Low-Energy Nuclear Experiments
Lecture 3: ‘Probing’ Wavefunctions

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The (Third) Plan

• Investigating level schemes
  • Decay spectroscopy

• Details of nuclear wavefunctions
  • Single particle ‘occupancies’ and spectroscopy with nuclear reactions
  • Excited state lifetimes and transition probabilities

• Example – planning an experiment
  • What, where, why?
Level schemes – collective vs. single particle

Level Schemes Contain Structural Information

Collective Rotation

Single Particle Alignment
• The majority of nuclides on the chart decay via $\beta^+$ or $\beta^-$ decay
  o $n \rightarrow p + \beta^- + \nu_e$
  o $p \rightarrow n + \beta^+ + \bar{\nu}_e$

• We can consider $\beta$-decay (and other decays) as a tool to populate excited states in daughter nuclei, but with a unique selectivity
**Beta-Delayed Gamma Spectroscopy**

- Gamma rays following decay events provide information on low-level structure of daughter nuclei.

**Isomeric Decay**

- Depending on the production mechanism, nuclei may be produced in long-lived excited states (isomeric states).
- A TAC for implantation-gamma provides the possibility for isomer lifetime determination, if you look for gammas following an implantation.
The use of highly-segmented detectors (usually Si) allows temporal and spatial correlations between implanted nuclei, and their subsequent decays → detect the implant and the decay to obtain half-lives and information on levels in the daughter relative to the parent ground state.
β-decay spectroscopy set-up: NSCL
A. Look at the gamma-rays in coincidence with the nucleus of interest ($^{56}$Sc) implantations – by fitting half-lives of the isomer, and through gamma-gamma correlations, build up a level scheme, and can get relative spin-parities for the states in $^{56}$Sc.

HLC et al., PRC 82, 014311 (2010).
Looking at gamma-gated half-lives provides information on parent. In this case, two distinct lifetimes indicates two beta-decaying states in $^{56}$Sc.

<table>
<thead>
<tr>
<th>$E_{\gamma}$ (keV)</th>
<th>$T_{1/2}$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>591.6 ± 0.3</td>
<td>78 ± 20</td>
</tr>
<tr>
<td>689.5 ± 0.3</td>
<td>74 ± 9</td>
</tr>
<tr>
<td>751.0 ± 0.4</td>
<td>28 ± 5</td>
</tr>
<tr>
<td>1128.7 ± 0.3</td>
<td>57 ± 6</td>
</tr>
<tr>
<td>1160.5 ± 0.3</td>
<td>72 ± 8</td>
</tr>
<tr>
<td>1203.6 ± 0.3</td>
<td>71 ± 19</td>
</tr>
<tr>
<td>1467.3 ± 0.4</td>
<td>103 ± 20</td>
</tr>
<tr>
<td>1495.0 ± 0.3</td>
<td>141 ± 42</td>
</tr>
<tr>
<td>1711.8 ± 0.5</td>
<td>26 ± 11</td>
</tr>
</tbody>
</table>

By looking at decays and correlating with $^{56}$Sc implants, see gamma-rays in $^{56}$Ti – build up level scheme.

$\beta$-decay spectroscopy: complex example
D. Gate on implantations that came in coincidence with isomer gamma-rays and look at half-life → determine which state the isomer populates, and fix the spin/parity.
Alpha decay

- $\alpha$ decay occurs only in heavier systems on the nuclear chart.
- Alpha decay however probes different aspects of the nuclear forces.

- Different selectivity in the process --> favour low $L$ alpha emission.
Efficiency is critical!! Gas-filled separators ‘collect’ charge states, high efficiency separation, and Si box-type arrays provide high efficiency for detecting residues.
Spectroscopy from element 115

Probing wavefunctions
Beyond excitation energies and spins?

Can we probe the details of the wavefunction ‘directly’?

Is there a way to tell where the particles are in terms of single-particle states (even within a specific model)?

- Protons
  - $d_{3/2}$
  - $s_{1/2}$
  - $d_{5/2}$

- Neutrons
  - $p_{3/2}$
  - $f_{7/2}$
  - $d_{3/2}$
  - $s_{1/2}$
  - $d_{5/2}$

$^{47}\text{Ca}$

Excitation energies:
- 2014 keV
- 3562 keV
- 3999 keV
- 437 keV
- 4403 keV
- 4811 keV

Additional energies:
- 2578 keV
- 2599 keV
- 2875 keV
- 3562 keV
- 3999 keV
- 437 keV
- 4403 keV
- 4811 keV
- 564 keV
- 585 keV
- 862 keV
- 47Ca

0 keV
Direct nucleon removal (or addition)

- Information regarding the ‘occupancy’ of single-particle states can be investigated within a model framework.
- Two energy regimes → low-energy transfer experiments and intermediate energy knockout.

\[
\begin{array}{c}
\text{A}Z_N \\
\hline
\text{8} \\
\hline
\text{d}_{5/2} \\
\text{d}_{3/2} \\
\text{s}_{1/2} \\
\text{f}_{7/2} \\
\text{p}_{3/2}
\end{array}
\quad \rightarrow \quad
\begin{array}{c}
\text{A}^{-1}Z_{N-1} \\
\hline
\text{8} \\
\hline
\text{d}_{5/2} \\
\text{d}_{3/2} \\
\text{s}_{1/2} \\
\text{f}_{7/2} \\
\text{p}_{3/2}
\end{array}
\]
Direct nucleon removal (or addition)

- Information regarding the ‘occupancy’ of single-particle states can be investigated within a model framework.
- Two energy regimes --> low-energy transfer experiments and intermediate energy knockout.

\[
\begin{align*}
\text{AZ}_N & \hspace{1cm} 28 \hspace{1cm} 20 \hspace{1cm} 8 \\
& \quad \text{d}^{5/2} \quad \text{d}^{3/2} \quad \text{s}^{1/2} \\
\text{A}^{-1}\text{Z}_{N-1} & \hspace{1cm} 28 \hspace{1cm} 20 \hspace{1cm} 8 \\
& \quad \text{d}^{5/2} \\
\end{align*}
\]

\[
\begin{align*}
& \quad \text{p}^{3/2} \\
& \quad \text{f}^{7/2} \\
\end{align*}
\]
Direct nucleon removal (or addition)

- Information regarding the ‘occupancy’ of single-particle states can be investigated within a model framework
- Two energy regimes --> low-energy transfer experiments and intermediate energy knockout

\[
\begin{align*}
\text{Before:} & \quad A^Z_N \\
\text{After:} & \quad A^{-1}^Z_{N-1}
\end{align*}
\]
Direct nucleon removal (or addition)

- Information regarding the ‘occupancy’ of single-particle states can be investigated within a model framework
- Two energy regimes --> low-energy transfer experiments and intermediate energy knockout

$$A Z_N$$

$$A + 1 Z_{N+1}$$
Selectivity of the reaction mechanism

- Knockout / nucleon removal
- Fusion – evaporation
- Transfer
- Deep inelastic
- Scattering (elastic / inelastic)
- Capture
Fusion evaporation vs. direct transfer

- \( A + b = C \rightarrow D + X \)
  - \(^{12}\text{C}(^{18}\text{O},3\text{n})^{27}\text{Si}^\ast\)
- Compound system has NO memory of its formation
- Evaporated particle energies give excitation energies of final states
- Two-body \( A(b,c)D \)
  - \(^{16}\text{O}(d,p)^{17}\text{O}^\ast\)
- Outgoing particle DO retain knowledge of transferred particles
Knockout reaction vs. direct transfer

- A + b = c – Xn - Xp
  - $^9\text{Be}({}^{44}\text{S},-1p1n)^{42}\text{P}^*$
  - Momentum distribution of recoil reflects orbital momentum transfer

- Two-body A(b,c)D
  - $^{16}\text{O}(d,p)^{17}\text{O}^*$
  - Outgoing particle DO retain knowledge of transferred particles
Transfer reactions

**Single-nucleon**
- [e.g., (d,p), (\(^3\)He,d), (\(\alpha\),t)]
  - Single-particle states

**Two-nucleon**
- [e.g., (t,p), (\(^3\)He,p), (\(\alpha\),d)]
  - Pair transfer (2n, d, etc.)

**Charge exchange**
- [e.g., (p,n), (\(^3\)He,t), (t,\(^3\)He)]
  - Gamow Teller Strengths
  - Isobaric analog states

**Surrogate reactions**
- [e.g., (\(^6\)Li,d), (\(^7\)Li,t), (d,n)]
  - Mimics the analogous particle transfer

**Heavy Ion**
- [e.g., (\(^{13}\)C,\(^{12}\)C), (\(^{12}\)C,\(^{10}\)Be), (\(^{14}\)C,\(^{10}\)C)]
  - Highly selective
  - Exploratory
Transfer reactions: measured quantities

- Momenta and angles of outgoing light particles [or heavy-ion recoils]

Reaction: \(A(b,c)D\)  
[e.g., \(^{208}\text{Pb}^{(3}\text{He},d)^{209}\text{Bi}\)]

\[
\text{BE}_D = M_D + E^*_D = \sqrt{M_c^2 + E_{cm}^2 - 2 \cdot E_{cm} \cdot E'_c} 
\]

\[
E'_c = f(E_c, \theta_c) 
\]

\[
Q = (\text{BE}_c + \text{BE}_D) - (\text{BE}_A + \text{BE}_b) 
\]
Transfer reactions: measured quantities

\[ [Q(g.s) = +2.92 \text{ MeV}] \]

\[ \begin{align*}
48_{\text{Ca}} \text{(d,p)} 49_{\text{Ca}} \\
E_d &= 20.0 \text{ MeV} \\
\theta &= 57.5^\circ
\end{align*} \]

\[ Q = (\text{BE}_C + \text{BE}_D) - (\text{BE}_A + \text{BE}_B) \]
Transfer reactions: measured quantities

Cross sections – Yields as a function of angle
[differential cross section: millibarns per ster radians (mb/sr)]

Rutherford Scattering
[V = Coulomb]
\[
\frac{d\sigma}{d\Omega} = \frac{(zZe^2)^2}{(4\pi\varepsilon_0)^2(4E_km)^2}\sin^4\left(\frac{\theta}{2}\right)
\]

Transfer Reaction
[V = Nuclear + Coulomb]
Cross section vs. incident beam energy
Transfer reactions: extracted quantities

Sensitivity of the differential cross sections to orbital angular momenta ($l$) of transferred nucleon(s)

Fig. 1. Theoretical angular distributions for $(d,p)$ and $(d,\pi)$ reactions for different angular momentum transfers to the initial nucleus.

- $l_n = 0$
- $l_n = 1$
- $l_n = 2$

$\sigma(\theta)$

$\frac{d\sigma}{d\Omega}$

$\theta$ in single-particle transfer [e.g., $(d,p)$]

$l$ if incoming particle is polarized
[analyzing power]

$L$ of pair in two-particle transfer [e.g., $(t,p)$]
Transfer reaction: extracted quantities

Experimental spectroscopic factor

[Relative values are typically reliable (<25%)]
[absolute values can be tricky (>30%)!]

\[ S_{ij} = \frac{d\sigma}{d\Omega} \mid_{\text{Meas}} = g S_{ij} \frac{d\sigma}{d\Omega} \mid_{\text{DWBA}} \]

Statistical factor

Calculated cross section for “pure” single-particle like state

Amount of overlap between initial and final states
Spectroscopic Factor

\(^{18}\text{O}(d,p)^{18}\text{O} \text{ at } 10 \text{ MeV/u}\)

\( l = 0 \)
\( l = 2 \)
Low-energy transfer experiments

Detection systems depend on kinematics of the reaction
--> ‘normal kinematics’ with a light beam on a heavy target – spectrographs can analyze the light outgoing particle
--> ‘inverse kinematics’ with a heavy beam on a light target – detect the light outgoing particle, or analyze the beam-like particle
Nucleon knockout reactions

Intermediate energy beams (> 50 MeV/nucleon)
  - Sudden approximation + eikonal approach for reaction theory

Spectroscopic strengths --> exclusive cross-sections
  - Populated states in A-1 residue provide detailed measure of beam structure

Theoretical cross-section

$$
\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^N C^2 S(j^\pi)\sigma_{sp}(j, S_N + E_x[j^\pi])
$$

Structure theory
Neutron knockout – $^{9}\text{Be}(^{34}\text{Ar},^{33}\text{Ar})X$

- $^{33}\text{Ar}$
  
  - 1358(6) keV
  
  - 1795(7) keV
  
  - 2460(9) keV

Energy (keV)

Counts / 13 keV

<table>
<thead>
<tr>
<th>BR (%)</th>
<th>$\sigma_{\text{exp}}$ (mb)</th>
<th>$C^2S_{\text{exp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^+$</td>
<td>30.2(46)</td>
<td>4.7(9)</td>
</tr>
<tr>
<td>$3/2^+$</td>
<td>20.2(44)</td>
<td>3.2(8)</td>
</tr>
<tr>
<td>$5/2^+$</td>
<td>31.7(31)</td>
<td>4.9(7)</td>
</tr>
<tr>
<td>$(5/2^+)$</td>
<td>17.9(30)</td>
<td>2.8(6)</td>
</tr>
</tbody>
</table>

A. Gade et al., PRC 69, 034311 (2004).
Excited state lifetimes
Lifetimes and transition probabilities

Transition probability for gamma-decay relates strongly to specific nuclear matrix elements --> provide a stringent test of theoretical wavefunctions

Consider the case of the first 2+ states in even-even nuclei

$$\tau_\gamma = 40.81 \times 10^{13} E^{-5} [B(E2) \uparrow/e^2b^2]^{-1}$$

$$B(E2 : J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \langle \psi_f | E2 | \psi_i \rangle^2$$

Lifetimes are of order ps --> how do we measure these lifetimes?
The lifetime of excited states in the range of 10-100s of ps can be measured by populating the state via Coulomb excitation or knock-out reactions, and observing the Doppler-shift of the decay gamma-ray.

Figure: Adapted from K. Starosta
Lifetime in $^{72,74}$Kr

Lifetimes are related to the reduced transition probabilities $B(E2)$, which are an indicator for collectivity in the nuclear structure.

Here, the irregular behaviour for the 4+ and 2+ states suggest a rapid shape evolution in $^{72}$Kr

Coulomb excitation
Collectivity: \( B(E2) \) from excitation probability

Coulomb excitation:
- purely Coulomb interaction causes excitation of the nucleus of interest
- well described interaction, and cross-section relates to transition matrix element, i.e. \( B(E2) \) for \( 0^+ \rightarrow 2^+ \) in even-even nuclei.

\[
\sigma_{\pi\lambda} \approx \left( \frac{Z_{\text{pro}}e^2}{\hbar c} \right)^2 \frac{\pi}{e^2b_{\text{min}}^{2\lambda-2}} B(\pi\lambda, 0 \rightarrow \lambda) \begin{cases} 
\frac{1}{(\lambda - 1)} & \text{for } \lambda \geq 2 \\
2\ln(b_{\text{a}}/b_{\text{min}}) & \text{for } \lambda = 1 
\end{cases}
\]
Pear shaped nuclei and atomic EDM

\[ \langle I' | E_\lambda | I \rangle = \sqrt{(2I' + 1)(2\lambda + 1) / 16\pi I'0\lambda0|I0} Q_\lambda \]

Intermediate-energy Coulex

- In conventional (low-energy) Coulomb excitation, bombarding energies are well below the Coulomb barrier
- At high energies (~100 MeV/A), nuclear contribution can be significant for small impact parameters, but for b > R_{int} Coulomb dominates

- At a given beam velocity, b relates to the scattering angle \( \theta \), so restricting analysis to forward scattering angles ensures ‘safe’ Coulex
Neutron-rich Fe and Cr

And what have I skipped?

- ‘Exotic’ decay modes
  - 1p and 2p decay at the proton dripline
  - Neutron decay --> recent sequential 2n decay at NSCL
- Resonance spectroscopy – properties of unbound states (beyond the proton and neutron driplines)
- Reactions for spectroscopy and more --> deep inelastic reactions, multi-nucleon transfer, charge-exchange, etc.
- And much, much more...
Example: Designing an experiment to access the physics
We read this theory paper...

J.D. Holt, J. Menendez, A. Schwenk, private communication.
Can we inform this physics question?

• Theory tells us there is a difference in spectroscopic factor for removal of neutrons in \(^{50}\text{Ca}\) to states in \(^{49}\text{Ca}\)
  
  o Is this observable? Can we design a measurement to test the different predictions? What could we do? What would our experiment observables be?
  
  o Where could we do this type of experiment? What facility could we use? What type of equipment?
  
  o What exactly would we **measure**? How would we have to interpret the data? Do we need theory to interpret the data?