Fundamental Symmetries: Overview

- Standard Model: Inadequacies
- Experimental Tests of Standard Model and Symmetries
  - Baryon Number Violation: Proton Decay
  - Parity Violation: MOLLER at JLab
  - Charged Lepton Flavor Violation: \( \mu N \rightarrow eN \)
  - Electric Dipole Moment Searches: \( e, \mu, n, p, \) nuclei
  - Precision Test of the Standard Model: Muon g-2
- Summary and Outlook

- My experience: experimentalist, worked on polarized deep-inelastic scattering, muonium hyperfine structure (test of bound state QED), muon g-2, electron EDM searches in polar diatomic molecules, polarized proton-proton scattering with PHENIX collaboration at RHIC - to measure \( \Delta g \) and \( \Delta \bar{u} \) and \( \Delta \bar{d} \), new muon g-2
Standard Model: Inadequacies

• What is origin of the observed matter-antimatter asymmetry?
  • SM prediction off by $>6$ orders of magnitude
• SM doesn’t explain 1/3 relation between quark and lepton charges
• What is the origin of neutrino mass?
• What is dark matter? What is dark energy?
• Can we explain the extreme hierarchy of masses and strengths of forces?
• Why are there 3 families? Can the electroweak and strong forces be unified?

⇒ What about gravity ???

• Is Standard Model a low-energy limit of a more fundamental theory ??
Proton Decay, Grand Unified Theories, and Supersymmetry

• Noether: \( \exists \) conserved quantity for every continuous symmetry of Lagrangian

• Baryon number: conserved by \( U(1)_B \) symmetry in SM, but broken by non-perturbative weak effects ('t Hooft, PRL 37, 8 (1976))

\[ \Rightarrow \text{Proton can annihilate with neutron: } p + n \rightarrow e^+ + \bar{\nu}_\mu, \ p + n \rightarrow \mu^+ + \bar{\nu}_e \]

\[ \Rightarrow \text{SM proton decay rate contains pre-factor } e^{-4\pi \sin^2 \theta_W/\alpha_{\text{QED}}} \approx e^{-4\pi/0.0335}, \]

so \( \Gamma \propto 10^{-163} \text{ s}^{-1} \Leftrightarrow \tau_{\text{proton}} > 10^{150} \text{ years}! \)

• But: baryon number violation \textit{required for creation of matter in universe} (\textit{i.e.} matter-antimatter asymmetry)

• Ultimate end of universe depends on proton stability

• Proton decay predicted in many Grand Unified Theories (GUTs)

• Scale at which forces unify, \( M_G \approx 10^{16} \text{ GeV}, \) well beyond EW scale \( G_F^{-1/2} \approx 250 \text{ GeV} \)

\[ \Rightarrow \text{Proton decay fantastic probe of profound physics, far beyond reach of accelerators} \]
Why unify forces?

⇒ Standard model described by groups $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ with 3 distinct couplings - can this be simplified?

⇒ Even electroweak unification doesn’t predict relative EM and weak couplings

⇒ Why are there 3 generations of fermions? Why large hierarchy of masses? $m_{\text{top}} > 10^5 m_e$

⇒ What is the origin of neutrino mass? Are neutrinos their own anti-particles?

⇒ What is the origin of the matter-antimatter asymmetry in the universe?

⇒ Quarks and lepton charged weak current doublets identical, $
\begin{pmatrix}
\nu_e \\
e
\end{pmatrix}_L, \n\begin{pmatrix}
u_e \\
e
\end{pmatrix}_L$

Are they related at more fundamental level?

⇒ Why is charge quantized? Why is $Q(e) + Q(p) = 0$? Why is $Q(d) = Q(e)/3$? Why not $Q(d) = Q(e)/5$?

⇒ Higgs hierarchy problem: radiative corrections should push Higgs mass to $M_P \approx 10^{19}$ GeV. Explained by SUSY?

⇒ Gravity - not explained. Dark energy, dark matter, also unexplained, ...

⇒ Many of us will measure zero or consistency with SM for many years - but great new physics is almost certainly there, waiting to be discovered
**SU(N) Groups**

- Elements of $SU(N)$ groups are $n \times n$ unitary matrices $U$ with $\det 1 \ (U^\dagger U = 1, \ \det(U)=1)$
- Matrix elements are complex so nominally $2 \times n \times n$ elements; but $U^\dagger U = 1$ implies $n$ constraints on diagonal elements, $n^2 - n$ constraints on off-diagonal, 1 constraint to make $\det(U)=1 \Rightarrow \ n^2 - 1$ independent parameters
- For $SU(2)$ there are three independent parameters: $\alpha, \beta, \gamma$; think of Euler angles
  
  $$U(\alpha, \beta, \gamma) = \begin{pmatrix}
  e^{-i(\alpha+\gamma)/2} \cos \beta/2 & -e^{-i(\alpha-\gamma)/2} \sin \beta/2 \\
  e^{i(\alpha-\gamma)/2} \sin \beta/2 & e^{i(\alpha+\gamma)/2} \cos \beta/2
\end{pmatrix}$$

- Can write $U = e^{iH}$ for $H$ Hermitian ($H = H^\dagger$, $U^\dagger U = (e^{iH})^\dagger(e^{iH}) = e^{i(H-H^\dagger)} = 1$)
- Can pick $n^2 - 1$ Hermitian matrices $G_i$ so any element $U$ of $SU(N)$ can be written as:
  
  $$U = \exp \left( \sum_{i=1}^{n^2-1} i\theta_i G_i \right),$$

- $\theta_i$ are real parameters, $G_i$ are the generators of the group ($n^2 - 1$ of them)
- For $SU(2)$, can pick three Pauli matrices $\sigma_i$ as generators
- Finally: $U = e^G$, $\det(e^G) = e^{\text{Tr}G}$, so $\det(U)=1$ implies generators $G_i$ traceless, Hermitian
- (See G. Kane, Modern Elementary Particle Physics or J.-Q. Chen, Group Representation Theory for Physicists)
Georgi and Glashow, “Unity of All Elementary-Particle Forces”, PRL 32, 438 (1974) propose a minimal $SU(5)$ as a possible GUT (minimal $\iff$ smallest Higgs sector)

Fermions in $\bar{5}$ and $10$ representations (versus SM singlets, doublets, triplets)

$\bar{5} = \begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ e^- \\ -\nu_e \end{pmatrix}_L$, $10 = \begin{pmatrix} 0 & \bar{u}_b & -\bar{u}_g & -u_r & -d_r \\ -\bar{u}_b & 0 & \bar{u}_r & -u_g & -d_g \\ \bar{u}_g & -\bar{u}_r & 0 & -u_b & -d_b \\ u_r & u_g & u_b & 0 & e^+ \\ d_r & d_g & d_b & -e^+ & 0 \end{pmatrix}_L$

$10$ is antisymmetric, 15 particles total, $SU(5)$ gauge bosons enable transitions between multiplet members (like $SU(2)_L$ mixes doublet: $u + W^- \to d$, $e^- + W^+ \to \nu_e$)

$SU(N)$ generators are traceless $\iff$ sum of eigenvalues is 0

Electric charge $Q$ is linear combination of generators from $SU(2)_L$ and $U(1)_Y$: $Q = T_3 + Y/2$

$\Rightarrow$ In $SU(5)$, $Q$ is a (traceless) generator so sum of electric charges in a representation is zero

$\Rightarrow Q(\nu_e) + Q(e^-) + 3Q(\bar{d}) = 0 \Rightarrow Q(\bar{d}) = \frac{1}{3}Q(e^-)!$

$\Rightarrow$ Electric charge of quarks is related to number of flavors, $Q(e^-) \equiv -Q(p)$ atoms neutral, charge quantized!

$\Rightarrow$ Explain a remarkable amount, very appealing to think forces are unified
• What about $SU(N)$ gauge bosons?

• For $SU(5)$ should be $N^2 - 1 = 5^2 - 1 = 24$ bosons, versus $(3^2 - 1) + (2^2 - 1) + 1 = 12$ for SM

• Displayed in matrix form as (see G. Ross, Grand Unified Theories):

\[
V_{SU(5)} = \begin{pmatrix}
gr_{\bar{r}r} - \frac{2}{\sqrt{30}}B & g_{r\bar{g}} & g_{r\bar{b}} & X_1 & Y_1 \\
g_{g\bar{r}} & g_{g\bar{g}} - \frac{2}{\sqrt{30}}B & g_{g\bar{b}} & X_2 & Y_2 \\
g_{b\bar{r}} & g_{b\bar{g}} & g_{b\bar{b}} - \frac{2}{\sqrt{30}}B & X_3 & Y_3 \\
X_1 & X_2 & X_3 & \frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B & W^+ \\
\bar{Y}_1 & \bar{Y}_2 & \bar{Y}_3 & W^- & -\frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B
\end{pmatrix}
\]

• Color group $SU(3)$ operates in first 3 rows and columns, $SU(2)$ on last two

• Twelve new gauge bosons $X_i$, $\bar{X}_i$, $Y_i$, $\bar{Y}_i$, $i = 1, 2, 3$

• New bosons mediate transitions between quarks and leptons
Unification of Forces

- Interaction part of $SU(5)$ Lagrangian (see C. Quigg):

$$
\mathcal{L}_{\text{int}} = -\frac{g_5}{2} G^a_\mu \left( \bar{u} \gamma^\mu \lambda^a u + \bar{d} \gamma^\mu \lambda^a d \right) - \frac{g_5}{2} W^i_\mu \left( \bar{L} \gamma^\mu \tau^i L + \bar{E} \gamma^\mu \tau^i E \right) - \frac{g_5}{2} B^A_\mu \sum_{\text{fermions}} \bar{f} \gamma^\mu Y_f - \frac{g_5}{\sqrt{2}} \left[ X^-_{\mu,\alpha} \left( \bar{d}^\alpha_R \gamma^\mu e^c_R + \bar{d}^\alpha_L \gamma^\mu e^c_L + \epsilon_{\alpha\beta\gamma} \bar{u}^c_L \gamma^\mu u^\beta_L \right) + H.C. \right] + \frac{g_5}{\sqrt{2}} \left[ Y^-_{\mu,\alpha} \left( \bar{d} d^\alpha_R \gamma^\mu \nu^c_R + \bar{u}^\alpha_L \gamma^\mu e^c_L + \epsilon_{\alpha\beta\gamma} \bar{u}^c_L \gamma^\mu \nu^\beta_R \right) + H.C. \right]
$$

- Doublets $L$ given by $L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L$, $L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$

- First three terms are from SM, though now with single coupling $g_5$

- Color $SU(3)$ $a = 1...8$, $SU(2)$ $i = 1, 2, 3$, $\alpha = r, g, b$, $c$ indicates anti-particle

- $X$ bosons (electric charge $-4/3$) and $Y$ (electric charge $-1/3$) mediate quarks $\leftrightarrow$ leptons

- $X, Y$ boson exchange will allow baryon number violation $\Rightarrow$ proton decay
Proton decay in $SU(5)$

- See possible decay mode: $p \rightarrow e^+ + \pi^0$
- What about proton lifetime? Estimate similar to $\tau_\mu$

$$
\tau_\mu = \left( \frac{M_W}{m_\mu g_w} \right)^4 \frac{12 \hbar (8\pi)^3}{m_\mu c^2 m_\mu^5} M_W^4 \propto \frac{M_W^4}{m_\mu^5}
$$

So expect $\tau_p \propto \frac{M_X^4}{m_p^5}$

- What do we use for new gauge boson masses $M_X, M_Y$?
• Coupling strength depends on momentum transfer of virtual gauge bosons
• EM force increases at smaller length scale \( (\alpha_1) \)
• Weak and strong force weaken at higher energy scales \( (\alpha_2, \alpha_3) \)
• Quickly review origin of this behavior
Unification of Forces (see Kane, Quigg, ...)
Unification of Forces (see Kane, Quigg, ...)

\[ I(q^2) = \frac{\alpha_0}{3\pi} \int_{M^2}^{\infty} \frac{dp^2}{p^2} - \frac{2\alpha_0}{\pi} \int_0^1 dx x(1-x) \ln \left[ 1 - \frac{q^2 x (1-x)}{M^2} \right] \]

\approx \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{M^2} - \frac{\alpha_0}{3\pi} \ln \frac{-q^2}{M^2}; \text{ for large } \frac{q^2}{M^2}, \text{ cutoff } \Lambda, \quad \alpha_0 \equiv \frac{e_0^2}{4\pi} \]

\[ = \frac{\alpha_0}{3\pi} \frac{\Lambda^2}{(-q^2)} \]

- So, amplitude describing diagram below is proportional to:

\[ \mathcal{M} \propto e_0 \left[ 1 - \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)} \right] \left[ \bar{u}(k') \gamma^\mu u(k) \right] \epsilon_\mu \]
Unification of Forces (see Kane, Quigg, ...)

- Can keep adding more loops

\[
M \approx e_0^2 \left[ 1 - \left( \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)} \right) + \left( \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)} \right)^2 + \ldots \right] \left( [\bar{u}(k')\gamma^\mu u(k)] [\bar{u}(p')\gamma_\mu u(p)] \right)
\]

\[
M \approx e_0^2 \left[ 1 - \epsilon_0 + \epsilon_0^2 - \epsilon_0^3 + \ldots \right] \left( [\bar{u}(k')\gamma^\mu u(k)] [\bar{u}(p')\gamma_\mu u(p)] \right)
\]

\[
M \approx \left[ \frac{e_0^2}{1 + \epsilon_0} \right] \left[ \bar{u}(k')\gamma^\mu u(k) \right] \times \left[ \bar{u}(p')\gamma_\mu u(p) \right], \text{ where } \epsilon_0 = \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)}
\]

\[
M \approx \left[ \frac{e_0^2}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)}} \right] \left[ \bar{u}(k')\gamma^\mu u(k) \right] \times \left[ \bar{u}(p')\gamma_\mu u(p) \right]
\]
Unification of Forces (see Kane, Quigg, ...)

- Include higher order diagrams by replacing “bare” $e_0$ with $q^2$-dependent coupling:

$$e_0^2 \Rightarrow e^2(q^2) = \frac{e_0^2}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{(-q^2)}}$$

- So coupling $\alpha$ measured at $\mu^2$ includes all loops, given by:

$$\alpha(\mu^2) = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{\mu^2}}$$

- Use measurement of $\alpha(\mu^2)$ at $\mu^2$ to determine $\alpha$ at any other momentum transfer $q^2$:

$$\alpha(q^2) = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \left[ \frac{\Lambda^2}{-q^2} \right]}$$

$$= \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \left[ \frac{\Lambda^2}{\mu^2} \cdot \frac{\mu^2}{-q^2} \right]}$$

$$\Rightarrow \alpha(q^2) = \frac{\alpha(\mu^2)}{1 + \frac{\alpha(\mu)^2}{3\pi} \ln \left[ \frac{\mu^2}{-q^2} \right]}$$

- No more dependence on cut-off $\Lambda$ or unmeasurable $\alpha_0$, just depends on one finite, measured value $\alpha(\mu^2)$. Also see $\alpha(q^2)$ increases as momentum transfer increases.
Unification of Forces (see Kane, Quigg, ...)

- Result above for $e^\pm$ in loops: need to include $\mu$, $\tau$, and quarks
- Should include contributions from all charged particles for which $|q^2| >> m^2$
- Multiply coefficient of correction by: $n_l + 3\left(\frac{4}{9}\right)n_u + 3\left(\frac{1}{9}\right)n_d$
- $n_l$ is number of leptons, $n_u$ is number of quarks with $Q = 2/3$, factor 3 is for three colors
  ⇒ Contribution depends on charge$^2$ since couple to $\gamma$ on each side of loop
  ⇒ Each family contributes factor 8/3
  ⇒ Need to include loops with $W^\pm$ when $|q^2| >> M_W^2$
  ⇒ How much stronger is $\alpha$ at $q^2 = M_W^2$ versus $\alpha(4M_e)^2 \approx 1/137$?
  ⇒ Number particles in loops $n_l = n_d = 3$, $n_u = 2$ gives factor 20/3 ($n_u = 2$ since $M_{top} > M_W$, no contribution from top)

$$\frac{\alpha(M_W^2)}{\alpha(4M_e^2)} \approx \frac{1}{1 - \frac{20/3}{3\pi \times 137} \ln \left[ \frac{M_W^2}{4M_e^2} \right]} \approx 1.066$$

$$\Rightarrow \alpha(M_W^2) \approx \frac{1}{128}$$

⇒ Running of coupling sensitive to particle content
Unification of Forces (see Kane, Quigg, ...)

- For QCD, similar effects but: no lepton contribution, quark color charges are the same, gluons self-couple.

- For quark loops $\alpha(\mu^2)/3\pi \Rightarrow \alpha_3(\mu^2)/6\pi$ for each flavor.

- Gluon loops lead to contribution with opposite sign, larger in magnitude.

- Gluon loops lead to anti-screening, weakening with $q^2$, asymptotic freedon:

\[
\frac{\alpha(\mu^2)}{3\pi} \Rightarrow \frac{\alpha_3(\mu^2)}{4\pi} \left(\frac{2}{3}n_f - 11\right)
\]

\[
\alpha_3(q^2) = \frac{\alpha_3(\mu^2)}{1 + \frac{\alpha_3(\mu^2)}{12\pi} (33 - 2n_f) \ln \left[\frac{-q^2}{\mu^2}\right]}
\]

- Since $(33 - n_f) = (33 - 2 \times 6) > 0$, QCD coupling decreases as momentum transfer increases $\Rightarrow$ asymptotic freedom.

- At very large $q^2$, $\alpha_3(q^2)$ independent of $\alpha_3(\mu^2)$.

- For small $q^2$, denominator approach zero as $q^2 \Rightarrow \Lambda_{QCD}$

\[
\Lambda_{QCD} \approx \mu \exp \left(-\frac{6\pi}{(33 - 2n_f)\alpha_3(\mu^2)}\right) \approx 170 \text{ MeV}
\]

- Using $\mu \approx 10 \text{ GeV}$, $\alpha_3(\mu^2) \approx 0.2$, $n_f = 5$.

- Sets the approximate scale for bound states of strongly interacting particles.
Unification of Forces (see Kane, Quigg, ...)

- For weak interaction: exchanged boson is $Z$, gauge bosons in loops ($W^\pm$, $Z$, $H$) dominate over fermions since weak charge larger
- Running of weak coupling like strong coupling: gets weaker as momentum transfer increases
- Grand Unification: if 3 forces emerge from breaking symmetry of a simpler gauge group - reunification occurs at some high scale (for instance $SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$)
- Strong and weak force, non-Abelian gauge groups, decrease in strength; EM has Abelian group, increases: could unite
- Can write EM and weak couplings reflecting normalization from EW unification:

\[
\alpha_1 \equiv \frac{5}{3} \frac{g'^2}{4\pi} = \frac{5\alpha_{\text{QED}}}{3 \cos^2 \theta_W}
\]
\[
\alpha_2 \equiv \frac{g^2}{4\pi} = \frac{\alpha_{\text{QED}}}{\sin^2 \theta_W}
\]
\[
\alpha_3 \equiv \frac{g_3^2}{4\pi}, \text{ so}
\]
\[
\frac{1}{\alpha_i(q^2)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{4\pi} \ln \left[ \frac{q^2}{\mu^2} \right]
\]
where $b_i = [-41/10, 19/6, 7]$

(see A.V. Gladyshev and D.I. Kazakov, arXiv:1212.2548 [hep-ph])
• If forces unify, expect: \( \alpha_5 = \alpha_1(M_G^2) = \alpha_2(M_G^2) = \alpha_3(M_G^2) \)

\[
\frac{1}{\alpha_2(\mu^2)} + \frac{b_2}{4\pi} \ln \left[ \frac{M_G^2}{\mu^2} \right] = \frac{1}{\alpha_3(\mu^2)} + \frac{b_3}{4\pi} \ln \left[ \frac{M_G^2}{\mu^2} \right]
\]

\[
\frac{1}{\alpha_2(\mu^2)} - \frac{1}{\alpha_3(\mu^2)} = \frac{2(b_3 - b_2)}{4\pi} \ln \frac{M_G}{\mu}
\]

where \( b_3 - b_2 = 11 - \frac{22}{3} = \frac{11}{3} \), depends on gauge bosons only

\[
\ln \frac{M_G}{\mu} = \frac{6\pi}{11} \left( \frac{1}{\alpha_2(\mu^2)} - \frac{1}{\alpha_3(\mu^2)} \right)
\]

at \( \mu = M_Z \), \( \alpha_2(M_Z) \approx 0.034 \), \( \alpha_3(M_Z) \approx 0.118 \), so

\[
\ln \frac{M_G}{M_Z} \approx 35.8 \Rightarrow M_G \approx 10^{17}
\]

• Result exponentially sensitive to measurement of couplings, affected by higher order corrections

• More exact treatment gives \( M_G \approx 10^{15} \text{ GeV} \), \( \tau_p \approx \frac{1}{\alpha_5^2 M_G^4} \approx 10^{30 \pm 1.5} \text{ years} \)

• Minimal \( SU(5) \) ruled out by IMB experiment, \( \tau_p > 5.5 \times 10^{32} \text{ years} \) for \( p \rightarrow e^+ + \pi^0 \)

In SM, strength of EM and weak forces are independent, even though theory “unified”

\[ \alpha_1 = \frac{e^2}{4\pi}, \quad g_1 = e / \sin \theta_W, \quad g_2 = e / \cos \theta_W, \quad \sin^2 \theta_W \approx 0.23 \]

In GUTs, the mixing angle is predicted.

In SM, \( Q = T_3 - Y/2 \), in \( SU(5) \), expect \( Q = T_3 + cT_1 \), \( c \) depends on group

Can write covariant derivative in \( SU(5) \) in terms of \( SU(5) \) gauge bosons \( V_{a \mu} \) and single coupling \( g_5 \):

\[
\partial^\mu - ig_5 T_a V_{a \mu} = \partial^\mu - ig_5 \left( T_3 W_3^\mu + T_1 B^\mu + \ldots \right), \quad \text{now recall SM relation}
\]

\[
B^\mu = A^\mu \cos \theta_W + Z^\mu \sin \theta_W,
\]

\[
W_3^\mu = -A^\mu \sin \theta_W + Z^\mu \cos \theta_W
\]

\[
\Rightarrow -g_5 T_3 \sin \theta_W + g_5 T_1 \cos \theta_W = -g_5 \sin \theta_W (T_3 - \cot \theta_W T_1)
\]

\[
= eQ \text{ which is the coupling (charge) to photon } A^\mu
\]

So charge \( e = g_5 \sin \theta_W, \quad c = -\cot \theta_W \)

Try to solve for \( c \) : \( Tr(Q^2) = Tr(T_3 + cT_1)^2 = Tr T_3^2 + Tr T_1^2 \)

But \( Tr T_3^2 = Tr T_1^2 \) so \( 1 + c^2 = Tr Q^2 / Tr T_3^2 \)

From 5 multiplet : \( Tr Q^2 = 0 + 1 + 3(1/9) = 4/3 \quad Tr T_3^2 = \frac{1}{4} + \frac{1}{4} + 0 + 0 + 0 = 1/2 \)

So \( 1 + c^2 = 8/3 \), and \( c^2 = 5/3 \)
From this we predict:

\[ \sin^2 \theta_W = \frac{g_1^2}{g_1^2 + g_2^2} = \frac{1}{1 + c^2} = \frac{3}{8} = 0.375 \text{ at unification scale} \]

We can run couplings down to lower scale using \( \alpha_5 = c^2 \alpha_1, \alpha_2 = \alpha_5 \):

\[ \sin^2 \theta_W = \frac{\alpha_1}{\alpha_1 + \alpha_2} = \frac{1}{1 + \alpha_2/\alpha_1} \]

\[ = \frac{1}{1 + \frac{0.033}{0.009}} \approx 0.21 \text{ at } M_W, \text{ big change from } 3/8 \]

Was strong motivation to pursue these ideas
Unification of Forces

- Coupling strength depends on momentum transfer of virtual gauge bosons
- Familiar plot shows that in SM the couplings don’t “unify”

Unification of the Coupling Constants in the SM and the minimal MSSM

Figure 5: Evolution of the inverse of the three coupling constants in the Standard Model (left) and in the supersymmetric extension of the SM (MSSM) (right). Only in the latter case unification is obtained. The SUSY particles are assumed to contribute only above the effective SUSY scale $M_{\text{SUSY}}$ of about 1 TeV, which causes a change in the slope in the evolution of couplings. The thickness of the lines represents the error in the coupling constants [15].

where 

$$\alpha_{\text{GUT}} = \frac{g_2}{\frac{5}{4} \pi}.$$ 

The first error originates from the uncertainty in the coupling constant, while the second one is due to the uncertainty in the mass splittings between the SUSY particles.

The $\chi^2$ distributions of $M_{\text{SUSY}}$ and $M_{\text{GUT}}$ are shown in Fig. 6 [15], where

$$\chi^2 = 3 \sum_{i=1}^{3} \left( \frac{1}{\alpha_i - 1} - \frac{1}{\alpha_{\text{GUT}}} \right)^2 \sigma_i^2.$$ 

(2.10)

Figure 6: The $\chi^2$ distributions of $M_{\text{SUSY}}$ and $M_{\text{GUT}}$.
Unification of Forces

- For couplings to unify, slopes need to change - need new particle between 100 GeV scale and $10^{17}$ GeV: SUSY introduces many new gauge bosons
- Coefficients (slope parameters) $b_i = [-41/10, 19.6, 7] \rightarrow [-33/5, -1, 3]$

![Graph showing the change of slope at thresholds for MSSM particles.](image)

- Notice change of slope at thresholds for MSSM particles
- $M_{SUSY} \approx 10^{3.4 \pm 0.9 \pm 0.4}$ GeV
- $M_{GUT} \approx 10^{15.8 \pm 0.3 \pm 0.1}$ GeV
- $\alpha_{GUT}^{-1} \approx 26.3 \pm 1.9 \pm 1.0$

- Uncertainties from couplings, SUSY mass splittings
- SUSY GUTs solve Higgs hierarchy problem: ordinarily get contributions to Higgs mass of order $M_{X,Y}$
- In SUSY GUTs, superpartners contribute to $M_H$ with same magnitude, opposite sign

SUSY increases $M_{GUT}$ by a rough factor of 10 compared to $SU(5)$, so $\tau_p$ increases by $10^4$.

SUSY also predicts $\sin^2 \theta_W = 0.233 \pm 0.003$, agrees with measurement $0.23116(12)$.


SUSY decay mode: $p \rightarrow \bar{\nu}K^+$
• 50 ktons water, 22.5 ktons fiducial volume, in Kamioka, Japan
• $7.5 \times 10^{33} \, p + 6 \times 10^{33} \, n$
• Stainless steel tanks, 39.3 m diameter, 41.4 m tall
• 1000 m rock overburden
• Inner detector: 20% coverage with 5182 20” PMTs
• Detect Cherenkov radiation from decay products, PID determines if $e$-like ($e$ shower, multiple overlapping Cherenkov rings in diffuse cone) or $\mu$-like (well defined circular ring)
Proton Decay: $p \rightarrow e^+ + \pi^0$ Detection in Super-K

- Good events: fully contained in fiducial volume, 2-3 rings consistent with EM shower
- Reconstructed $\pi^0$ mass of 85-185 MeV$/c^2$, no $e$ from $\mu$ decay
- Total mass range 800-1050 MeV$/c^2$
- Net momentum $< 250$ MeV$/c$ (can have momentum from Fermi motion of nucleon in $^{16}O$ nucleus, meson-nucleon interactions (elastic scattering, charge exchange, absorption): modeled carefully, include nuclear de-excitation with $\gamma$
- Efficiency $\approx 44\%$, mainly limited by $\pi^0$ absorption in $^{16}O$ nucleus
- Background from atmospheric neutrinos: $\bar{\nu}_e + p \rightarrow e^+ + \pi^0 + n$
- Invariant mass of backgrounds typically less than for $p$ decay, momentum range larger
Proton Decay: $p \rightarrow e^+ + \pi^0$ Detection in Super-K

- Set limits on nucleon decay to charged anti-lepton ($e^+$ or $\mu^+$) and light mesons ($\pi^0$, $\pi^-$, $\eta$, $\rho^0$, $\rho^-$, $\omega$)
- No signals observed, backgrounds typically due to atmospheric neutrino interactions. Limits from $3.6 \times 10^{31}$ to $8.2 \times 10^{33}$ years at 90% C.L. depending on mode
- Exposure 49.2 kiloton-years, for $p \rightarrow e^+ + \pi^0$, background $0.11 \pm 0.02$ events, no candidates, lifetime $8.2 \times 10^{33}$ years at 90% C.L.
• Minimal $SU(5)$ ruled out from $p \rightarrow e^+ + \pi^0$

• Improving $p \rightarrow e^+ + K^0$, $p \rightarrow \mu^+ + K^0$ by order of magnitude would have big impact

• Plans to get to beyond $\tau_p > 10^{35}$ years
Proton Decay: Prospects

- Achieving another order of magnitude or more in $\tau$ very important
- Super Kamiokande will continue to run with improved analysis, searches in new channels
- Next generation detectors will be necessary

(Figure from Ed Kearns, Boston University)

$\tau/B = 5e34$
- 17 evts
- 1 BG

$\tau/B = 1e34$
- 9 evts
- 0.3 BG

10 year sample points
Proton Decay: Future Approaches

- Achieving another order of magnitude or more in $\tau$ very important
- Super Kamiokande will continue to run with improved analysis, searches in new channels
- Next generation detectors will be necessary

<table>
<thead>
<tr>
<th>Technique</th>
<th>Examples</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Cherenkov</td>
<td>22.5 kton Super-K</td>
<td>Best for $e^+\pi^0$</td>
</tr>
<tr>
<td></td>
<td>560 kton Hyper-Kamiokande</td>
<td>Good for all modes</td>
</tr>
<tr>
<td>Liquid Argon</td>
<td>34 kton LBNE LAr TPC</td>
<td>Best for $K^+\nu$</td>
</tr>
<tr>
<td></td>
<td>20 kton LBNO 2-phase TPC</td>
<td>Good for many other modes</td>
</tr>
<tr>
<td>Scintillator</td>
<td>50 kton LENA</td>
<td>Specific to $K^+\nu$</td>
</tr>
<tr>
<td></td>
<td>Next gen. reactor (DB2) ?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Water-based LSc ?</td>
<td></td>
</tr>
</tbody>
</table>

(from Ed Kearns, Boston University)
Electroweak interactions tested extensively, consistency at 0.1% level

No compelling discrepancies between electroweak observables and Standard Model

$$\sin^2 \hat{\theta}(M_Z)(\overline{MS}) = 0.231 \pm 0.0016(12),$$ known at $5 \times 10^{-4}$ level

Direct searches for new particles and new physics at LHC complemented by precision measurements

Look for deviations from Standard Model predictions at lower center of mass energies, through radiative corrections

Compelling theoretical arguments for new physics at TeV scale
MOLLER Experiment at JLab: Precision Test of Electroweak Physics

- Proposes a measurement of parity-violating asymmetry $A_{PV}$ in longitudinally polarized $e^-$ off unpolarized $e^-

$$A_{PV} \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

- $\sigma_R (\sigma_L)$ is scattering cross-section for incident right (left) handed electrons

- $A_{PV} \neq 0$ violates parity

- At $Q^2 << M_Z^2$ parity nonconservation comes from interference between EM and weak amplitudes

  ![Diagram of electron scattering processes involving photons and Z bosons.]

- The unpolarized cross-section is dominated by photon exchange, given by:

  $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m_e E} \frac{(3 + \cos^2 \theta)^2}{\sin^4 \theta} = \frac{\alpha^2}{4m_e E} \frac{1 + y^4 + (1 - y)^4}{y^2(1 - y)^2}$$

  - $\alpha$ is fine structure constant, $E$ incident beam energy, $\theta$ scattering angle, $y \equiv 1 - E'/E$, $E'$ energy of scattered $e$
• $A_{PV}$ due to interference between photon and $Z^0$ exchange diagrams

• Remember - $e$ coupling to $Z^0$ is different for left and right-handed $e$

• See E. Derman and W.J. Marciano, Annals Phys. 121, 147 (1979)

$$A_{PV} = m_e E \frac{G_F}{\sqrt{2\pi\alpha}} \frac{4\sin^2\theta}{(3 + \cos^2\theta)^2} Q^e_W m_e E \frac{G_F}{\sqrt{2\pi\alpha}} \frac{2y(1-y)}{1+y^4 + (1-y)^4} Q^e_W$$

• $Q^e_W$ proportional to product of electron vector and axial-vector coupling to $Z^0$

• $Q^e_W$ weak charge of the electron

• At leading order $Q^e_W = 1 - 4\sin^2\theta_W$; modified at 1-loop and beyond $\rightarrow 1 - 4\sin^2\theta_W(Q^2)$

• At $M_Z$, $\sin^2\theta_W(M_Z) \approx 0.23116(12)$, $Q^e_W \approx 0.075$

$\Rightarrow$ At $Q^2 \approx 0.0056$ GeV$^2$ of MOLLER experiment, $\sin^2\theta_W \approx 3\%$ larger $Q^2_W \approx 0.0469 \pm 0.0006$, change of 40% compared to tree level value at $M_Z$

• Very sensitive to running of $\sin^2\theta_W$
MOLLER Experiment at JLab: Precision Test of Electroweak Physics

- $A_{PV} \approx 35$ ppb, goal of MOLLER is measurement with statistical precision 0.73 ppb, 2.3% measurement of $Q_W^e$ (Spokesperson Krishna Kumar, thanks for material)
- Determines $\delta(\sin^2 \theta_W) \pm 0.00029$ (0.1%); comparable to single best measurements from LEP and SLC
- Would use 11 GeV polarized $e^-$ beam in Hall A

- What is physics motivation for a precision measurement of $\sin^2 \theta_W$?
- Electroweak theory provides precise predictions with negligible uncertainty - corrections at 1-loop level all known
- Comparison with precise experimental result ($\approx 10^{-3} \cdot G_F$) sensitive to new physics at TeV scale
- Uniquely sensitive to purely leptonic amplitudes at $Q^2 << M_Z^2$

MOLLER Experiment : Sensitivity to Contact Interactions

- Express amplitudes of new high energy dynamics as contact interaction between leptons:

\[
\mathcal{L}_{e_1e_2} = \sum_{i,j=L,R} \frac{g_{ij}^2}{2\Lambda^2} \bar{e}_i \gamma_\mu e_i \bar{e}_j \gamma^\mu e_j. \tag{1}
\]

- \(e_{L/R} = \frac{1}{2}(1 \mp \gamma_5)Y_e\) chiral projections of electron spinor, \(\Lambda\) mass scale of new interaction, \(g_{ij} = g_{ij}^*\) are new couplings, \(g_{RL} = g_{LR}\)

- For 0.023 measurement of \(Q_e^e\), sensitivity to new interactions (like lepton compositeness):

\[
\frac{\Lambda}{\sqrt{|g_{RR}^2 - g_{LL}^2|}} = \frac{1}{\sqrt{2G_F|\Delta Q_e^e|}} \approx \frac{246 \text{ GeV}}{\sqrt{0.023Q_e^e}} = 7.5 \text{ TeV} \tag{2}
\]

- For \(\sqrt{|g_{RR}^2 - g_{LL}^2|} = 2\pi\), \(\Lambda = 47\) TeV, electron structure probed at \(4 \times 10^{-21}\) m

- Best contact interaction limits on leptons from LEP, on quarks from Tevatron and LHC.

- But LEP only sensitive to \(g_{RL}^2\) and \(g_{RR}^2 + g_{LL}^2\) - insensitive to PV combination \(g_{RR}^2 - g_{LL}^2\)

- New \(Z'\) bosons, like \(Z_\chi\) from \(SO(10)\), predict PV couplings:

\[
\sqrt{|g_{RR}^2 - g_{LL}^2|} = \sqrt{\frac{4\pi\alpha}{3\cos^2\theta_W}} \approx 0.2 \Rightarrow Z_\chi \approx 1.5 \text{ TeV}
\]

- Get sensitivity up to \(Z_{LR} \approx 1.8\) TeV from left-right symmetric models
- New particles in Minimal Supersymmetric Standard Model (MSSM) enter $A_{PV}$ through radiative loops
- Effects from MSSM as large as $\pm 8\%$ on $Q^e_W$, can be measured to significance of 3.5 $\sigma$
- If R-parity violated, $Q^e_W$ can shift by -18%, an 8 $\sigma$ effect
- MOLLER can help distinguish between R-parity conserving and violating SUSY; RPC lightest SUSY particle could be dark matter candidate (plot below from DOE proposal)

Figure 3: Relative shifts in the electron and proton weak charges due to SUSY effects. Dots indicate the range of allowed MSSM-loop corrections. The interior of the truncated elliptical regions give possible shifts due to R-parity violating (RPV) SUSY interactions, where (a) and (b) correspond to different assumptions on limits derived from first row CKM unitarity constraints.
MOLLER : Measurement of $\sin^2 \theta_W$

- Plot from DOE proposal, shows 3 planned measurements with projected sensitivity, arbitrary central values
- Notice: some tension between left-right asymmetry in $Z$ production at SLC $A_{LR}(had)$ vs forward backward asymmetry in $Z$ decays to b-quarks $A_{FB}(b)$ at LEP
- MOLLER will achieve similar 0.1% accuracy, potentially influence world average
**MOLLER Experiment Design (K. Kumar)**

![Diagram of the experiment design](image-url)

### Parameters and Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ [GeV]</td>
<td>$\approx 11.0$</td>
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<tr>
<td>$E'$ [GeV]</td>
<td>1.8 - 8.8</td>
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<tr>
<td>$\theta_{cm}$</td>
<td>$46^\circ$ - $127^\circ$</td>
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<td>$\theta_{lab}$</td>
<td>$0.23^\circ$ - $1.1^\circ$</td>
</tr>
<tr>
<td>$\langle Q^2 \rangle$ [GeV$^2$]</td>
<td>0.0056</td>
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<tr>
<td>Maximum Current [$\mu$A]</td>
<td>85</td>
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<tr>
<td>Target Length (cm)</td>
<td>150</td>
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<tr>
<td>$\rho_{tgt}$ [g/cm$^3$] (T= 20K, P = 35 psia)</td>
<td>0.0715</td>
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<tr>
<td>Max. Luminosity [cm$^{-2}$ sec$^{-1}$]</td>
<td>$3.4 \cdot 10^{39}$</td>
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<td>$\sigma$ [$\mu$Barn]</td>
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<td>Møller Rate [GHz]</td>
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<td>Statistical Width (2 kHz flip) [ppm/pair]</td>
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<td>Target Raster Size [mm]</td>
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<tr>
<td>$\Delta A_{raw}$ [ppb]</td>
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<td>Background Fraction</td>
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<td>$P_{beam}$</td>
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<td>$\langle A_{pv} \rangle$ [ppb]</td>
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<tr>
<td>$\Delta A_{stat}/\langle A_{expt} \rangle$</td>
<td>2.1%</td>
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<tr>
<td>$\delta(\sin^2 \theta_W)_{stat}$</td>
<td>0.00026</td>
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</tbody>
</table>
Technical Challenges

- ~ 150 GHz scattered electron rate
  - Design to flip Pockels cell ~ 2 kHz
  - 80 ppm pulse-to-pulse statistical fluctuations
- 1 nm control of beam centroid on target
  - Improved methods of “slow helicity reversal”
- > 10 gm/cm² liquid hydrogen target
  - 1.5 m: ~ 5 kW @ 85 µA
- Full Azimuthal acceptance with $\theta_{lab} \sim 5$ mrad
  - novel two-toroid spectrometer
  - radiation hard, highly segmented integrating detectors
- Robust and Redundant 0.4% beam polarimetry
  - Pursue both Compton and Atomic Hydrogen techniques

- **MOLLER Collaboration**
  - ~ 100 authors, ~ 30 institutions
  - Expertise from SAMPLE A4, HAPPEX, GO, PREX, Qweak, E158
  - 4th generation JLab parity experiment

- 20M$ proposal to DoE NP
- 3-4 years construction
- 2-3 years running

- **First data ≈ 2017**