Neutron Stars
Birth, Structure, Cooling
Edward Brown
Textbook view of massive stars

*Essential Cosmic Perspective*, Bennett et al.
Massive stars: advanced stages of burning with *MESA*, Paxton et al. 2010
A note about silicon “burning”

For $T \gtrsim 10^9$ K, photodissociation reactions such as $(\gamma, n), \ (\gamma, p)$ become possible. Forward and inverse reaction flows come into balance, and material relaxes to Nuclear Statistical Equilibrium. Most bound nuclei around Fe-peak.
Collapse

For a relativistic ideal gas,

\[
P = \frac{1}{4} n\mu = \frac{1}{4} (3\pi^2)^{1/3} \hbar c \left( \frac{Y_e \rho}{m_u} \right)^{4/3}.
\]

How does radius scale with mass for \( P \sim M^2 R^{-4}, \rho \sim MR^{-3} \)?

A relativistic gas has a characteristic mass scale, the Chandrasekhar mass

\[
M_{\text{Ch}} = 1.4(2Y_e)^2 M_\odot.
\]
Energy considerations

One can use the virial system to show that if \( P \sim \rho^\gamma \), then the total (gravitational and thermal) energy of the star is

\[
E = - \frac{3(\gamma - 1) - 1}{5\gamma - 6} \frac{GM^2}{R}.
\]
Core (~1 \( M_{\text{sun}} \)) becomes unstable, dynamically collapses

What is the timescale for collapse? Construct an estimate.
5. The super-nova process

We have tentatively suggested that the super-nova process represents the transition of an ordinary star into a neutron star. If neutrons are produced on the surface of an ordinary star they will "rain" down towards the center if we assume that the light pressure on neutrons is practically zero. This view explains the speed of the star's transformation into a neutron star. We are fully aware that our suggestion carries with it grave implications regarding the ordinary views about the constitution of stars and therefore will require further careful studies.

W. Haage
F. Zwicky

Mt. Wilson Observatory and
California Institute of Technology, Pasadena.
May 28, 1934.
FIG. 10: The main stages of evolution of a neutron star, from Ref. [133]. Shading indicates approximate relative temperature, a delayed collapse to a black hole is still possible during this epoch.

Following the onset of neutrino transparency, the core continues to cool by neutrino emission, but the star's crust remains warm and cools less quickly. The crust serves as an insulating blanket which prevents the star from coming to complete thermal equilibrium and keeps the surface relatively warm ($T \approx 3 \times 10^6 \text{ K}$) for up to 100 years (stage V). The temperature of the surface after the interior of the star becomes isothermal (stage VI) is determined by the rate of neutrino emission in the star's core and the composition of the surface.

B. Theoretical expectations

To understand what aspects of the EOS and structure can be probed by neutrinos, we examine some analytic models for proto-neutron star evolution [133, 136]. For clarity and simplicity, we employ Newtonian gravitation, as this does not affect the qualitative conclusions we will draw. We will assume that the neutrino distribution function is well-approximated by a Fermi-Dirac distribution, so the neutrino number density is

$$n_{\nu} = \int_{0}^{\infty} n_{\nu}(E_{\nu}) dE_{\nu},$$

where

$$n_{\nu}(E_{\nu}) = \frac{E_{\nu}^2}{\pi^3(\hbar c)^3 f_{\nu}(E_{\nu})},$$

and

$$f_{\nu}(E_{\nu}) = \left[1 + e^{(E_{\nu} - \mu_{\nu})/T}\right]^{-1}.$$
Supernova 1987a: neutrinos detected by Kamiokande!
neutron star basics

A solar mass consists of $\sim 10^{57}$ nucleons. If they are separated by typical inter-nucleon distances, what would the radius of the volume containing them be?
neutron star basics

\[ R \sim 1 \text{ fm} \cdot (10^{57})^{1/3} \sim 10 \text{ km} \]

\[ \frac{GM}{Rc^2} \approx 0.2 \approx \frac{\Delta \lambda}{\lambda} \]

\[ \frac{GMm_H}{R} \approx 200 \text{ MeV} \]
Exercise

What is the dynamical time of a solar-mass neutron star (density is \( \approx 2 \cdot 10^{14} \) g/cm\(^3\); density of sun is \( \approx 1 \) g/cm\(^3\))? 

Recall that we defined a dynamical time 

\[
\tau_{\text{dyn}} = (G\bar{\rho})^{-1/2}
\]
Discovery!

radio pulsations discovered (Hewish, Bell, et al. 1968)

Gold (1968): explained as due to rotation of a neutron star (WHY?)
Crab Nebula: Remnant of supernova in 1054
Discovery!

radio pulsations discovered (Hewish, Bell, et al. 1968)
Gold (1968): explained pulsations as being due to rotation of a magnetized neutron star (WHY?)
Unlike white dwarfs, can’t assume ideal Fermi gas EOS
neutron stars and nuclear physics

Nuclear Astrophysics

- The Origin of the Elements
- Explosive Nucleosynthesis

- Composition of Neutron Stars—There are roughly one billion neutron stars in our galaxy, yet their internal structure and the composition of their crusts are poorly understood. ... a FRIB can study the central questions concerning the composition and energetics of their upper mantles.—*Scientific Opportunities with a Rare-Isotope Facility in the United States*, National Research Council (2006)
neutron stars and astrophysics

“What is the nature of dense matter?” is one of the top unanswered questions for the 21st century

-Connecting Quarks with the Cosmos, Nat’l Academies Press

“Measuring neutron star masses and radii yields direct information about the interior composition [of neutron stars] that can be compared with theoretical predictions.”

-New Worlds, New Horizons in Astronomy and Astrophysics (Decadal survey of astronomy)
Neutron stars are a unique probe of dense matter
Thermodynamics near saturation density

see review by Lattimer & Prakash

Let’s examine properties of $npe$ matter near saturation density $n = 0.16 \text{ fm}^{-3}$. The proton fraction is

$$x = n_p / (n_n + n_p),$$

and charge neutrality requires that

$$n_e = n_p = xn.$$

We write the energy per nucleon as

$$\varepsilon(n, x) = \varepsilon_S(n) + \varepsilon_A(n)(1 - 2x)^2.$$
Why so neutron-rich?

In $\beta$-equilibrium,

\[
\mu_e = \mu_n - \mu_p = \left( \frac{\partial \epsilon}{\partial n_n} \right)_{n_p} - \left( \frac{\partial \epsilon}{\partial n_p} \right)_{n_n} \]

\[= -\frac{\partial \epsilon}{\partial x} = 4\varepsilon_A (1 - 2x).\]

For relativistic electrons, $\mu_e = (3\pi^2 n_e)^{1/3} \hbar c$, so we can solve for the proton fraction

\[
x = \left[ 6 + \frac{3\pi^2}{64} \left( \frac{\hbar c}{\varepsilon_A} \right)^3 n \right]^{-1} \approx 0.04
\]

for $\varepsilon_A = 30$ MeV.
What is the pressure at saturation?

\[ P = n^2 \frac{\partial \varepsilon}{\partial n} + \frac{n_e \mu_e}{4} \]

\[ = n^2 \frac{\partial \varepsilon_s}{\partial n} + n^2 \frac{\partial \varepsilon_A}{\partial n} (1 - 2x)^2 + \varepsilon_A (1 - 2x) x n \]

\[ = n(1 - 2x) \left[ n \frac{\partial \varepsilon_A}{\partial n} (1 - 2x) + x \varepsilon_A \right]. \]
equation of state from heavy nucleus collisions
Danielelewicz et al. (2002) Science

ongoing projects at NSCL, RIKEN (SAMURAI/TPC)
Relativistic stellar structure equations (non-rotating)

Tolman; Oppenheimer & Volkoff 1939; Thorne 1967

\[
\begin{align*}
\frac{dm}{dr} &= 4\pi r^2 \rho \\
\frac{dP}{dr} &= -\rho \frac{G m(r)}{r^2}
\end{align*}
\]

\[
\begin{align*}
\frac{dm}{dr} &= 4\pi r^2 \rho \\
\frac{dP}{dr} &= -\rho \frac{G m}{r^2} \left\{ \frac{(1 + 4\pi r^3 P/mc^2)}{(1 - 2Gm/rc^2)} \right\}
\end{align*}
\]
Detection: Isolated neutron stars

about 1700 pulsars detected; about 50 are in binary systems with some mass information

very precise mass information;
but, no radius information

fastest spin is 716 Hz (Hessels et al. 2006; faster than household blender!)
Neutron stars can reach $2 \, M_{\odot}$! Demorest et al. 2010
A neutron star cooling from its fiery birth

*ROSAT* Image of thermal emission from neutron star in Puppis A supernova remnant
cooling: the Urca process
Gamow & Schoenberg 1941

In npe-matter, maintain $\beta$-equilibrium via

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$p + e^- \rightarrow n + \nu_e$$

Integration of the rate over phase space gives a $T^6$ dependence of the cooling rate.
but this is blocked...

Chiu & Salpeter, Bahcall & Wolff

mom. cons. \[ p_{F,n} < p_{F,e} + p_{F,p} \]

\( \beta \)-equil. \[ \mu_e = \mu_n - \mu_p \]

charge neut. \[ n_e = n_p \]

Need a “bystander” particle, e.g., \( n + n \rightarrow n + p + e^- + \bar{\nu}_e \).
This rate is > \( 10^6 \) times slower at typical \( T < 10^8 \) K.

If \( n_p/n > 0.11 \), direct process can go.
Also if other channels, e.g. hyperons, are available.

A high symmetry energy implies that neutron stars should cool rapidly!
—Lattimer & Prakash 2007
Neutron stars cooling from their fiery birth: “fast” and “slow” neutrino emissivities

plot from Yakovlev et al. ‘02; see also Page et al. ‘04, ’09
Cooling without PBF vector channel suppression

Neutron $^3P_2$ gap = "b"

Heavy element envelopes

Light element envelopes

Fig. 9.—Comparison of predictions of the minimal cooling scenario with data; all models are for 1.4 $M_\odot$ stars built using the EOS of APR (Akmal, Pandharipande & Ravenhall 1998). In the right panels the suppression of the vector channel in the Cooper-pair neutron emission is fully taken into account whereas, for comparison, in the left panels the suppression has been omitted. In each row, the two panels have the same neutron $^3P_2$ gap, from a vanishing gap in the upper row to our model gaps "a" and "b" (following the notations of Figure 10 in Paper I) in the next two rows. In each panel two sets of cooling trajectories, either with light or with heavy element envelopes, are shown which include 25 curves corresponding to 5 choices of the neutron $^1S_0$ and of the proton $^1S_0$ gaps covering the range of predictions about the sizes of these gaps.

Equation 13), as in our models "b" and "c". In the extreme case that the neutron $^3P_2$ gap is vanishingly small and also that all observed young cooling neutron stars have light element envelopes, then nearly all of them, with the possible exception of PSR B0538+2817, are observed to be too cold to be compatible with minimal cooling predictions. In the less extreme possibility of a heterogeneity in chemical composition and a vanishingly small neutron $^3P_2$ gap, we still find that more than half (seven out of twelve) of the observed young cooling neutron stars are too cold to be compatible with minimal cooling. (Notice that among the remaining five, out of twelve stars, the compact objects in Cas A and the Crab still have only upper limits.) If these conditions on the $T_c$ curve are not satisfied for a particular model of superfluidity in dense matter, then that model also requires enhanced cooling beyond the minimal cooling paradigm. These results highlight the importance of the neutron $^3P_2$ gap in more precise terms than discussed in Paper I.

Our conclusion regarding the need for heterogeneity in the chemical composition of the atmosphere is consistent with the results of Kaminker, et al. (2006), who had to employ both light and heavy element atmospheres in their cooling models to match the data of most stars.

That it is apparently possible to explain the majority of the normally-emitting neutron stars with the minimal cooling
Accreting neutron stars
≈ solar mass star

$P_{\text{orb}} = \text{minutes–hours}$

Each accreted H releases

$$\approx \frac{GMm_H}{R} \approx 200 \text{ MeV}.$$ 

Fusing H to He releases

$$\approx 7 \text{ MeV}$$

per nucleon.