{TT} \cos 2\varphi_x ]
\]

with

\[
\sigma_M = \frac{\alpha^2 \cos^2 \theta/2}{4e^2 \sin^4 \theta/2}
\]

\[
v_L = \left( \frac{Q^2}{q^2} \right)^2 \quad v_T = \frac{Q^2}{2q^2} + \tan^2 \theta/2
\]

\[
v_{LT} = \frac{Q^2}{2q^2} \quad v_{TT} = \frac{Q^2}{q^2} \sqrt{\frac{Q^2}{q^2}} + \tan^2 \theta/2
\]

4 structure (or response) functions for unpolarized (e,e’h) – this would have been 2 for (e,e’).
INCLUDING ELECTRON AND RECOIL PROTON POLARIZATIONS

Many more structure (or response) functions for fully polarized \((e,e'p)\) (and in general \((e,e'h)\) or \((p,2p)\)) case.

\[
\left( \frac{d^6\sigma}{d\Omega_e d\Omega_p dp d\omega} \right)_{LAB} = \frac{pE}{(2\pi)^3} \sigma_M \left[ \nu_L (R_L + R^n_L S_n) + \nu_T (R_T + R^n_T S_n) \right. \\
+ \nu_{LT} \left[ (R_{LT} + R^n_{LT} S_n) \cos \varphi_x + (R^l_{LT} S_l + R^t_{LT} S_t) \sin \varphi_x \right] \\
+ \nu_{TT} \left[ (R_{TT} + R^n_{TT} S_n) \cos 2\varphi_x + (R^l_{TT} S_l + R^t_{TT} S_t) \sin 2\varphi_x \right] \\
+ \hbar \nu_{LT'} \left[ (R_{LT'} + R^n_{LT'} S_n) \sin \varphi_x + (R^l_{LT'} S_l + R^t_{LT'} S_t) \cos \varphi_x \right] \\
+ \hbar \nu_{TT'} (R^l_{TT'} S_l + R^t_{TT'} S_t) \right] \\
\text{with} \quad \nu_{LT'} = \frac{Q^2}{q^2} \tan \theta/2 \quad \nu_{TT'} = \tan \theta/2 \sqrt{\frac{Q^2}{q^2} + \tan^2 \theta/2} \\
\text{and other } \nu'\text{'s defined as before}
Plane Wave Impulse Approximation (PWIA)

\[ q - p = p_{A-1} = p_m = -p_0 \]
THE SPECTRAL FUNCTION

In nonrelativistic PWIA:

\[
\frac{d^6 \sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} S(p_m, \varepsilon_m)
\]

e-p cross section

nuclear spectral function

For bound state of recoil system:

\[
\rightarrow \frac{d^5 \sigma}{d\omega d\Omega_e d\Omega_p} = K' \sigma_{ep} |\Phi(p_m)|^2
\]
THE SPECTRAL FUNCTION, CONT’D.

\[ S(\vec{p}_0, E_0) = \sum_f \left| \left\langle B_f \left| a(\vec{p}_0)\right| A \right\rangle \right|^2 \delta(E_0 - \varepsilon_m) \]

where \( \vec{p}_0 = -\vec{p}_m = \) initial momentum
\[ E_0 = E - \omega = \) initial energy
\[ \varepsilon_m = \) missing energy

Note: \( S \) is not an observable!
\[ p_m = E_e - E_{e'} - p = q - p \]

\[ E_m = \nu - T_p - T_A - 1 = E_{sep} + E_{exc} \]
WHERE DOES THE “MISSING” STRENGTH GO?

One possibility:

Detected

populates high $\varepsilon_m$

recoils
Proton Momenta in the nucleus

Short-range repulsive core gives rise to high proton momenta

Potential between two nucleons

\( V(r) \)

\( r \text{ [fm]} \)

\( \sim 1 \text{ fm} \)

Similar shapes for few-body nuclei and nuclear matter at high \( k (=p_m) \).

\( \beta = 0.75 \)

\( ^2\text{H} \)

\( ^3\text{He} \)

\( ^4\text{He} \)

0 200 400 600 800 1000

\( p \) (MeV/c)

\( n(k \text{[fm}^3]) \)
DEEP INELASTIC SCATTERING

→ Precision microscope with superfine control

\[ Q^2 \rightarrow \text{Measure of resolution} \]
\[ y \rightarrow \text{Measure of inelasticity} \]
\[ x \rightarrow \text{Measure of momentum fraction of the struck quark in a proton} \]

\[ Q^2 = S \times y \]

Inclusive events: \( e+p/A \rightarrow e'+X \)
Detect only the scattered lepton in the detector

Semi-Inclusive events: \( e+p/A \rightarrow e'+h(\pi,K,p,jet)+X \)
Detect the scattered lepton in coincidence with identified hadrons/jets in the detector

Exclusive events: \( e+p/A \rightarrow e' + p'/A' + h(\pi,K,p,jet) \)
Detect every things including scattered proton/nucleus (or its fragments)
Deep Inelastic Scattering (DIS) and the Parton Model

Parton Model: Feynman; Bjorken, Paschos

Bjorken Limit: $Q^2 \to \infty, \nu \to \infty$

(Infinite Momentum Frame)

$x = \frac{Q^2}{2P \cdot q}$

$(x = \text{momentum fraction})$

$\xi = x$

$F_2(x) = \sum_i e_i^2 xq_i(x)$

Empirically, DIS region is where logarithmic scaling is observed

$Q^2 > 1 \text{ GeV}^2 \text{, } W^2 > 4 \text{ GeV}^2$
History: Structure Function by Different DIS Probes

- \( F_2^{\mu D} \)  \( \mu \) DIS
- \( F_2^{\nu N} x \frac{5}{18} \)  \( \nu \) DIS

The Classic Example
- Extending the scaling found by the venerable SLAC (electron scattering) experiment.
- Confirming the basis of the Quark-Parton Model: the expected 5/18 charge weighting works well.
- Logarithmic Scaling Violations as anticipated from a renormalizable field theory (QCD) are clearly shown, with both muon and neutrino probes.
Elastic electron scattering determines charge and magnetism of nucleon

Approx. sphere with $<r> \approx 0.85$ Fermi

The proton contains quarks, as well as dynamically generated quark-antiquark pairs and gluons.

The proton spin and mass have large contributions from the quark-gluon dynamics.
Proton Viewed in High Energy Electron Scattering: 1 Longitudinal Dimension

- Viewed from boosted frame, length contracted by
  \[ \gamma_{\text{Breit}} = \sqrt{1 + \frac{Q^2}{4M^2}} \]
- Internal motion of the proton’s constituents is slowed down by time dilation – the instantaneous charge distribution of the proton is seen.
- In boosted frame \( x \) is understood as the longitudinal momentum fraction
  - valence quarks: \( 0.1 < x < 1 \)
  - sea quarks: \( x < 0.1 \)

J. Bjorken, SLAC-PUB-0571
March 1969

Lorentz Invariants
- \( E_{\text{CM}}^2 = (p+k)^2 \)
- \( Q^2 = -(k-k')^2 \)
- \( x = Q^2/(2p \cdot q) \)
High Energy Electron Scattering

Snapshots where $0 < x < 1$ is the shutter exposure time

$x \approx 10^{-4}$
Probe non-linear dynamics
short exposure time

$x \approx 10^{-2}$
Probe rad. dominated
medium exposure time

$x \approx 0.3$
Probe valence quarks
long exposure time

R. Milner

Shutter speed:
- Freeze action: 1/1000, 1/500, 1/250
- Hand hold: 1/125, 1/60
- Movement blur - tripod needed: 1/30, 1/15, 1/8, 1/4, 1/2, 1, 2, 4, 8
PARTON DISTRIBUTION FUNCTIONS (PDFS)

PDF $q(x)$: probability that a quark (or gluon) has fraction $x$ of proton’s momentum

proton: $uud + u\bar{u} + d\bar{d} + \ldots$
HERA collider at DESY (Germany)

Gives tremendous reach in measurements of $F_2$ structure functions. Result: the gluons start running wild…
What about the Nucleus?

$Q^2 = 1 \text{ GeV}^2$

- $1/Q \sim \text{spatial resolution}$
- $g \sim \text{gluons}$
- $S \sim \text{quark sea}$
- $u, d \sim \text{up, down valence quarks}$

R. Yoshida  
C. Gwenlan
High energy lore: gluon splits in quark and antiquark pair & $m_u \sim m_d$: $\bar{d} = \bar{u}$

Low energy: Neutron has no charge, but does have a charge distribution: $n = p + \pi^-$, $n = ddu$, duud$\bar{u}$

Even at high energies … The low-energy signatures persist
QCD

Asymptotic Freedom

Small Distance
High Energy

Perturbative QCD
High Energy Scattering

Gluon Jets
Observed

Confinement

Large Distance
Low Energy

Strong QCD

no signature of gluons???

Hadron Spectrum
The low- and high-energy side of nuclei

The Low Energy View of Nuclear Matter
- nucleus = protons + neutrons
- nucleon ↔ quark model
- (valence) quark model ↔ QCD

The High Energy View of Nuclear Matter
The visible Universe is generated by quarks, but dominated by gluons!
But what influence does this have on hadron structure?

Remove factor 20
The strange quark is at the boundary - both emergent-mass and Higgs-mass generation mechanisms are important.

Emergent mass of the visible universe
The Structure of the Proton

Naïve Quark Model: \( \text{proton} = \text{uud} \) (valence quarks)

QCD: \( \text{proton} = \text{uud} + \text{u} \overline{u} + \text{d} \overline{d} + \text{s} \overline{s} + \ldots \)

The proton sea has a non-trivial structure: \( \overline{u} \neq \overline{d} \) & gluons are abundant

- The proton is **far more** than just its up + up + down (valence) quark structure
- Gluon \( \neq \) photon: Radiates and recombines:
Nuclear Femtography – Subatomic Matter is Unique

- Localized mass and charge centers – vast “open” space:
  
  **Molecule:**
  
  **Crystall:**
  
  **Nanomaterial:**

- Interactions and structure are mixed up in nuclear matter: Nuclear matter is made of quarks that are bound by gluons that also bind themselves. Unlike with the more familiar atomic and molecular matter, the interactions and structures are inextricably mixed up, and the observed properties of nucleons and nuclei, such as mass & spin, emerge out of this complex system.

- Not so in proton structure!
In other sciences, imaging the physical systems under study has been key to gaining new understanding.

Structure mapped in terms of:

- \( b_T \) = transverse position
- \( k_T \) = transverse momentum