Hydrodynamics in Heavy Ion Collisions

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Hydrodynamics of QGP
How does one describe a system of \( N \gg 1 \) bodies?

Depends on how much “information” one wants.

Worst case: \( \psi(x_1, \cdots, x_N) \)

Best case: Minimum information content. I.e. maximum entropy. Need only a handful of numbers such as temperature and chemical potential.

\[
n(p) = 1/\left( e^{E_p/T - \mu/T} - 1 \right)
\]

These quantities are the Langrange multipliers that constraints conserved quantities such as energy and charge.
Non-Equilibrium

- How does one describe a system of $N \gg 1$ bodies?
- Depends on how much “information” one wants.
- Worst case: $\psi(x_1, \cdots, x_N)$
- Best case: Minimum information content. I.e. maximum entropy. Need only a handful of numbers such as temperature, and chemical potential. But locally.
- $n(p, x) = 1/(e^{p \mu u / T(x)} - \mu(x) / T(x) \mp 1)$
- You only need to know few functions: $T(t, x), \mu(t, x)$ as well as the collective velocity $u(t, x)$
- These quantities are the Langrange multipliers that constraints conserved quantities such as energy, momentum and charge.
System is made up of “fluid cells”.

Each fluid cell feels a force according to the pressure difference (gradient) w.r.t. its neighbors.

System evolves by “flowing” towards lower pressure.

NR: \( \mathbf{F} = ma \)

\[-\nabla P = n_m \partial_t \mathbf{u} \]

where \( n_m \): mass density,
\( P \): pressure,
\( \mathbf{u} \): flow velocity
What is the nature of the initial condition?
Do we reach local equilibrium in heavy ion collisions?
How hot is it?
How viscous is QGP?
(Is there a phase transition? If so what kind?)
Information content of single particle spectra

\[
\frac{dN_i}{dy\,d^2p_T} = \frac{dN_i}{2\pi p_T dp_T dy} \left( 1 + \sum_{n=1}^{\infty} 2v_{i,n}(p_T, y) \cos(n\phi) \right)
\]

“Flow”: \(v_{i,n}(p_T)\)

Came from

\[
\varepsilon(x_T, \eta) = \varepsilon(r_T, \eta) \left( 1 + \sum_{n=1}^{\infty} 2\epsilon_n(r_T, \eta) \cos(n\phi) \right)
\]

- \textit{Pressure} converts it into \(v_{i,n}(p_T)\)
- History matters
- \(\epsilon_n \rightarrow v_{i,n}\) conversion contains information on the medium and its evolution
Elliptic Flow – $\cos(2\phi)$ component

Spatial anisotropy  Pressure does the conversion  Momentum anisotropy
Physics from Hydrodynamics

- Triangular Flow – $\cos(3\phi)$ component

Spatial anisotropy  Pressure does the conversion  Momentum anisotropy

Flows are about: **Pressure** converting
Spatial morphology $\epsilon_n(r_T, \eta) \implies$ Momentum space morphology $\nu_n(p_T, y)$

This is sensitive to
- Initial Conditions
- Flow dynamics ($\eta/s$)
- Equation of State (to a less extent)
Why is initial condition important?

- Initial temperature (distribution) \( T_0 > T_c \)
- Beginning time of hydro (\( \sim \) thermalization time) \( \tau_0 \)
- The size of the hot spots \( \sigma_0 \)
- What happens *before* the hydro stage?

\( v_2 \) alone cannot determine all these \( \implies v_3, v_4, \cdots \)
Why is $\eta/s$ important?

- One of the central properties of QGP
- Calculable in perturbative QCD $\eta/s \sim 1/g^4 \ln(1/g)$
- Calculable in AdS/CFT $\eta/s = 1/4\pi$
- If $\eta/s \sim 1/4\pi$, QGP must be sQGP
Equations for $T, u$ – Conservation laws

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

Stress-energy tensor $T^{\mu\nu}$ has only 10 d.o.f. Cons. laws provide 4 constraints $\Longrightarrow$ 6 d.o.f. left.

No dynamical content yet.

Energy density and flow vector

$$T^{\mu\nu} u_\nu = -\varepsilon u^\mu$$

$u^\mu$: Time-like eigenvector of $T^{\mu\nu}$. Normalized to $u^\mu u_\mu = -1$.

$\varepsilon$: Local energy density

This is always possible since $T^{\mu\nu}$ is real and symmetric.
Hydrodynamics

- So far:

\[ T^{\mu\nu} = \varepsilon u^\mu u^\nu + W^{\mu\nu} \]

with

\[ W^{\mu\nu} u_\nu = 0 \]

This is just math. No physics input except that \( \varepsilon \) is the energy density and \( u^\mu \) is the velocity of the energy flow.

- Physics - Small scale physics is thermal \( \Rightarrow \) Local equilibrium \( \Rightarrow \) Equation of state (i.e. \( P = P(\varepsilon) \))

  - \( W^{\mu\nu} = (g^{\mu\nu} + u^\mu u^\nu)P(\varepsilon) + \pi^{\mu\nu}[\varepsilon, u] \) with \( \pi^{\mu\nu} u_\nu = 0 \)

  - Ideal Hydro: \( \pi^{\mu\nu} = 0 \) gives \( \partial_i ((\varepsilon + P)u) = -\nabla P \) for small \( u \)

  - Viscous Hydro:

\[ \pi^{ij} = -\frac{\eta}{2} \left( \partial^i u^j + \partial^j u^i - g^{ij}(2/3)\nabla \cdot u \right) - \zeta g^{ij} \nabla \cdot u \]
Validity of Hydrodynamics

- $\partial_\mu T^{\mu\nu} = 0$: This is an operator statement. This is valid no matter what.

- $\partial_\mu \langle T^{\mu\nu} \rangle = 0$: This is a statement about average. This is valid no matter what.

**Ideal Hydro**

$$T^{\mu\nu}_{\text{id}} = \epsilon u^\mu u^\nu + P(g^{\mu\nu} + u^\mu u^\nu)$$

This assumes that the the system has isotropized $\implies$ Ideal Hydrodynamics is valid only after the system has isotropize. But this is not enough.

- $P(x) = P(\varepsilon(x))$: Equation of state. Valid only if local equilibrium is reached. Recent most complete characterization of QCD thermalization process: 1107.5050 by Moore and Kurkela.

$$t_{eq} \sim \alpha^{-2} Q^{-1}.$$
Validity of Hydrodynamics

Viscous Hydro

- \( \pi_{ij} = -\eta \partial \langle i u_j \rangle \) (tranceless, symmetric and transverse to \( u^\mu \))

- Gradient expansion must be valid \( \implies \) Higher derivatives are smaller.

- This means local equilibrium is established in the length scale much longer than the microscopic mean free path.

- In fact, \( \pi_{ij} = -\eta \partial \langle i u_j \rangle \) induces unphysical faster-than-light propagations.

  \[ \implies \text{Second order Israel-Stewart formalism: } \pi_{ij} \text{ relaxes towards} \]

  \[ -\eta \partial \langle i u_j \rangle \]

  \[ \frac{d}{d\tau} \pi_{ij} = -\frac{1}{\tau_r} \left( \pi_{ij} - (-\eta \partial \langle i u_j \rangle) \right) \]
Ideal Hydro

- Stress-energy tensor

\[ T^{\mu \nu} = \varepsilon u^{\mu} u^{\nu} + P(u^{\mu} u^{\nu} - g^{\mu \nu}) \]

- Energy momentum conservation

\[ 0 = u^{\mu} \partial_{\mu} \varepsilon + (\varepsilon + P)(\partial_{\mu} u^{\mu}) \]

and

\[ (\varepsilon + P) u^{\mu} \partial_{\mu} u_{\alpha} = \partial_{\alpha} P - u_{\alpha} u_{\nu} \partial^{\nu} P \]

- One can easily show that entropy is conserved

\[ \partial_{\mu}(s u^{\mu}) = 0 \]

using \( sT = \varepsilon + P \) and \( TdS = dU + PdV \rightarrow Tds = d\varepsilon \)
Solving Hydro – Need for $\tau, \eta$

- Idealized physical picture: Two infinitely energetic ($v = c$) pancakes pulling away from each other

- Can’t distinguish the two cases $\implies$ Boost invariance if $E_{\text{beam}} = \pm \infty$. 
Need for $\tau, \eta$

- Dynamic rapidity $y$ is defined by:
  \[
  E = \sqrt{m^2 + p_T^2 \cosh(y)} \\
p_z = \sqrt{m^2 + p_T^2 \cosh(y)}
  \]

- Ends with $\pm c$ are at $y = \pm \infty$ $\implies$ The system occupies the whole rapidity axis.

- With $\gamma = \cosh \Delta y$ and $\gamma v = \sinh \Delta y$, Lorentz boost is just a translation in the rapidity space
  \[
  E' = \gamma E + \gamma vp_z = m_T \cosh(y + \Delta y) \\
p_z' = \gamma p_z + \gamma vE = m_T \sinh(y + \Delta y)
  \]

- The system must be homogeneous in $y$ $\implies$ Independent of $y$
Solving Hydro – Need for $\tau, \eta$

- Space-time rapidity $\eta$ defined by
  \[
  t = \tau \cosh \eta \\
  z = \tau \sinh \eta
  \]

- Lorentz boost is just a translation in the rapidity space
  \[
  t' = \gamma t + \gamma vz = \tau \cosh(\eta + \Delta y) \\
  z' = \gamma z + \gamma vt = \tau \sinh(\eta + \Delta y)
  \]

- A boost invariant system is independent of $\eta$ as well.
Simplify some more – No dependence on $x, y$. No dissipation.

The only thing a boost can do: Lorentz transform the fluid velocity $u^\mu$.

Boost invariance: Fluid velocity can only be $u^\mu = \left(\frac{t}{\tau}, 0, 0, \frac{z}{\tau}\right) = \left(\cosh \eta, 0, 0, \sinh \eta\right)$.

Let

$$\varepsilon = \varepsilon(\tau)$$
$$P = P(\tau)$$

The energy-momentum conservation becomes

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P}{\tau}$$
\[
\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P}{\tau}
\]
can be rewritten as
\[
\frac{ds}{d\tau} = -\frac{s}{\tau}
\]
using \( Ts = \varepsilon + P \) and \( TdS = dU + PdV \).
Also,
\[
\frac{d\varepsilon}{d\tau} = -\left(1 + v_s^2\right)\varepsilon
\]
using \( v_s^2 = \frac{\partial P}{\partial\varepsilon} \)

\textbf{Solutions}

\[
s(\tau) = s_0 \left(\frac{\tau_0}{\tau}\right)
\]

and

\[
\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau_0}{\tau}\right)^{1 + v_s^2}
\]
At RHIC: $y_{\text{max}} \approx \pm 5.4$

At LHC: $y_{\text{max}} \approx \pm 8.0$

Not $\pm \infty$, but big enough

More technical reason: Hard to contain this system in $t - z$ as the boundary of the system linearly increases with time.

In $\tau, \eta$, $\eta_{\text{max}} > y_{\text{max}}$ is enough.

Price to pay: $\partial_\mu T^{\mu \nu} = 0$ becomes complicated.
Including Viscosity

- Generalized Israel-Stewart
- For example, shear viscosity: Baier, Romatschke, Son, Starinets, Stephanov (0712.2451)

\[
\Delta^\mu_\alpha \Delta^\nu_\beta D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi} \left( \pi^{\mu\nu} - 2\eta \nabla^\langle \mu \rangle u^\mu \right) + \frac{4}{3} \tau_\pi \pi^{\mu\nu} (\partial_\alpha u^\alpha) \]

Jeon (McGill)
Physics Issue 1: Initial state
What we want to do:

Study how initial state spatial anisotropy $$\varepsilon(x_T, \eta) = \varepsilon(r_T, \eta) \left( 1 + \sum_{n=1}^{\infty} 2\epsilon_n(r_T, \eta) \cos(n\phi) \right)$$

turns into the final state momentum anisotropy

$$\frac{dN_i}{dy \ d^2p_T} = \frac{dN_i}{2\pi p_T dp_T dy} \left( 1 + \sum_{n=1}^{\infty} 2v_{i,n}(p_T, y) \cos(n\phi) \right)$$

Wants to get history of physical quantities $$P, u, \eta/s, \cdots$$ from the flow coefficients $$v_{i,n}(p_T)$$ – Need many different measurements
Thickness function

\[ T_A(s) = \int dz \rho_A(s, z) \]

Overlap function:

\[ T_{AB}(s, b) = T_A(s)T_B(b + s) \]
Participants: $N_{\text{part}}(s, b) \propto T_A(s) + T_B(b + s)$

Binary scatterings: $N_{\text{bin}}(s, b) \propto T_{AB}(s, b)$

Initial energy density

$$\varepsilon(s, b) = c_1 [T_A(s) + T_B(b + s)] + c_2 T_{AB}(s, b)$$
Smooth Geometry

Ultimately, initial geometry determines the initial conditions and the final flow pattern.

Initial geometry also determines number of jets at s and the path conditions for those jets.
Smooth initial states have up-down, left-right symmetry: Initial states only has \( \cos(2n\phi) \) components such as \( v_2, v_4, v_6, \cdots \).
What determines the initial shape?

- Averaged smooth initial condition $\implies$ Only $\nu_{\text{even}}$’s survive.
What determines the initial shape?

- Fluctuating initial condition \( \implies \) All \( v_n \) are non-zero.
Why go beyond $\nu_2$?

- $\nu_{\text{odd}} \neq 0$ due to fluctuations is obvious once you see it

- $\nu_2$ and $\nu_3$ are sensitive to the different features of the initial condition

  - Elliptic flow: Sensitive to the overall almond shape
  - Triangular flow: Less so. More local in the sense that average initial condition gives zero $\nu_3$.

- Viscosity effect on different features is different
  - Viscosity smears out lumps.
  - Viscosity reduces differential flow - Triangle is “rounder” than ellipse
Effect of viscosity

eta = 0
No friction

The relative velocity of the two layers does not change.

eta ≠ 0
Friction between the layers

The velocities eventually become the same.
Effect of viscosity

- $\eta = 0$ means $u_1 < u_2 < u_3$ is maintained for a long time.
- $\eta \neq 0$ means that $u_1 \approx u_2 \approx u_3$ is achieved more quickly.
- Shear viscosity smears out flow differences (it’s a diffusion).
- Shear Viscosity reduces non-sphericity.
This causes elliptic flow. It is harder to destroy this than
\( \varepsilon_n \)

\[ \text{this (v_3) ...} \]
or this \((v_4)\) ...
or this ($v_{10}$) ...
Initial Conditions

Differences in models:

- Position of the energy deposite (collision sites)
- Energy deposit at each collision sites \( (xN_{\text{part}} + (1 − x)N_{\text{coll}}) \)
- Size and spread of the initial lump
Collision Geometry

- \( \mathbf{b} \): Impact parameter. Vector between two centers in transverse space.
- \( \mathbf{r}_\perp \): Position vector from the center of the target nucleus.
- \( \mathbf{b} + \mathbf{r}_\perp \): Position vector from the center of the projectile nucleus.
Initial Conditions

**MC-Glauber**

- Sample Wood-Saxon thickness function

\[
T_A(r_\perp) = \int dz \frac{\rho_0}{1 + e^{(R-r)/a}}
\]

\[
T_A(b + r_\perp) = \int dz' \frac{\rho_0}{1 + e^{(R-r')/a}}
\]

for nucleon positions

- **NN** collision occurs if two nucleons are within \( D = \sqrt{\sigma_{NN}/\pi} \)

- For each wounded nucleon, deposit \( \epsilon_0 e^{-\frac{(x-x_c)^2}{2\sigma_0^2}} \)
Detour: Saturation

[BFKL, JIMWLK, BK]

- Gluon distributions for protons for $Q^2 = 10$ GeV$^2$ and $Q^2 = 100$ GeV$^2$.
- Looks like growing indefinitely: Unphysical
Leading order BFKL equation (evolution in $x$) takes into account splitting, but not recombination.

When density is high, recombination must be taken into account → JIMWLK & BK

Density is high: Classical field limit

Recombination: Non-linear effect
Detour: Saturation

[BFKL, JIMWLK, BK]

- Saturation (or Recombination) scale
  - Transverse gluon density
    \[ \rho \sim \frac{xg_A(x, Q^2)}{S_\perp} \sim \frac{Axg(x, Q^2)}{A^{2/3}} \sim A^{1/3}xg(x, Q^2) \]

- Recombination cross-section
  \[ \sigma_{gg\rightarrow g} \sim \frac{\alpha_s^2}{Q^2} \]

- Saturation when
  \[ \rho\sigma_{gg\rightarrow g} \sim 1 \]

- Saturation scale
  \[ Q_s^2 = \alpha_s(Q_s)A^{1/3}xg(x, Q_s^2) \]
Detour: Saturation

- Classical field equation of QCD

\[ D_\mu G^{\mu\nu} = J^\nu \]

where

\[ D_\mu = \partial_\mu - igA^a_\mu T_a \]

and

\[ G^{a\mu\nu} = \partial_\mu A^{a\nu} - \partial_\nu A^{a\mu} + gf_{abc}A^b_\mu A^c_\nu \]

- \( J^\mu_a = \rho_A \delta^{\mu+} + \rho_B \delta^{\mu-} \): Color source

- Gluon field

\[ A^\mu = A^{\mu\Lambda}_A + A^{\mu\Lambda}_B + A^{\mu\Lambda}_P \theta(\tau) \]

The produced field \( A_P \) after the collision is what we are after.
Static color charges

Classical gluon field

High energy photons

Hard Scatterings occur at this stage
Jets are produced

Nuclei are breaking up

Gluon fields are grabbing each other

Jets propagating

Photons are produced

Nucleus remnant

Entropy is produced.

Pre-equilibrium mix of streaming quarks, gluons and classical gluon field.
Initial Conditions

**MC-KLN** Drescher and Nara, PRC 75:034905

- Sample Wood-Saxon thickness function

\[
T_A(r_\perp) = \int dz \frac{\rho_0}{1 + e^{(R-r)/a}}
\]

\[
T_A(b + r_\perp) = \int dz' \frac{\rho_0}{1 + e^{(R-r')/a}}
\]

for nucleon positions
**MC-KLN** Drescher and Nara, PRC 75:034905

- Calculate thickness function again:

$$t_A(r_\perp) = \frac{\text{# of nucleons in the tube at } r_\perp}{S}$$

$$t_A(b + r_\perp) = \frac{\text{# of nucleons in the tube at } b + r_\perp}{S}$$

where $S = \sigma_{NN}$ is the cross-section of the tube
**Initial Conditions**

*MC-KLN* Drescher and Nara, PRC 75:034905

- Calculate the **saturation scale**

\[
Q_{s, A}^2(x, r_\perp) = 2 \text{ GeV}^2 \left( \frac{t_A(r_\perp)}{1.53} \right) \left( \frac{0.01}{x} \right)^\lambda
\]

- Calculate the unintegrated gluon density function

\[
\phi(x, k_{\perp}^2; r_\perp) = \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2}{\text{max}(Q_s^2, k_{\perp}^2)}
\]
**MC-KLN** Drescher and Nara, PRC 75:034905

- Deposited energy with an approximate the $g_A g_B \rightarrow g_P$ process

\[
\frac{dE_g}{d^2r_\perp dyd^2p_\perp} = \frac{4N_c}{N_c^2 - 1} \frac{1}{|\mathbf{p}_\perp|} \int d^2k_\perp \alpha_s \phi_A((\mathbf{p}_\perp + \mathbf{k}_\perp)^2/4) \phi_A((\mathbf{p}_\perp - \mathbf{k}_\perp)^2/4)
\]
Initial Conditions

[Tribedy & Venugopalan, Nucl.Phys.A850 136; Tribedy & Venugopalan, PLB710 125; Schenke, Tribedy & Venugopalan, PRL108 252301; Gale, Jeon, Schenke, Tribedy & Venugopalan, PRL110 012302]

**IP-Glasma**
- Sample the position of the nucleons.
- Calculate the saturation momentum for each nucleon using the IP-Sat model (Kowalski & Teaney, PRD68 114005)
- Calculate the color charge density by summing over all $Q_s$ at the given global position

$$g_{\mu}(x, b) = c \sum_i Q_s(x, b_i)$$
Initial Conditions

[Tribedy & Venugopalan, Nucl. Phys. A850 136; Tribedy & Venugopalan, PLB710 125; Schenke, Tribedy & Venugopalan, PRL108 252301; Gale, Jeon, Schenke, Tribedy & Venugopalan, PRL110 012302]

IP-Glasma

- Sample the color charge distribution of each nucleus using the Gaussian distribution
  \[ W_A[\rho] = \exp \left( -\frac{\rho_a \rho_b}{(g^2 \mu_A^2)} \right) \]

- Solve the Classical Yang-Mills equation
- After evolving for \( \tau_0 \) calculate \( T^{\mu\nu} \)
- Connect it to Hydro
Comparison


MC-Glauber

MC-KLN

IP-Glasma $\tau=0.01$ fm/c

IP-Glasma $\tau=0.2$ fm/c
Initial conditions

- Different size and distribution of $\epsilon_n$
- Test: Need to get the $\nu_n(p_T)$ for various centralities
- Test: Need to get the e-by-e distribution of integrated $\nu_n$
Physics Issue 2: Viscosity
Effect of viscosity

\[ \eta = 0 \]

The relative velocity of the two layers does not change.

\[ \eta \neq 0 \]

Friction between the layers

The velocities eventually become the same.
Effect of viscosity

\[ \eta = 0 \] means \( u_1 < u_2 < u_3 \) is maintained for a long time.

\[ \eta \neq 0 \] means that \( u_1 \approx u_2 \approx u_3 \) is achieved more quickly.

Shear viscosity smears out flow differences (it’s a diffusion).

Shear Viscosity reduces non-sphericity.
Interaction Strength and Viscosity

Weak coupling allows rapid *momentum diffusion*

Large $\eta/s$: $u_\mu(x)$ changes due to pressure gradient and diffusion
Strong coupling \textit{does not} allow momentum diffusion.

Small $\eta/s$: $u_\mu(x)$ changes due to pressure gradient only.
Kinetic Theory estimate

\( u_z \): Flow velocity
\( v_x \): Average speed of microscopic particles

Rough estimate (fluid rest frame, or \( u_z(x) = 0 \))

- The momentum density: \( T_{0z} = (\epsilon + P)u_0u_z \) diffuses in the \( x \) direction with \( v_x = u_x/u_0 \). Net change:

\[
\langle \epsilon + P \rangle |v_x| u_0 (u_z(x - l_{mfp}) - u_z(x + l_{mfp})) \\
\approx -2 \langle \epsilon + P \rangle |v_x| u_0 l_{mfp} \partial_x u_z(x) \\
\sim -\eta u_0 \partial_x u_z
\]

Here \( l_{mfp} \): Mean free path

- Recall thermo. id.: \( \langle \epsilon + P \rangle = sT \)

\[
\eta \sim \langle \epsilon + P \rangle l_{mfp} \langle |v_x| \rangle \sim sT l_{mfp} \langle |v_x| \rangle
\]
Perturbative estimate

High Temperature limit: $\langle |v_x| \rangle = O(1)$

- $\eta/s \approx Tl_{mf} \approx \frac{T}{n\sigma} \sim \frac{1}{T^{2\sigma}}$

- The only energy scale: $T$

$$\sigma \sim \frac{(\text{coupling constant})^\#}{T^2}$$

Hence

$$\frac{\eta}{s} \sim \frac{1}{(\text{coupling constant})^\#}$$

- Perturbative QCD partonic 2-2 cross-section

$$\frac{d\sigma_{el}}{dt} = C \frac{2\pi\alpha_s^2}{t^2} \left(1 + \frac{u^2}{s^2}\right)$$
Naively expect

\[ \eta/s \sim \frac{1}{\alpha_s^2} \]

Coulomb enhancement (cut-off by \(m_D\)) leads to

\[ \eta/s \sim \frac{1}{\alpha_s^2 \ln(1/\alpha_s)} \]
QCD $\eta$ calc

 Relevant processes

\begin{align*}
(A) & \quad (B) & \quad (C) & \quad (D) & \quad (E) \\
(F) & \quad (G) & \quad (H) & \quad (I) & \quad (J)
\end{align*}

\(\sim 80\%\)

Use kinetic theory

\[ \frac{df}{dt} = C_{2\leftrightarrow 2} + C_{1\leftrightarrow 2} \]

Complication: 1 $\leftrightarrow$ 2 process needs resummation (LPM effect, AMY)
QCD Estimates of $\eta/s$

- Danielewicz and Gyulassy [PRD 31, 53 (1985)]:
  - $\eta/s$ bound from the kinetic theory: Recall: $\eta \sim s \ T \ l_{\text{mfp}} \ \langle |v_x| \rangle$ Use $l_{\text{mfp}} \ \langle |v_x| \rangle \sim \Delta x \Delta p / m$ to get
    \[
    \frac{\eta}{s} \gtrsim \frac{1}{12} \approx 0.08 \approx (1/4\pi)
    \]
  - QCD estimate in the small $\alpha_S$ limit with $N_f = 2$ and $2 \rightarrow 2$ only (min. at $\alpha_S = 0.6$):
    \[
    \eta \approx \frac{T}{\sigma_\eta} \approx \frac{0.57 \ T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.2s \approx (2.5/4\pi)s
    \]

- Baym, Monien, Pethick and Ravenhall [PRL 64, 1867 (1990)]
  \[
  \eta \approx \frac{1.16 \ T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.4s \approx (5/4\pi)s
  \]

- M. Thoma [PLB 269, 144 (1991)]
  \[
  \eta \approx \frac{1.02 \ T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.4s \approx (5/4\pi)s
  \]
Full leading order calculation of $\eta/s$

- Arnold-Moore-Yaffe (JHEP 0305, 051 (2003)) [Plots: Guy]:

Minimum $\eta/s \approx 0.6 \approx 7.5/4\pi$ for $\alpha_s \approx 0.3$

NB: Approximate formula $\eta/s \approx \frac{1}{15.4\alpha_s^2 \ln(0.46/\alpha_s)}$ is not good for $\alpha_s > \frac{1}{4\pi(1+N_f/6)}$
Shear viscosity in $\mathcal{N}=4$ SYM

Son, Starinets, Policastro, Kovtun, Buchel, Liu, ...

- Strong coupling limit, 4 ingredients
  - Kubo formula
    \[
    \eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt \, d^3x \, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle
    \]

- Gauge-Gravity duality
  \[
  \sigma_{\text{abs}}(\omega) = \frac{8\pi G}{\omega} \int dt \, d^3x \, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle
  \]

  \[
  \lim_{\omega \to 0} \sigma_{\text{abs}}(\omega) = A_{\text{blackhole}}
  \]

  - Entropy of the BH : $s = A_{\text{blackhole}} / 4G$

Therefore, (including the first order correction)

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + \frac{7.12}{(g^2 N_c)^{3/2}} \right)
\]

Correction is small if $g \gg 1$ (10% at $g = 2.4$).
Perturbative calculation and the strong coupling calculation behave very differently.

$N = 4$ SYM

$\eta/s$ vs. Coupling $\lambda = N_c g^2$

Experimental Evidence for \( \eta/s \sim 1/4\pi \)

- Theoretical situation:
  - Perturbative calculations: \( \eta/s \geq 7.5/(4\pi) \)
  - AdS/CFT in the infinite coupling limit: \( \eta/s = 1/(4\pi) \)
  - Roughly an order of magnitude difference \( \implies \) Testable!

- A relativistic heavy ion collision produces a complicated system \( \implies \) Need a hydrodynamics simulation suite

- We use MUSIC (3+1D e-by-e viscous hydrodynamics)

- Viscosity measurement is through the flow coefficients

\[
\frac{dN}{dyd^2p_T} = \frac{dN}{2\pi dy p_T dp_T} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\phi - \psi_n)) \right)
\]

- \( v_n \) is a translation of the eccentricities \( \epsilon_n \) via pressure gradient
MUSCL: Monotone Upstream-centered Schemes for Conservation Laws
Current MUSIC (and MARTINI) Team

- Charles Gale (McGill)
- Sangyong Jeon (McGill)
- Björn Schenke (Formerly McGill, now BNL)
- Clint Young (Formerly McGill, now UMN)
- Gabriel Denicol (McGill)
- Matt Luzum (McGill/LBL)
- Sangwook Ryu (McGill)
- Gojko Vujanovic (McGill)
- Jean-Francois Paquet (McGill)
- Michael Richard (McGill)
- Igor Kozlov (McGill)
3+1D Event-by-Event Viscous Hydrodynamics

- 3+1D parallel implementation of new Kurganov-Tadmor Scheme in $(\tau, \eta)$ with an additional baryon current (No need for a Riemann Solver. Semi-discrete method.)
- Ideal and Viscous Hydro
- Event-by-Event fluctuating initial condition
- Sophisticated Freeze-out surface construction
- Full resonance decay (3+1D version of Kolb and Heinz)
- Many different equation of states including the newest from Huovinen and Petreczky
- New Development: Glasma Initial Conditions & UrQMD after-burner
Fluctuating Initial Condition

Each event is *not* symmetric: Fluctuating initial condition $\Rightarrow$ All $v_n$ are non-zero.
Ideal vs. Viscous

[Movies by B. Schenke]

\[ \tau = 0.4 \text{ fm/c} \]

\[ \tau = 6.0 \text{ fm/c, ideal} \]

\[ \tau = 6.0 \text{ fm/c, } \eta /s = 0.16 \]
Ideal vs. Viscous
Magnitude of higher harmonics, $v_3, v_4, \ldots$, (almost) independent of centrality – Local fluctuations dominate

Higher harmonics are easier to destroy than $v_2$ which is a global distortion – Viscosity effect.

To get a good handle on flow: Both fluctuations and viscosity are essential
E-by-E MUSIC vs LHC Data

[Schenke, Jeon and Gale, Phys. Rev. C 85, 024901 (2012)]
Best value $\eta/s = 0.16 = 2/(4\pi)$.
Glasma Initial Condition

[Gale, Jeon, Schenke, Tribedy and Venugopalan, arXiv:1209.6330]

Best value $\eta/s = 0.2 = 2.5/(4\pi)$.
Glasma Initial Condition

[Gale, Jeon, Schenke, Tribedy and Venugopalan, arXiv:1209.6330]

E-by-E distributions

\[ \frac{v^2}{\langle v^2 \rangle}, \frac{\varepsilon^2}{\langle \varepsilon^2 \rangle} \]

\[ p_T > 0.5 \text{ GeV} \]

|\( |\eta| < 2.5 \) |

0-5% \( \varepsilon^2 \) IP-Glasma

\( \varepsilon^2 \) IP-Glasma+MUSIC

\( \varepsilon^2 \) ATLAS

20-25% \( \varepsilon^2 \) IP-Glasma

\( \varepsilon^2 \) IP-Glasma+MUSIC

\( \varepsilon^2 \) ATLAS

0-5% \( \varepsilon^3 \) IP-Glasma

\( \varepsilon^3 \) IP-Glasma+MUSIC

\( \varepsilon^3 \) ATLAS

20-25% \( \varepsilon^3 \) IP-Glasma

\( \varepsilon^3 \) IP-Glasma+MUSIC

\( \varepsilon^3 \) ATLAS

0-5% \( \varepsilon^4 \) IP-Glasma

\( \varepsilon^4 \) IP-Glasma+MUSIC

\( \varepsilon^4 \) ATLAS
\( \nu_2 \) at RHIC (Midrapidity). In each centrality class: 100 UrQMD times 100 MUSIC events. [Ryu, Jeon, Gale, Schenke and Young, arXiv:1210.4558]

- \( \eta/s = 1/4\pi \)
- Using previous MUSIC parameters that were tuned to reproduce PHENIX \( \nu_n \)
In each centrality class: 100 MUSIC times 10 UrQMD events. 
\[ \eta/s = 2/(4\pi) \]. ALICE data from QM12.
LHC Flows

In each centrality class: 100 MUSIC times 10 UrQMD events

Jeon (McGill)
Soft
Stony Brook 2013
Conclusions and questions for $\eta/s$

- Strong flows: Strongest evidence that $\eta/s$ has to be small
- $\eta/s$ much larger than 0.2 cannot be accommodated within current understanding of the system.
- Perturbative result of $\eta/s = 0.4 - 0.6$ is out.
- Using the LQCD EoS.
- LQCD estimate $(\eta + 3\zeta/4)/s \approx 0.20 - 0.26$ between $1.58T_c - 2.32T_c$.
- Does this mean very large coupling?
Jet Quenching

- Fact: Jets lose energy (ATLAS images).
Jet Quenching

- Fact: Jets lose energy (ATLAS images).

\[
\frac{dE}{dx} \approx C_1 \pi \alpha_S^2 T^2 \left[ \log \left( \frac{E_p}{\alpha_S T} \right) + C_2 \right]
\]

\( C_{1,2} \): Depends on the process. \( O(1) \).

Radiational \( \propto \alpha_S^2 \) (Arnold, Moore, Yaffe, JHEP 0206, 030 (2002))
What we want to get at

- What $\alpha_S$ do we need for these?

$R_{AA} = \frac{(dN_{AA}/dp_T)}{(N_{coll} dN_{pp}/dp_T)}$

$A_j = (E_\uparrow - E_\downarrow)/(E_\uparrow + E_\downarrow)$
Tool

- Event generator
  - Jet propagation through evolving QGP medium.
- Several on the market. We use MARTINI.
MARTINI
**MARTINI**

- **Modular Algorithm for Relativistic Treatment of Heavy Ion Interactions**
- Hybrid approach
  - Calculate Hydrodynamic evolution of the soft mode (MUSIC)
  - Propagate jets in the evolving medium according to McGill-AMY
HIC Jet production scheme:

\[
\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \ni \times \frac{d\sigma_{ab\rightarrow cd}}{dt} \times P(x_c \rightarrow x'_c| T, u^\mu) \ni \times D(z'_c, Q)
\]

\(P(x_c \rightarrow x'_c| T, u^\mu)\): Medium modification of high energy parton property
\[
\frac{d\sigma_{AB}}{dt} = \int \text{geometry} \int_{abcdc'} d\sigma_{ab\rightarrow cd} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \\
\times P(x_c \rightarrow x'_c | T, u^\mu) \\
\times D(z'_c, Q)
\]

- Sample collision geometry using Wood-Saxon
\[
\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcda'} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\rightarrow cd}}{dt} \times \mathcal{P}(x_c \rightarrow x'_c|T, u^\mu) \times D(z'_c, Q)
\]

- PYTHIA 8.1 generates high $p_T$ partons
- Shadowing included
- Shower (Radiation) stops at $Q = \sqrt{p_T/\tau_0}$
\[
\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'} \left( f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \right. \\
\times \frac{d\sigma_{ab\rightarrow cd}}{dt} \\
\times \mathcal{P}(x_c \rightarrow x'_c | T, u^\mu) \\
\times D(z'_c, Q) 
\]

- Hydrodynamic phase (MUSIC)
- AMY evolution – MC simulation of the rate equ’s.
Parton propagation

Process include in MARTINI (all of them can be switched on & off):

- **Inelastic:**
  - [Diagram of inelastic process]

- **Elastic:**
  - [Diagram of elastic process]

- **Conversion:**
  - [Diagram of conversion process]

- **Photon:** emission & conversion
Parton propagation

Resummation for the inelastic processes included:

- All such graphs are leading order (BDMPS)
- Full leading order SD-Eq (AMY): (Figure from G. Qin)
Parton propagation

\[ (PYTHIA \, 8.1) \]

An example path in MARTINI. (Figure from B. Schenke)
While this is happening in the background ...
Projection on to the longitudinal plane
Projection onto the transverse plane
Pion production

[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

- $\pi^0$ spectra and $R_{AA}$

For RHIC, $\alpha_S = 0.29$
Photon production

[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

- Spectra and $R_{AA}^\gamma$

- $\alpha_s = 0.29$
Azimuthal dependence of $R_{AA}$

$R_{AA}(p_T, \Delta \phi)$

$\alpha_S = 0.29$

AMY+elastic + 3+1D hydro, $b=7.5$ fm, $\Delta \phi = 0-15^\circ$
AMY+elastic + 3+1D hydro, $b=7.5$ fm, $\Delta \phi = 75-90^\circ$
PHENIX 20-30% $\Delta \phi = 0-15^\circ$
PHENIX 20-30% $\Delta \phi = 75-90^\circ$

Jeon (McGill)
MARTINI – LHC $dN/dA$

[Young, Schenke, Jeon, Gale, Phys. Rev. C 84, 024907 (2011)].

- $A = (E_t - E_a)/(E_t + E_a)$
- This is with ideal hydro with a smooth initial condition
- Full jet reconstruction with FASTJET
- $\alpha_S = 0.27$ seems to work.

ATLAS, PRL 105 (2010) 252303

[Young, Schenke, Jeon, Gale, Phys. Rev. C 84, 024907 (2011)].

- \( A = \frac{(E_t - E_a)}{(E_t + E_a)} \)
- This is with ideal hydro with a smooth initial condition
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ATLAS, QM 2011

**Not the full story**

### [Clint Young’s HP2012 Proceedings]

- $R_{AA}$ – For LHC, constant $\alpha_S$ suppresses jets too much.
- Need to incorporate finite length effect (Caron-Huot-Gale) and running $\alpha_S$. This is with maximum $\alpha_S = 0.27$.
- Don’t quite get azimuthal dependence yet. $\Delta \phi$ broadening may be due to the background fluctuations → Need to combine UrQMD background?

**Graphs:**

- **Left Graph:**
  - $R_{AA}$ of all charged particles
  - $p_T$ vs. $R_{AA}$
  - $\alpha_s=0.27$, 0-5% centrality, finite-size dependence, running coupling
  - LHC

- **Right Graph:**
  - $dN/d\phi$ vs. $\phi$
  - $p+p$ PYTHIA+fastjet
  - $\alpha_s=0.3$
  - $\alpha_s=0.27$
  - $\alpha_s=0.25$
  - ATLAS pp, 7 TeV
  - ATLAS Pb+Pb, 0-10%
Few last words

- So many nuclear experiments are being done/planned. – RHIC, LHC, Raon, FRIB, FAIR, JPARC, Dubna, HIRFL-CSR ...

- There never have been a time in history when so much information is so readily available.

- This is a great time to be/become a nuclear physicist.


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