Lattice QCD at non-zero temperature and density

Frithjof Karsch
Bielefeld University & Brookhaven National Laboratory

- QCD in a nutshell, non-perturbative physics, lattice-regularized QCD, Monte Carlo simulations
- the phase diagram on strongly interacting matter, chiral symmetry restoration, the equation of state
- finite density QCD, cumulants of conserved charge fluctuations, thermal masses & transport properties
Phases of strong-interaction matter

**Chiral phase transition** (crossover)

**Phase transitions** are related to the spontaneous breaking/restoration of global symmetries

**Phase structure:**

Order parameter:

\[
\frac{\langle \bar{\psi} \psi \rangle_l}{T^3} = \frac{1}{VT^3} \frac{1}{4} \frac{\partial \ln Z}{\partial m_l/T}
\]

Chiral susceptibility:

\[
\frac{\chi_l}{T^2} = \frac{\partial \langle \bar{\psi} \psi \rangle_l/T^3}{\partial m_l/T}
\]
Symmetries of QCD

\[ \mathcal{L}_E = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi}_{j,a} \left( \sum_{\nu=0}^{3} \gamma_{\nu} \left( \partial_{\nu} - \frac{ig}{2} A_{\nu}^a \lambda^a \right) + m_j \right)^{a,b} \psi_{j,b} \]

- symmetries of QCD: \( U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f) \)

- chiral decomposition: \( \psi \equiv (\psi_1, \ldots \psi_{n_f}) = \psi_L + \psi_R \)

\[ P_\epsilon = \frac{1}{2} (1 + \epsilon \gamma_5) , \ \epsilon = \pm 1 \]
\[ P_\epsilon^2 = P_\epsilon , \ P_+ P_- = 0 \]
\[ \psi_L = P_+ \psi , \ \psi_R = P_- \psi \]
\[ \bar{\psi}_L = \bar{\psi} P_- , \ \bar{\psi}_R = \bar{\psi} P_+ \]

\[ \mathcal{L}_F \sim \bar{\psi}_L \slashed{D}_\mu \psi_L + \bar{\psi}_R \slashed{D}_\mu \psi_R - m_q (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \]

\( U_V(1) : \) baryon number \( \psi^\Theta = e^{i\Theta} \psi , \ \bar{\psi}^\Theta = \bar{\psi} e^{-i\Theta} \)

\( U_A(1) : \) axial symmetry \( \psi^\Theta = e^{i\Theta \gamma_5} \psi , \ \bar{\psi}^\Theta = \bar{\psi} e^{i\Theta \gamma_5} \)

\( SU_{L/R}(n_f) : \) flavor symmetry \( G_\epsilon \equiv P_{-\epsilon} \cdot 1 + P_\epsilon U_\epsilon , \ U_\epsilon \in U(n_f) \)

\[ G \equiv G_+ (U_+) G_- (U_-) \]
\[ \psi' = G \psi , \ \bar{\psi}' = \bar{\psi} G^\dagger \]
Chiral phase transition

Which symmetry is restored?

\[ U_L(n_f) \times U_R(n_f) \Leftrightarrow U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f) \]

exact: baryon number conservation

axial anomaly

\[ n_f = 2 \ (u, d) : O(4) \]

\[ n_f = 2 \] :

- standard scenario: \( U_A(1) \) remains broken, chiral limit controlled by \( O(4) \)
- alternative scenario: \( U_A(1) \) "effectively" restored, first order transition possible

Chiral symmetry breaking and restoration

staggered (or Kogut-Susskind) fermions do have a global 
U(1)xU(1) symmetry (remnant of the chiral SU(nf)xSU(nf))

\( U(1) \times U(1) \): independent phase transformations on 
even and odd sites of the lattice

\[ \begin{align*}
\psi'_e &= e^{i\theta_1} \psi_e , \quad \bar{\psi}'_e = e^{-i\theta_2} \bar{\psi}_e \\
\psi'_o &= e^{i\theta_2} \psi_o , \quad \bar{\psi}'_o = e^{-i\theta_1} \bar{\psi}_o
\end{align*} \]

one parameter, continuous global symmetry

its spontaneous breaking generates one Goldstone pion

\( m_\pi \sim m^2_l \)
Universality and the Chiral Phase Transition

close to the chiral limit thermodynamics in the vicinity of the QCD transition is controlled by a universal O(4) scaling function

\[ \frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, t) = -f_s(t/h^{1/\beta\delta}) - f_r(V, T) \]

critical point:
\[ t \equiv 0, \ h \equiv 0 \]

\[ t \sim \frac{T - T_c}{T_c}, \ h \sim \frac{m_l}{T} \]

\[ M_b \equiv \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = \frac{1}{VT^3} \frac{m_s}{T} \frac{1}{2} \frac{\partial \ln Z}{\partial m_l/T} = h^{1/\delta} f_G(z) \]

\[ \frac{\chi_l}{T^2} = \frac{\partial \langle \bar{\psi}\psi \rangle_l}{T^3} = \frac{\partial m_l}{T} \sim h^{1/\delta - 1} f_\chi(t/h^{1/\beta\delta}), \ z = t/h^{1/\beta\delta} \]

\[ \frac{df_\chi(z)}{dz} = 0 \iff z_{max} \]

- defines pseudo-critical \( T_c(m_l) \)
- scaling: \( \chi_l(m_l)/T^2 \sim m_l^{1/\delta - 1} \)

\[ \alpha = -0.213 \]
\[ \beta = 0.380 \]
\[ \delta = 4.824 \]

\[ (2 - \alpha)/\beta\delta = 1 + 1/\delta \]
Chiral phase transition

Chiral susceptibility

\[ \chi_l(t, h) \sim \frac{\partial M}{\partial h} = h^{1/\delta-1} f_x(t/h^{1/\beta\delta}) + \text{regular} \]

Exploring the chiral limit in (2+1)- and 3-flavor QCD

A. Bazavov et al, arXiv:1701.03548

\[ (\chi_l/T^2)^{\text{max}} \sim m_l^{1/\delta-1} \]

\[ (\chi_l/T^2)^{\text{max}} \sim (m_l - m_c)^{1/\delta-1} \]

\[ m_{\pi}^{\text{crit.}} \sim 3 \text{ GeV} \]

\[ m_{\pi}^{\text{crit.}} \leq 50 \text{ MeV} \]
3-flavor QCD:

<table>
<thead>
<tr>
<th>Action</th>
<th>$N_t$</th>
<th>$m_\pi^c$</th>
<th>Year</th>
</tr>
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<tr>
<td>standard staggered</td>
<td>4</td>
<td>$\sim 290$ MeV</td>
<td>2001</td>
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<tr>
<td>p4 staggered</td>
<td>4</td>
<td>$\sim 67$ MeV</td>
<td>2004</td>
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<tr>
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<td>$\lesssim 50$ MeV</td>
<td>2017$^1$</td>
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<td>stout staggered</td>
<td>4-6</td>
<td>could be zero</td>
<td>2014</td>
</tr>
<tr>
<td>Wilson-clover</td>
<td>6-8</td>
<td>$\sim 300$ MeV</td>
<td>2014</td>
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<td>Wilson-clover</td>
<td>4-10</td>
<td>$\sim 100$ MeV</td>
<td>2016</td>
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<tr>
<td>Wilson-clover</td>
<td>4-10, cont. extrap.</td>
<td>$\lesssim 170$ MeV</td>
<td>2017$^2$</td>
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</tbody>
</table>

1) A. Bazavov et al., arXiv:1701.03548  
2) X.-J. Jin et al., arXiv:1706.01178
2 and (2+1)-flavor QCD: \textbf{O(4) scaling?}

- scaling analysis in (2+1)-flavor QCD with HISQ fermions

\[ \chi_{\text{dis}} / T^2 \sim m_\pi^2 (1/\delta - 1) \]

\[ M = m_s \langle \bar{\psi} \psi \rangle / T^4 \]

magnetic equation of state: \( M = h^{1/\delta} f_G(z) \)

\( m_\pi^{\text{crit}} < 80 \text{MeV} \)

not yet sensitive to O(4) scaling in the chiral limit vs. Z(2) critical behavior at \( m_c > 0 \)

staggered fermions: O(2) instead of O(4) for non-zero cut-off
The QCD crossover transition – extracting the pseudo-critical temperature –

Crossover transition temperature

![Graph showing the crossover transition temperature](image)

**Critical temperature from location of peak in the fluctuation of the chiral condensate (order parameter):**

\[
\chi_l = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial m_l^2} = \chi_l,\text{disc} + \chi_l,\text{con}
\]

**Chiral susceptibility**

- \( T_c = (154 \pm 9) \text{ MeV} \)
- well defined pseudo-critical temperature
- quark mass dependence of susceptibilities consistent with O(4) scaling


**lattice:** \( N_\sigma^3 \cdot N_\tau \)

**temperature:** \( T = 1/N_\tau a \)

consistent with Y. Aoki et al, JHEP 0906 (2009) 088
Symmetries and in-medium properties of hadrons

Which symmetries are restored at $T_c$?

- Thermal hadron correlation functions

Greens functions $G$ of quark-antiquark pair in different quantum number channels $H$, controlled by operators $J$

$$J_H(x) = \bar{q}(x)\Gamma_H q(x)$$

$\Gamma_H = 1, \gamma_5, \gamma_{\mu}, \gamma_{\mu}\gamma_5$

Scalar, pseudo-scalar, vector, axial-vector

$q(\bar{q}) = u(\bar{u}), d(\bar{d}), \ldots \Rightarrow$

$\bar{q}q = \bar{u}u$  flavor singlet

$\bar{q}q = \bar{u}d$  flavor non-singlet

$$G_H(\tau, \vec{x}) = \langle J_H(\tau, \vec{x}) \ J_H^{\dagger}(0, \vec{0}) \rangle \sim e^{-m_H\tau}$$

at $T=0$
Thermal modification of the hadron spectrum

quark propagator: \( \bar{q}(x)q(0) = M_q^{-1}(x,0) \)

\[
G_\pi(x) = \langle \text{Tr} \gamma_5 M_l^{-1}(x,0) \gamma_5 M_l^{-1}(0,x) \rangle 
\]

\[
G_\eta(x) = G_\pi(x) - \langle \text{Tr} \left[ \gamma_5 M_l^{-1}(x,x) \right] \text{Tr} \left[ \gamma_5 M_l^{-1}(0,0) \right] \rangle 
\]

\[
G_\delta(x) = -\langle \text{Tr} M_l^{-1}(x,0) M_l^{-1}(0,x) \rangle 
\]

\[
G_\sigma(x) = G_\delta(x) + \langle \text{Tr} M_l^{-1}(x,x) \text{Tr} M_l^{-1}(0,0) \rangle 
- \langle \text{Tr} M_l^{-1}(x,x) \rangle \langle \text{tr} M_l^{-1}(0,0) \rangle 
\]

hadronic susceptibilities

\[
\chi_\pi = \sum_x G_\pi(x) \equiv \chi_{5,\text{con}} \quad , \quad \chi_\delta = \sum_x G_\delta(x) = \chi_{\text{con}} 
\]

\[
\chi_\eta = \sum_x G_\eta(x) \equiv \chi_{5,\text{con}} - \chi_{5,\text{disc}} 
\]

\[
\chi_\sigma = \sum_x G_\sigma(x) = \chi_{\text{con}} + \chi_{\text{disc}} 
\]
Thermal modification of the hadron spectrum

\[ T < T_c : \] broken chiral symmetry is reflected in the hadron spectrum

\[ T \geq T_c : \] restoration of symmetries is reflected in the (thermal) hadron spectrum

\[ SU(2)_L \times SU(2)_R : (\pi, \sigma), (a_1, \rho) \text{ degenerate} \]

\[ U(1)_A : (\pi, \delta) \text{ degenerate} \]
Symmetry restoration and correlation functions

\[ C_H(z) = \sum_{x,y,z,\tau} G_H(x, y, z, \tau) \]

Thermodynamics with domain wall fermions, hotQCD, arXiv:1205.3535

Chiral flavor symmetry is restored at

\[ T \gtrsim 160 \text{ MeV} \quad m_\pi \approx 200 \text{ MeV} \]

No continuous extrapolation.
Restoration of the axial symmetry

\[ T < T_c : \] broken chiral symmetry is reflected in the hadron spectrum

\[ T \geq T_c : SU(2)_L \times SU(2)_R \text{ restored} \]

\[ \chi_{5,\text{con}} = \chi_{\text{con}} + \chi_{\text{disc}} \]

\[ U(1)_A \text{ restored} \]

\[ \chi_{\text{disc}} = 0 \iff \chi_{\pi}(x) - \chi_{\delta}(x) = 0? \]
\( U(1)_A \) remains broken

The difference of the scalar (\( \delta \)) and pseudo-scalar (\( \pi \)) drops by an order of magnitude but stays non-zero

Above \( T_c \) (but still for \( m>0 \)):

\[
\frac{\chi_{\pi} - \chi_\delta}{T^2} = \frac{\chi_{disc}}{T^2} = \frac{\chi_{5,disc}}{T^2} > 0
\]

Thermodynamics with domain wall fermions

hotQCD, arXiv:1205.3535

Nonetheless, chiral limit remains controversial

S. Aoki et al., PR D86 (2012) 114512
Lattice QCD at non-zero baryon number density \( \mu > 0 \)

**THE PROBLEM in QCD Thermodynamics**

The partition function again:

\[
Z(V, T, \mu) = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} \ e^{\bar{\psi} M(A, m_q, \mu) \psi} \ e^{-S_G}
\]

\[
= \int \mathcal{D}A \ \text{det} M(A, m_q, \mu) \ e^{-S_G}
\]

\( \mu > 0 \)

The fermion determinant – is no longer positive definite

standard simulation techniques fail

\[
\text{det} M(A, m_q, \mu) = e^{i\theta(\mu)} |\text{det} M(A, m_q, \mu)|
\]
Lattice QCD at non-zero baryon number density
– the infamous sign problem –

partition function:

\[ Z(V, T, \mu) = \int \mathcal{D}A \det M(A, m_q, \mu) \ e^{-S_G} \]

staggered fermion matrix:

\[
M(\mu) = m_q \delta_{i,j} + \frac{1}{2} \eta_i \left( \sum_{k=1}^{3} (U_{i,k} \delta_{i,j-k} - U_{i+\hat{k},k}^{\dagger} \delta_{i,j+\hat{k}}) + e^\mu U_{i,0} \delta_{i,j-\hat{0}} - e^{-\mu} U_{i-\hat{0},0}^{\dagger} \delta_{i,j+\hat{0}} \right)
\]

\[
= m_q \cdot 1 + \sum_{i=1}^{3} D_i + D_0(\mu)
\]

\[
= \begin{pmatrix}
  m_q & D_{eo} \\
  D_{oe} & m_q
\end{pmatrix}
\]
Lattice QCD at non-zero baryon number density
– the infamous sign problem –

schematic:

\[
\begin{pmatrix}
  m_q & e^\mu U_{i+\hat{0},0} \\
  -e^{-\mu} U_{i-\hat{0},0}^\dagger & m_q
\end{pmatrix}
\]

\[
\begin{pmatrix}
  m_q & D_{eo} \\
  D_{oe} & m_q
\end{pmatrix}
\]

\[\mu = 0:\quad D \text{ is anti-hermitian}
\]
\[\mu = 0:\quad \text{eigenvalues are purely imaginary}\]
\[\det M \geq 0\]

\[\mu^2 > 0:\quad D \text{ is no-longer anti-hermitian}
\]
\[\mu^2 > 0:\quad \text{eigenvalues are no longer purely imaginary}\]
\[\det M = e^{i\theta} |\det M| \quad \text{SIGN problem!!}\]

\[\mu^2 < 0:\quad D \text{ is anti-hermitian}
\]
\[\mu^2 < 0:\quad \text{eigenvalues are purely imaginary}\]
\[\det M \geq 0\]
Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the QCD pressure:
\[
\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)
\]

\[
\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

cumulants of net-charge fluctuations and correlations:
\[
\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_B,Q,S=0} , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}
\]

the pressure in hadron resonance gas (HRG) models:
\[
\frac{p}{T^4} = \sum_{m \in \text{meson}} \ln Z_m^b(T, V, \mu) + \sum_{m \in \text{baryon}} \ln Z_m^f(T, V, \mu)
\sim e^{-m_H/T} e^{(B\mu_B + S\mu_S + Q\mu_Q)/T}
\]

F. Karsch, NNPSS 2017
Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

\[
\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{BQS}^{i,j,k}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

the simplest case: $\mu_S = \mu_Q = 0$

\[
\frac{P(T, \mu_B)}{T^4} = \frac{P(T, 0)}{T^4} + \frac{\chi_2^B(T)}{2} \left( \frac{\mu_B}{T} \right)^2 + \frac{\chi_4^B(T)}{24} \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O}((\mu_B/T)^6)
\]

An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

HRG vs. QCD:

$\mathcal{O}((\mu_B/T)^4)$: difference is less than 3% at $\mu_B/T = 2$

$\mathcal{O}((\mu_B/T)^6)$: difference is less than 2% at $\mu_B/T = 3$
Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

\[
\frac{\Delta(T, \mu_B)}{T^4} = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left( \frac{\mu_B}{T} \right)^2 \left( 1 + \frac{1}{12} \frac{\chi_4^B}{\chi_2^B} \left( \frac{\mu_B}{T} \right)^2 \right)
\]

\begin{itemize}
  \item variance of net-baryon number distribution
  \item kurtosis*variance
\end{itemize}

fits: A. Bazavov et al. (Bielefeld-BNL-CCNU)
arXiv:1701.04325

data are updated:
Bielefeld-BNL-CCNU preliminary

\begin{itemize}
  \item leading and next-to-leading order corrections agree well with HRG for $T<150$ MeV
  \item already in the crossover region deviations from HRG can reach $\sim 40\%$ for $T\sim 165$ MeV
\end{itemize}
Equation of state of (2+1)-flavor QCD: \( \mu_B / T > 0 \)

\[
\frac{\Delta(T, \mu_B)}{T^4} = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi^B_2}{2} \left( \frac{\mu_B}{T} \right)^2 \left( 1 + \frac{1}{12} \frac{\chi^B_4}{\chi^B_2} \left( \frac{\mu_B}{T} \right)^2 \right) + \frac{1}{720} \frac{\chi^B_6}{\chi^B_2} \left( \frac{\mu_B}{T} \right)^6
\]

The EoS is well controlled for \( \mu_B / T \leq 2 \) or equivalently \( \sqrt{s_{NN}} \geq 20 \text{ GeV} \)
Searching for a critical point at $\mu_B > 0$

– signatures for a critical point: large fluctuations in e.g. the net baryon-number

break-down of Taylor series expansion $\rightarrow$ radius of convergence
Chiral transition, **hadronization** and **freeze-out**

- pseudo-critical temperature \( T_c = 154(9) \text{MeV} \)
- hadronization temperatures \( T_h = 164(3) \text{ MeV} \)
- freeze-out temperatures: \( T_{fo} = 156(3) \text{ MeV} \)
  \( T_{fo} = [164(5) - 168(4)] \text{ MeV} \)

**Where does hadronization set in?**

physics is quite different at lower and upper end of the current error bar on \( T_c \)

probed with net-charge correlations&fluctuations

crossover transition lines:
G. Endrodi et al., arXiv:1102.1356, O. Kaczmarek et al., arXiv:1011.31.30
C. Bonati et al., arXiv:1507.03571, P. Cea et al., arXiv:1403.0821
HRG vs. QCD
net baryon-number fluctuations

\[ \frac{\mu_B}{T} > 0 \]

for simplicity: \( \mu_Q = \mu_S = 0 \)

\[ \chi_2^B(T, \mu_B) = \chi_2^B + \frac{1}{2} \chi_4^B \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{24} \chi_6^B \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O}(\mu_B^6) \]

- agreement between HRG and QCD will start to deteriorate for \( T > 150 \text{ MeV} \)
- net baryon-number fluctuations in QCD always smaller than in HRG for \( T > 150 \text{ MeV} \)
HRG vs. QCD
net baryon-number fluctuations

\[ \frac{\mu_B}{T} > 0 \]
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\[ \chi_2^B(T, \mu_B) = \chi_2^B + \frac{1}{2} \chi_4^B \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{24} \chi_6^B \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O}(\mu_B^6) \]

- agreement between HRG and QCD will start to deteriorate for \( T > 150 \text{ MeV} \)
- net baryon-number fluctuations in QCD always smaller than in HRG for \( T > 150 \text{ MeV} \)

no evidence for enhanced net baryon-number fluctuations for \( T \geq 135 \text{ MeV} , \mu_B \leq 2T \)

no evidence for getting closer to a "critical region"
Taylor expansion of the pressure and critical point

\[
\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left( \frac{\mu_B}{T} \right)^n
\]

for simplicity: \( \mu_Q = \mu_S = 0 \)

estimator for the radius of convergence:

\[
\left( \frac{\mu_B}{T} \right)^{\chi}_{\text{crit},n} \equiv r_n^{\chi} = \sqrt{\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}}
\]

– radius of convergence corresponds to a critical point only, iff

\( \chi_n > 0 \) for all \( n \geq n_0 \)

forces \( P/T^4 \) and \( \chi_n^B(T, \mu_B) \) to be monotonically growing with \( \mu_B/T \)

at \( T_{CP} : \kappa_B \sigma_B^2 = \frac{\chi_4^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} > 1 \)

if not:

– radius of convergence does not determine the critical point

– Taylor expansion can not be used close to the critical point
estimates/constraints on critical point location

01/01/17:
based on ongoing calculations of 6\textsuperscript{th} order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
A. Bazavov et al., arXiv:1701.04325
estimates/constraints on critical point location

01/01/17:
Based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
A. Bazavov et al., arXiv:1701.04325

Strongly disfavored as
\[ \chi_6^B < 0 \]
estimates/constraints on critical point location

not accessible in BES@RHIC collider mode

01/01/17:
based on ongoing calculations of 6\textsuperscript{th} order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
A. Bazavov et al., arXiv:1701.04325

\[ \chi_6^B < 0 \]
Explore the **structure of matter** close to the QCD transition temperature using fluctuation of **conserved charges**

**baryon number, strangeness, electric charge**

### High T: ideal gas

**ideal quark (fermi) gas, m=0**

- **fractional charges**
  - baryon number: \( B = +/- \frac{1}{3} \)
  - electric charge: \( Q = +/- \frac{1}{3}, +/- \frac{2}{3} \)
  - strangeness: \( S = 0, +/- 1 \)

### Low T: HRG

**hadron resonance gas**

- **integer charges**
  - baryon number: \( B = +/- 1 \)
  - electric charge: \( Q = 0 = +/- 1, +/- 2 \)
  - strangeness: \( S = 0, +/- 1, +/- 2, +/- 3 \)
Correlations and Fluctuations of conserved charges

- construct QCD observables that would project onto specific quantum numbers, if QCD = HRG

- obtain fluctuations of quantum numbers and correlations between them from the grand canonical potential (~pressure)

\[
\frac{P}{T^4} = \ln Z(T, V, \mu_B, \mu_Q, \mu_S, \ldots)
\]

charge fluctuations

\[
\chi_n^X = \left. \frac{\partial^n \ln Z(T, V, ..\mu_X..)}{\partial \mu_X^n} \right|_{\mu=0}
\]

\[n = 2: \quad \chi_2^X = \langle X^2 \rangle - \langle X \rangle^2\]

charge correlations:

\[
\chi_{nm}^{XY} = \left. \frac{\partial^{n+m} \ln Z(T, V, ..\mu_X, \mu_Y..)}{\partial \mu_X^n \partial \mu_Y^m} \right|_{\mu=0}
\]

\[n = m = 1: \quad \chi_{11}^{XY} = \langle XY \rangle - \langle X \rangle \langle Y \rangle\]
Net baryon-number fluctuations

ratio of 4\textsuperscript{th} and 2\textsuperscript{nd} order cumulants:

BNL-Bielefeld-CCNU:
Net baryon-number fluctuations

ratio of 4\textsuperscript{th} and 2\textsuperscript{nd} order cumulants:

appearance of fractional charges

BNL-Bielefeld-CCNU:
Net baryon-number fluctuations

ratio of $4^{th}$ and $2^{nd}$ order cumulants:

$$\frac{\chi_4^{B}}{\chi_2^{B}}$$

appearance of fractional charges

transition temperature

$$T = 154(9)\,\text{MeV} \simeq 3 \times 10^{12} \, ^{\circ}\text{C}$$

BNL-Bielefeld-CCNU:

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

PDG-HRG: uses experimentally known hadron spectrum listed by the Particle Data Group
QM-HRG: uses additional hadrons predicted to exist in Quark Model calculations

evidence for experimentally not yet observed strange baryons?
Probing the hadron spectrum using QCD thermodynamics

Lattice QCD

$$P_{tot} = \sum \frac{P_h}{E/m_q}$$

$h$ = all hadrons

**strange baryons**

more strangeness = larger fluct.


Ouark Model

$$\Lambda [\text{GeV}]$$

$$\Sigma [\text{GeV}]$$

experimentally established states

F. Karsch, NNPSS 2017
Probing the hadron spectrum using QCD thermodynamics

Lattice QCD

\[ P_{tot} = \sum_{h=\text{all hadrons}} P_h \]

Quark Model

\( \Lambda_c \) [GeV]

\( \Sigma_C \) [GeV]

M. Padmanath et al., arXiv:1311.4806

Correlations and Fluctuations: HRG vs. LQCD

– construct QCD observables that would project onto specific quantum numbers, if QCD = HRG

E.g.: HRG pressure:

\[
\frac{P}{T^4} = \sum_{m \in \text{mesons}} \ln Z^b_m(T, V, \mu) + \sum_{m \in \text{baryons}} \ln Z^f_m(T, V, \mu)
\]

HRG charmed-charged-baryon density:

\[
\chi_{211}^{BQC} = \sum_{m \in \text{QC-baryons}} \frac{\partial^4 \ln Z^f_m(T, V, \mu)}{\partial \mu_B^2 \partial \mu_Q \partial \mu_C}
\]

sum "knows" about spectrum

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart}
\end{figure}
\end{center}
Evidence for many charmed baryons in thermodynamics

close to Tc charmed baryon fluctuations are about 50% larger than expected in a HRG based on known charmed baryon resonances (PDG-HRG)

all charmed baryons/mesons
charged charmed baryons/mesons
strange charmed baryons/mesons

including resonances predicted in quark model calculations and observed in lattice QCD calculations allows for a HRG model (QM-HRG) description of lattice QCD results on conserved charge fluctuations and correlations

Evidence for many charmed baryons in thermodynamics

close to $T_c$ charmed baryon fluctuations are about 50% larger than expected in a HRG based on known charmed baryon resonances (PDG-HRG)

observation of 5 new charmed baryons by LHCb arXiv:1703.04639

all charmed baryons

charged charmed baryons/mesons

strange charmed baryons/mesons

including resonances predicted in quark model calculations and observed in lattice QCD calculations allows for a HRG model (QM-HRG) description of lattice QCD results on conserved charge fluctuations and correlations

Thank you for your attention and the many interested/interesting questions you asked during the lectures and the breaks.