Unravelling the neutron star interior: Prospects and challenges
Unravelling the neutron star interior: Prospects and challenges

Sanjay Reddy
INT, Univ. of Washington
Cold Compression
Cold Compression

\[ x \rightarrow 10^{-4} \ x \]
Cold Compression

\[ x \Rightarrow 10^{-4} x \]

\[ x \Rightarrow 10^{-10} x \]
Cold Compression

\[ x \Rightarrow 10^{-4} x \]

\[ x \Rightarrow 10^{-10} x \]

\[ x \Rightarrow (1-\varepsilon) x \]
Getting to $\sim 2.5 \times 10^{14}$ g/cm$^3$: Nuclear Physics

Properties of nuclei - almost get us there!
<table>
<thead>
<tr>
<th>Density</th>
<th>Energy</th>
<th>Phenomena</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3 - 10^6 \text{ g/cm}^3$</td>
<td>Electron Chemical Pot. $\mu_e = 10 \text{ keV– MeV}$</td>
<td>Ionization</td>
</tr>
<tr>
<td>$10^6 - 10^{11} \text{ g/cm}^3$</td>
<td>Electron Chemical Pot. $\mu_e = 1-25 \text{ MeV}$</td>
<td>Neutron-rich Nuclei</td>
</tr>
<tr>
<td>$10^{11} - 10^{14} \text{ g/cm}^3$</td>
<td>Neutron Chemical Pot. $\mu_n = 1-30 \text{ MeV}$</td>
<td>Neutron-drip</td>
</tr>
<tr>
<td>$10^{14} - 10^{15} \text{ g/cm}^3$</td>
<td>Neutron Chemical Pot. $\mu_n = 30-1000 \text{ MeV}$</td>
<td>Nuclear matter Hyperons or Quarks?</td>
</tr>
</tbody>
</table>
Frustration in Neutron Matter

Too many down quarks

Hyperon Matter

Kaon Condensed Matter

Glendenning (1991)

Kaplan & Nelson (1986)
Strangeness in Dense Matter: Theory very uncertain.

\[ E_\Lambda(p = 0) = M_\Lambda + V_{n\Lambda}(\rho) \leq \mu_B \]
\[ E_{K^-}(p = 0) = M_{K^-} + V_{nK^-}(\rho) \leq \mu_e \]

Interactions are poorly known
Rel. Fermi gas of u,d,s quarks

**Asymptotic Density**

Interactions are nearly perturbative - calculable.

Interactions lead to pairing and color superconductivity

Strongest attraction in color-antisymmetric channel: **Color-Flavor-Locking**

\[ \Delta \gg \frac{m_s^2}{4\mu} \]

Alford, Rajagopal, Wilczek (1999)
Asymptotic Density

Rel. Fermi gas of u,d,s quarks

Interactions are nearly perturbative - calculable.

Interactions lead to pairing and color superconductivity

Strongest attraction in color-antisymmetric channel:
Color-Flavor-Locking

\[ \Delta \gg \frac{m_s^2}{4\mu} \]

Alford, Rajagopal, Wilczek (1999)
Interactions are nearly perturbative - calculable.

Interactions lead to pairing and color superconductivity

Strongest attraction in color-antisymmetric channel: Color-Flavor-Locking

\[ \Delta \gg \frac{m_s^2}{4\mu} \]

Alford, Rajagopal, Wilczek (1999)
Quark Matter in Neutron Stars

Interactions are non-perturbative. Difficult to predict critical density.

\[ \Delta \approx \frac{m_s^2}{4 \mu} \]

• Difficult to predict ground state.
• Complicated spectrum of excitations (Strongly coupled quasi-particles)

Ground state is CFL.
• Low energy spectrum is simple (Goldstone modes - weakly coupled)
Neutron Star in Depth (km)

\[ \rho \text{(g/cm}^3\text{)} \]

- $^{56}\text{Fe}$ nuclei + e$^-$
- $^{62}\text{Ni}$
- neutron-rich nuclei relativistic electrons
- $^{66}\text{Ni}$
- $^{84}\text{Se}$
- $^{118}\text{Kr}$

- spherical nuclei + superfluid neutrons + e$^-$
- non-spherical nuclei or pasta phase
- liquid core neutron-rich matter center at 10 km

Layers:
- Atmosphere
- Ocean
- Crust
- Core
Phase & Composition

Crust:
Inner Crust:
(Solid-Superfluid)
nuclei, electrons, neutrons

Outer Crust:
(solid) nuclei, electrons

Inner Core
~ 3

Outer Core
(Superfluid-Superconductor)
nuclei, neutrons, protons, electrons
~ 10.5

M ~ 1.4 M☉
What can we observe?

- Orbital Characteristics in Binaries
- Surface Luminosity
- Spin

What can we infer?

- Explosions & Flares
- Neutrinos (Supernova)
- Gravity Waves (likely within 5 yrs!)

Hard Physics

- Mass
- Radius
- Crust thickness
- Oscillations frequencies
- Ground state EoS

Soft Physics

- Surface and interior temperature
- Neutrino cooling and scattering rates
- Electrical & Thermal Conductivities
- Damping rates
- Low energy fluctuations
The Nuclear Equation of State

\{
\begin{align*}
\epsilon &= \rho \, E(\rho) \\
\rho(E) &= \rho^2 \frac{\partial E(\rho)}{\partial \rho}
\end{align*}
\}

Nuclei exist here

\[ E(\rho_n, \rho_p) \text{ MeV} \]

\[ \frac{\rho}{\rho_0} = 1 \]

\[ \rho_0 = 0.16 \text{ fm}^{-3} \]

Nuclear saturation density
The Nuclear Equation of State

\[ \epsilon = \rho \ E(\rho) \]

\[ P(\epsilon) = \rho^2 \frac{\partial E(\rho)}{\partial \rho} \]

Nuclei exist here

Neutron Stars

\[ x_P = 0 \]

\[ \frac{\rho}{\rho_0} = 1 \]

\[ \rho_0 = 0.16 \text{ fm}^{-3} \]

Nuclear saturation density
The Nuclear Equation of State

\[
\begin{align*}
\epsilon &= \rho \, E(\rho) \\
P(\epsilon) &= \rho^2 \frac{\partial E(\rho)}{\partial \rho}
\end{align*}
\]

Nuclei exist here

\[E(\rho_n, \rho_p) \text{ MeV}\]

\[\rho_0 = 0.16 \text{ fm}^{-3}\]

\[S(\rho_0) = 32 \pm 2 \text{ MeV}\]
Nuclear Many Body Theory

\[ H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \cdots \]

Phenomenological potentials (Argonne etc) tuned to fit scattering and light nuclei.

Chiral potentials and softer low energy potentials obtained using RG.

\[ E(\rho_n, \rho_p) : \text{Energy per particle} \]
Neutron Matter & 3N Forces

Figure 1: The energy per particle of neutron matter for different values of the nuclear symmetry energy $E_{\text{sym}}$(MeV). For each value of $E_{\text{sym}}$, the corresponding band shows the effect of different spatial and spin structures of the three-neutron interaction. The inset shows the linear correlation between $E_{\text{sym}}$ and its density derivative $L$.

- Our assumption is that the symmetry energy is defined as the difference between the energy per particle in symmetric nuclear and neutron matter at nuclear density and is denoted by $E_{\text{sym}} = E_{\text{neutron}}(\rho_0) - E_{\text{nuclear}}(\rho_0)$. It is determined from model fits to nuclear masses favors $E_{\text{sym}} = 30.5$ MeV (NN).

- Although not proven, we make two reasonable assumptions: 1) relativistic effects in neutron matter show a similar density dependence to the short-range three-nucleon interaction as carefully studied in Ref. \textsuperscript{25}; and 2) four-nucleon force contributions are suppressed relative to the 3n force for densities up to $2-3 \rho_0$. This assumption can be justified at nuclear density by the high precision fits to light-nuclei obtained with only 3n forces \textsuperscript{21}, at higher density this model assumption can be tested by its predicted correlation between properties of neutron-rich nuclei and neutron stars.

- The inset shows the linear correlation between $E_{\text{sym}}$ and its density derivative $L$. The points represent different values of $E_{\text{sym}}$ and $L$. The lines are fit to the data points, showing the trend of the correlation.

- The Fermi-gas line is shown as a dashed purple line for reference. The Fermi-gas model provides a baseline for comparison with the more complex interactions.

Gandolfi, Carlson, Reddy (2010)
Neutron-rich Nuclei

- Nuclear masses are sensitive to the symmetry energy.
- Neutron distribution at the surface is sensitive to its density dependence.

Bethe-Weizsäcker formula:

\[
E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_a(A) \frac{(N-Z)^2}{A} + E_{\text{mic}}
\]

\[
\frac{1}{a_a(A)} = \frac{1}{a_a^V} + \frac{1}{a_a^S} A^{1/3}
\]

\[
\begin{align*}
& a_a^V \Rightarrow E_{\text{sym}} (S) \\
& a_a^S \Rightarrow \frac{\partial E_{\text{sym}}}{\partial \rho} (L/3)
\end{align*}
\]

Neutron-rich Nuclei

- Nuclear masses are sensitive to the symmetry energy.
- Neutron distribution at the surface is sensitive to its density dependence.

Bethe-Weizsäcker formula:

\[ E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_a(A) \frac{(N-Z)^2}{A} + E_{mic} \]

\[ \frac{1}{a_a(A)} = \frac{1}{a_a^V} + \frac{1}{a_a^S} A^{1/3} \]

\[ a_a^V \Rightarrow E_{sym} (S) \]

\[ a_a^S \Rightarrow \frac{\partial E_{sym}}{\partial \rho} (L/3) \]

Neutron-rich Nuclei

- Nuclear masses are sensitive to the symmetry energy.
- Neutron distribution at the surface is sensitive to its density dependence.

\[ \Delta R = R_n - R_p \]

Bethe-Weizsäcker formula:

\[ E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_a(A) \frac{(N-Z)^2}{A} + E_{\text{mic}} \]

\[ \frac{1}{a_a(A)} = \frac{1}{a_a^V} + \frac{1}{a_a^S} A^{1/3} \]

\{ \begin{align*}
  a_a^V & \Rightarrow E_{\text{sym}} (S) \\
  a_a^S & \Rightarrow \frac{\partial E_{\text{sym}}}{\partial \rho} (L/3)
\end{align*} \}

Nuclear experiments to measure S and L:

- Masses of very neutron-rich nuclei near the neutron-drip will reduce systematic errors in extracting S from model fits. (Facilities such as FRIB, FAIR, JPARC)
- Distribution of neutrons in the surface region of neutron-rich nuclei can measure L indirectly. Eg. PREX at Jefferson lab.

To extrapolate to high density we need a theory that can predict S & L.
Mass-Radius

\[ M(R) \leftrightarrow P(\epsilon) \]
Mass-Radius

$M(R) \leftrightarrow P(\epsilon)$
Mass-Radius

$M(R) \leftrightarrow P(\epsilon)$

![Graph showing the relationship between mass and radius for different models, including Fermi Gas and 2N.](image-url)
Mass-Radius

$M(R) \leftrightarrow P(\epsilon)$

2N

3N

Fermi Gas
Mass-Radius

- **Soft EoS:** low maximum mass and small radii
- **Stiff EoS:** high maximum mass and large radii

![Graph showing Mass-Radius relationship with curves for different EoS models and mass range](image-url)
Causality: $R > 2.9 \text{ (GM/c}^2\text{)}$

$\rho_{\text{central}} = 2 \rho_0$

$\rho_{\text{central}} = 3 \rho_0$

$\rho_{\text{central}} = 4 \rho_0$

$\rho_{\text{central}} = 5 \rho_0$

$E_{\text{sym}} = 30.5 \text{ MeV (NN)}$

$\rho_{\text{central}} = 1.4 M_{\odot}$

$\rho_{\text{central}} = 1.97(4) M_{\odot}$

The intersection with the orange lines show roughly the central densities realized in stars with different masses and radii. The dot-dashed lines show the masses of typical neutron star with $M = 1.4 M_{\odot}$ and the recently observed mass of neutron star of Ref. 

The yellow region is excluded by the causality constraint on the equation of state.
Figure 2: Mass-Radius relation for equations of state with three-neutron interactions corresponding to the bands for different $E_{\text{sym}}$ shown in Fig. 1. The intersection with the orange lines show roughly the central densities realized in stars with different masses and radii. The dot-dashed lines show the masses of typical neutron star with $M = 1.4 \, M_{\odot}$ and the recently observed mass of neutron star of Ref. [1]. The yellow region is excluded by the causality constraint on the equation of state.

The estimated error in the prediction for the neutron star radius with a canonical mass of $1.4 \, M_{\odot}$ leads to an uncertainty of about 3 km for the radius, while the error due to uncertainties in the short-distance structure of the 3n force predicts a radius uncertainty of less than 1 km. The blue band corresponds to the band of equations of state shown in Fig. 1 with same color. They all correspond to $E_{\text{sym}} = 33.7 \, \text{MeV}$. Similarly the green band corresponds to the green band of equations of state shown in Fig. 1 with $E_{\text{sym}} = 32.0 \, \text{MeV}$. The red curve is the prediction for neutron star mass and radius obtained without 3n interaction and the black curve is one for which the 3n is very strong with $E_{\text{sym}} = 35.1 \, \text{MeV}$ corresponding to the original Urbana IX 3n force.
Maximum Mass & Phase Transitions

Note differences at $\rho_0$!
Maximum Mass & Phase Transitions

note differences at $\rho_0$!
Maximum Mass & Phase Transitions

\[ P \text{ (MeV/fm}^3) \]
\[ \varepsilon \text{ (MeV/fm}^3) \]
\[ M \text{ (\( M_\odot \))} \]
\[ R \text{ (km)} \]

- Note differences at \( \rho_0 \)!

- 4\( \rho_0 \) Phase Transition

- Mass Measurements

Hybrid Star

APR

R = R_{\text{MS}} = 3 R_\odot
The 2 solar mass neutron star rules out a strong first-order transitions at supra-nuclear density
BEYOND MASS & RADIUS: TRANSPORT PHENOMENA

How does dense matter:

• Cool

• Conduct heat and electric currents

• Respond to angular momentum

• Oscillate when its perturbed?
• The rate of production and scattering of neutrinos (neutron star cooling, supernova), and scattering of electrons (thermal relaxation) are related to the thermal fluctuation spectrum.

Rate = Coupling $\times$ Kinematics $\times$ Response Function
The rate of production and scattering of neutrinos (neutron star cooling, supernova), and scattering of electrons (thermal relaxation) are related to the thermal fluctuation spectrum.

\[ q_0 = E_1 - E_3 \]

\[ q_0 = E_1 + E_3 \]

\[ q_0 = -E_1 - E_3 \]

Rate = Coupling \( \times \) Kinematics \( \times \) Response Function
Response of Interacting System

\[ \omega, q \]

\[ \nu \]

\[ \text{dense matter} \]

\[ \Delta R \]

\[ q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R} \]

\[ \text{emission} \]

\[ \omega = q \]

\[ \text{scattering} \]
Response of Interacting System

\[ q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R} \]

\[ q = \frac{2\pi}{\lambda} < \frac{1}{\Delta R} \]
Response of Interacting System

\[ \omega, q \]

\[ \Delta R \]

\[ q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R} \]

\[ q = \frac{2\pi}{\lambda} < \frac{1}{\Delta R} \]
Response of Interacting System

\[ \omega = \frac{2\pi}{\tau} > \frac{2\pi}{\tau_{\text{collision}}} \]

\[ q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R} \]

\[ q = \frac{2\pi}{\lambda} < \frac{1}{\Delta R} \]

\[ \omega = \frac{2\pi}{\tau_{\text{collision}}} \]

\[ \tau_{\text{collision}} = \text{Collision Time} \]
Weak Interaction Rates

\[ L = \frac{G_F}{2\sqrt{2}} l_\nu(x) j^\mu(x) \]

\[ l_\nu = \bar{\nu}(x) \gamma_5 (1 - \gamma_5) \nu(x) \]

\[ j^\mu = \bar{\psi}(x) (c_V \gamma^\mu - c_A \gamma^\mu_5 + iF_2 \sigma^{\mu\nu} \frac{q_\nu}{2M}) \psi(x) \]

\[
\frac{d^2 \sigma}{V \ d\cos \theta \ dE'} \approx G_F^2 \ \frac{E}{E'} \ \text{Im} \left[ L_{\mu\nu}(k, k + q) \ \Pi^{\mu\nu}(q) \right]
\]

\[ L_{\mu\nu} = \text{Tr} \left[ l_\mu(k) \ l_\nu(k + q) \right] \]

\[ \Pi^{\mu\nu} = \int \frac{d^4 p}{(2\pi)^4} \ \text{Tr} \left[ j^\mu(p) \ j^\nu(p + q) \right] \]
Neutrino-Nucleon Scattering

Neutrinos couple to density and spin

\[ j^\mu(x) = \bar{\psi}(x) \gamma^\mu (c_V - c_A \gamma_5) \psi(x) \]

NR\[ \rightarrow c_V \psi^+ \psi \delta^{\mu 0} - c_A \psi^+ \sigma^i \psi \delta^{\mu i} \]

\[
\frac{d\Gamma}{d\cos \theta dE'_\nu} = \frac{G_F^2}{4\pi^2} (1 - f_\nu(E'_\nu)) E'_\nu^2 \\
\times (c_V^2 (1 + \cos \theta) S(|q|, \omega) + c_A^2 (3 - \cos \theta) S^A(|q|, \omega))
\]

\[ S(|q|, \omega) = \int_{-\infty}^{\infty} dt \exp(i\omega t) \langle \rho(q, t) \rho(-q, 0) \rangle \]

\[ S^A(|q|, \omega) = \int_{-\infty}^{\infty} dt \exp(i\omega t) \delta_{ij} \langle \sigma_i(q, t) \sigma(-q, 0) \rangle \]

Iwamoto & Pethick (1982)
Response of a classical liquid

The density-density correlation for N particles is

\[ \langle \rho(q, 0) \rho(q, t) \rangle = \langle \sum_i e^{-i q \cdot r_i} \sum_j e^{-i q \cdot r_j(t)} \rangle \]

Positions at \( t=0 \)

Positions at \( t \)

Ensemble average

Need to specify equations of motion or \( r_j(t) \).

Classical limit:

\[ r_j(\Delta t) = r_j(0) + v_j \Delta t + \frac{1}{2m} \sum_{i \neq j} F_{ij} t^2 \]
Screening, Damping & Collective Modes

- Strong repulsive Coulomb forces affect the spatial distribution.
- A collective mode exists in the system.
- Response is pushed to high energy.
- Multi-particle excitations smears the response.
Response Functions: In Quantum Fluids

\[ S_q(\omega) = \sum_{\lambda, \lambda'} f_\lambda |\langle \lambda | A_q | \lambda' \rangle|^2 \delta(E_\lambda - E'_\lambda - \omega) \]

\[ = \int dt \ e^{i\omega t} \langle \lambda | A_q(t) A_q^\dagger(0) | \lambda \rangle \]

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]

\[ \omega \]
Response Functions: In Quantum Fluids

\[ S_q(\omega) = \sum_{\lambda,\lambda'} f_\lambda \left| \langle \lambda | A_q | \lambda' \rangle \right|^2 \delta(E_\lambda - E'_{\lambda} - \omega) \]

\[ = \int dt \ e^{i\omega t} \langle \langle \lambda | A_q(t) \ A_q^\dagger(0) | \lambda \rangle \rangle \]

Fermi Motion and Pauli Blocking.

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]
Response Functions: In Quantum Fluids

\[ S_q(\omega) = \sum_{\lambda, \lambda'} f_{\lambda} |\langle \lambda | A_q | \lambda' \rangle|^2 \delta(E_\lambda - E'_\lambda - \omega) \]

\[ = \int dt \ e^{i\omega t} \langle \langle \lambda | A_q(t) A_{q}^\dagger(0) | \lambda \rangle \rangle \]

Fermi Motion and Pauli Blocking.

Correlations and collisions.

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]
Response Functions: In Quantum Fluids

\[ S_q(\omega) = \sum_{\lambda, \lambda'} f_\lambda |\langle \lambda | A_q | \lambda' \rangle|^2 \delta(E_\lambda - E'_{\lambda} - \omega) \]

\[ = \int dt \ e^{i\omega t} \langle \langle \lambda | A_q(t) A_q^{\dagger}(0) | \lambda \rangle \rangle \]

Fermi Motion and Pauli Blocking.

Correlations and collisions.

Cooper Pairing & Superfluidity.

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]
Computing Correlation Functions

No exact methods exist in strongly coupled quantum systems.

In the Fermi gas:

In the mean field approximation:

Corrections to single particle energy

Vertex Corrections
Computing Correlation Functions

No exact methods exist in strongly coupled quantum systems.

In the Fermi gas:

In the mean field approximation:

Corrections to single particle energy

Vertex Corrections

RPA
Correlations in a nuclear liquid

\[
\frac{d\Gamma(E_1)}{d \cos \theta \ dq_0} = \frac{G_F^2}{4\pi^2} (E_1 - q_0)^2 \left[(1 + \cos \theta) S_{V}^{\text{RPA}}(q_0, q) + (3 - \cos \theta) S_{A}^{\text{RPA}}(q_0, q)\right]
\]

### Neutrino scattering can be significantly reduced
Can we measure correlation functions in neutron star matter?
Can we measure correlation functions in neutron star matter?

Yes.

This will require at least two of the following:

• New experiments with exotic targets.
• Temporal phenomena in neutron stars.
• Theoretical understanding of transport properties.
Neuron star matter.
Warning: Radioactive outside the pressure chamber

Missing target. Awaiting new collaborators.
Time Dependent Phenomena

- Thermal relaxation of the core.
- Neutron star cooling.
- Thermal relaxation of the crust.
Thermal Relaxation of the Core

Once in a lifetime we may detect a neutrino burst from a galactic supernova.

\[ \approx 10^4 \text{ neutrinos for SN @ 10 kpc} \]

\[ \nu_e, \bar{\nu}_e, \nu_X, \bar{\nu}_X \]
Supernova Neutrinos

Past:
SN 1987a: ~ 20 neutrinos ..in support of supernova theory

Future:
Can detect ~10,000 neutrinos from galactic supernova

\[ 3 \times 10^{53} \text{ ergs} = 10^{58} \times 20 \text{ MeV Neutrinos} \]

\[ \frac{dN_{\text{detect}}}{dt} \sim \frac{\sigma_{\text{ref}} \times n_p \times M_{\text{tons}}}{4\pi D^2} \frac{E_v^2}{m_e^2} \frac{dN_{\text{emit}}}{dt} \]

Pons et al. (2002)
• Neutrinos are trapped during core collapse. Collapse is nearly adiabatic.

• Gravitational binding energy is stored as thermal energy and lepton degeneracy energy.
Traversing the phase diagram

Traversing the phase diagram

T (MeV)

QGP

Novel High Density Phases: Hyperons, Kaons Quark Matter ..

Nuclear Matter

Nuclei

Neutron Stars

Nuclei

μ_B (MeV)

930

~1200 ?

?
Traversing the phase diagram

Novel High Density Phases: Hyperons, Kaons Quark Matter ..

QGP

Nuclear Matter

Neutron Stars

930 ~1200 ? ? μB (MeV)

T (MeV)

Collapse
Traversing the phase diagram

Novel High Density Phases: Hyperons, Kaons, Quark Matter...

QGP

PNS Evolution

Collapse

Nuclear Matter

Neutron Stars

Nuclei

T (MeV)

\( \mu_B \) (MeV)

930

\(~1200\)
Protoneutron Star Evolution

Neutrino diffusion cools the PNS.

Protoneutron Star Evolution

Neutrino diffusion cools the PNS.

Typical time-scales:

\[
T(t) \approx T(t = 0) \left(1 - \frac{t}{\tau_C}\right)
\]

\[
\tau_C \approx C_V \frac{R^2}{c \langle \lambda_\nu \rangle}
\]

Neutron Star Tomography

- Time structure of the neutrino signal maps the neutrino opacity as a function of depth.
- Opacity is directly related to spectrum of density and spin fluctuations in dense matter.
- Important to note that several other astrophysical effects can complicate this simple interpretation.
Late Time Cooling in X-Rays

- Cooling of isolated neutron stars.

- Thermal relaxation of accreting neutron stars.

Neutron Star in a Supernova Remnant

Cassiopeia A

Neutron star in X-ray Binary

KS1731-260

chandra.harvard.edu/photo/2011/casa/

chandra.harvard.edu/photo/2001/ks1731/
NEUTRON STAR COOLING

Crust cools by conduction

Isothermal core cools by neutrino emission

Surface photon emission dominates at late time $t > 10^6$ yrs

Basic neutrino reactions:

- $n \rightarrow p + e^- + \bar{\nu}_e$
- $e^- + p \rightarrow n + \nu_e$
- $n + n \rightarrow n + p + e^- + \bar{\nu}_e$
- $e^- + p + n \rightarrow n + n + \nu_e$

Fast: Direct URCA

Slow: Modified URCA
Table 1. Dominant neutrino emission processes.

<table>
<thead>
<tr>
<th>Name</th>
<th>Process</th>
<th>Emissivity† (erg cm⁻³ s⁻¹)</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified Urca cycle (neutron branch)</td>
<td>( n + n \rightarrow n + p + e^- + \bar{\nu}_e ) ( n + p + e^- \rightarrow n + n + \nu_e )</td>
<td>( \sim 2 \times 10^{21} \ R \ T_9^8 )</td>
<td>Slow</td>
</tr>
<tr>
<td>Modified Urca cycle (proton branch)</td>
<td>( p + n \rightarrow p + p + e^- + \bar{\nu}_e ) ( p + p + e^- \rightarrow p + n + \nu_e )</td>
<td>( \sim 10^{21} \ R \ T_9^8 )</td>
<td>Slow</td>
</tr>
<tr>
<td>Bremsstrahlung</td>
<td>( n + p \rightarrow n + p + \nu + \bar{\nu} ) ( p + p \rightarrow p + p + \nu + \bar{\nu} )</td>
<td>( \sim 10^{19} \ R \ T_9^8 )</td>
<td>Slow</td>
</tr>
<tr>
<td>Cooper pair formations</td>
<td>( n + n \rightarrow [nn] + \nu + \bar{\nu} ) ( p + p \rightarrow [pp] + \nu + \bar{\nu} )</td>
<td>( \sim 5 \times 10^{21} \ R \ T_9^7 ) ( \sim 5 \times 10^{19} \ R \ T_9^7 )</td>
<td>Medium</td>
</tr>
<tr>
<td>Direct Urca cycle (nucleons)</td>
<td>( n \rightarrow p + e^- + \bar{\nu}_e ) ( p + e^- \rightarrow n + \nu_e )</td>
<td>( \sim 10^{27} \ R \ T_9^6 )</td>
<td>Fast</td>
</tr>
<tr>
<td>Direct Urca cycle (( \Lambda ) hyperons)</td>
<td>( \Lambda \rightarrow p + e^- + \bar{\nu}_e ) ( p + e^- \rightarrow \Lambda + \nu_e )</td>
<td>( \sim 10^{27} \ R \ T_9^6 )</td>
<td>Fast</td>
</tr>
<tr>
<td>Direct Urca cycle (( \Sigma^- ) hyperons)</td>
<td>( \Sigma^- \rightarrow n + e^- + \bar{\nu}_e ) ( n + e^- \rightarrow \Sigma^- + \nu_e )</td>
<td>( \sim 10^{27} \ R \ T_9^6 )</td>
<td>Fast</td>
</tr>
<tr>
<td>( \pi^- ) condensate</td>
<td>( n + \langle \pi^- \rangle \rightarrow n + e^- + \bar{\nu}_e )</td>
<td>( \sim 10^{26} \ R \ T_9^6 )</td>
<td>Fast</td>
</tr>
<tr>
<td>( K^- ) condensate</td>
<td>( n + \langle K^- \rangle \rightarrow n + e^- + \bar{\nu}_e )</td>
<td>( \sim 10^{25} \ R \ T_9^6 )</td>
<td>Fast</td>
</tr>
<tr>
<td>Direct Urca cycle (u-d quarks)</td>
<td>( d \rightarrow u + e^- + \bar{\nu}_e ) ( u + e^- \rightarrow d + \nu_e )</td>
<td>( \sim 10^{27} \ R \ T_9^6 )</td>
<td>Fast</td>
</tr>
<tr>
<td>Direct Urca cycle (u-s quarks)</td>
<td>( s \rightarrow u + e^- + \bar{\nu}_e ) ( u + e^- \rightarrow s + \nu_e )</td>
<td>( \sim 10^{27} \ R \ T_9^6 )</td>
<td>Fast</td>
</tr>
</tbody>
</table>
Standard Cooling

\[ \dot{\epsilon} \approx e^{-2\Delta/T} \times 10^{27} T_9^6 \text{ erg cm}^{-3} \text{s}^{-1} \]

\[ \nu_e \rightarrow e^- \]

\[ n \rightarrow p \]

\[ \dot{\epsilon} \approx e^{-2\Delta/T} \times 10^{19} T_9^8 \text{ erg cm}^{-3} \text{s}^{-1} \]

\[ \nu \rightarrow \nu \]

\[ n \rightarrow n \]

\[ \dot{\epsilon} \approx e^{-2\Delta/T} \times 10^{21} T_9^6 \text{ erg cm}^{-3} \text{s}^{-1} \]

\[ \nu_e \rightarrow e^- \]

\[ n \rightarrow p \]

\[ n \rightarrow n \]

\[ \dot{\epsilon} \approx \mathcal{R}(T/T_c) \times 10^{21} T_9^7 \text{ erg cm}^{-3} \text{s}^{-1} \]

\[ n \rightarrow n \]
Standard Cooling

\[ \dot{\varepsilon} \approx e^{-2\Delta/T} \ 10^{27} \ T_9^6 \ \text{erg cm}^{-3}\text{s}^{-1} \]

\[ \nu_e \rightarrow e^- \]

\[ n \rightarrow p \]

\[ \dot{\varepsilon} \approx e^{-2\Delta/T} \ 10^{19} \ T_9^8 \ \text{erg cm}^{-3}\text{s}^{-1} \]

\[ \nu \rightarrow \nu \]

\[ n \rightarrow n \]

\[ \dot{\varepsilon} \approx e^{-2\Delta/T} \ 10^{21} \ T_9^8 \ \text{erg cm}^{-3}\text{s}^{-1} \]

\[ \nu_e \rightarrow e^- \]

\[ n \rightarrow p \]

\[ n \rightarrow n \]

\[ \dot{\varepsilon} \approx R(T/T_c) \ 10^{21} \ T_9^7 \ \text{erg cm}^{-3}\text{s}^{-1} \]

Operates only near $T_c$
Standard Cooling

\[ \dot{\varepsilon} \approx e^{-2\Delta/T} \cdot 10^{27} \cdot T_9^6 \text{ erg cm}^{-3}\text{s}^{-1} \]

\[ \nu_e \rightarrow e^- \]

Kinematically Forbidden

\[ \dot{\varepsilon} \approx e^{-2\Delta/T} \cdot 10^{19} \cdot T_9^8 \text{ erg cm}^{-3}\text{s}^{-1} \]

\[ \nu \rightarrow \nu \nu \]

Operates only near \( T_c \)

\[ \dot{\varepsilon} \approx e^{-2\Delta/T} \cdot 10^{21} \cdot T_9^8 \text{ erg cm}^{-3}\text{s}^{-1} \]

\[ \nu \rightarrow \nu \nu \]

\[ \langle nn \rangle \]
Neutron decay at the Fermi surface cannot conserve momentum if
\[ \chi_p \sim \left( \frac{p_{Fp}}{p_{Fn}} \right)^3 < 0.12-14 \]

- In the standard scenario only massive stars (\(M \sim 2 M_\odot\)) cool rapidly.

Neutron decay at the Fermi surface cannot conserve momentum if
\[ x_p \sim \left( \frac{p_{Fp}}{p_{Fn}} \right)^3 < 0.12-14 \]

- In the standard scenario only massive stars (M \sim 2 M\odot) cool rapidly.
- A large symmetry energy will allow direct URCA for typical NS (M \sim 1.4 M\odot).
- Recall a large symmetry energy also favors large radii.
\[ \frac{dE_{th}}{dt} \equiv C_V \frac{dT}{dt} = -L_\nu - L_\gamma \]

Standard or Slow Cooling

Rapid Cooling

Pairing

I. Too hot for electron pairing:

\[ T_c \approx \omega_p^{\text{ion}} \exp \left( -\frac{\nu Fe}{\alpha_{\text{em}}} \right) \]

Ginzburg (1969)

Relativistic electrons move too quickly to feel the phonon induced attraction.

II. Pairing between nucleons is inevitable.

\[ T_c \approx E_{Fn} \exp \left( -\frac{\pi}{2k_{Fn} a_{nn}} \right) \]

Bohr, Mottelson, Pines (1958)
Migdal (1959)

Typical energy scale is MeV (~10^{10} K)
Recall Response Function in Superfluid!

\[ S_q(\omega) = \sum_{\lambda, \lambda'} f_{\lambda} |\langle \lambda | A_q | \lambda' \rangle|^2 \delta(E_{\lambda} - E'_{\lambda} - \omega) \]

\[ = \int dt \ e^{i\omega t} \langle \langle \lambda | A_q(t) A^\dagger_q(0) | \lambda \rangle \rangle \]

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]

\[ S_q(\omega) \]
Recall Response Function in Superfluid!

\[ S_q(\omega) = \sum_{\lambda, \lambda'} f_\lambda |\langle \lambda | A_q | \lambda' \rangle|^2 \delta(E_\lambda - E'_{\lambda} - \omega) \]

\[ = \int dt \ e^{i\omega t} \langle \langle \lambda | A_q(t) \ A_q^\dagger(0) | \lambda \rangle \rangle \]

Fermi Motion and Pauli Blocking.

\[ S_q(\omega) \]

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]
Recall Response Function in Superfluid!

\[ S_q(\omega) = \sum_{\lambda, \lambda'} f_\lambda |\langle \lambda | A_q | \lambda' \rangle|^2 \delta(E_\lambda - E'_\lambda - \omega) \]

\[ = \int dt \ e^{i \omega t} \langle \langle \lambda | A_q(t) \ A^\dagger_q(0) | \lambda \rangle \rangle \]

Fermi Motion and Pauli Blocking.

Correlations and collisions.

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]
Recall Response Function in Superfluid!

\[ S_q(\omega) = \sum_{\lambda, \lambda'} f_\lambda \left| \langle \lambda | A_q | \lambda' \rangle \right|^2 \delta(E_\lambda - E'_{\lambda} - \omega) \]

\[ = \int dt \ e^{i\omega t} \langle \langle \lambda | A_q(t) \ A^\dagger_q(0) | \lambda \rangle \rangle \]

Fermi Motion and Pauli Blocking.

Correlations and collisions.

Cooper Pairing & Superfluidity.

\[ \omega = \frac{q^2}{2M} + \frac{\vec{p} \cdot \vec{q}}{M} \]

\[ \omega \]
S-wave pairing

• The nucleon-nucleon interaction is known up to relative momenta $\sim 350$ MeV.
• Perturbation theory fails, but Quantum Monte Carlo and lattice methods may be reliable.
• Best estimates for the gap indicate that it reaches a maximum value $\sim 1$ MeV in the crust.

Cold atom experiments help validate many-body theory of strong short-range interactions.

Bulgac, Carlson, Drut, Gandolfi, Forbes, Kaplan ...
**S-wave pairing**

- The nucleon-nucleon interaction is known up to relative momenta \( \sim 350 \text{ MeV} \).
- Perturbation theory fails, but Quantum Monte Carlo and lattice methods may be reliable.
- Best estimates for the gap indicate that it reaches a maximum value \( \sim 1 \text{ MeV} \) in the crust.
S-wave interaction is repulsive at high density. Attraction in spin 1 channel due to P-wave interaction.
Predictions based on the sign and magnitude of the interaction.

Screening, the multi-component nature, and other many-body effects can be important at high density.
Fluctuations near $T_c$ are efficient at producing neutrinos.

Flowers, Ruderman, Sutherland (1976)

Fluctuations in the $^3P_2$ superfluid are most efficient: (i) because neutrino’s couple to spin, (ii) conservation laws suppress the emission in the $^1S_0$ channel.

Leinson (2008)
Steiner & Reddy (2009)
• Fluctuations near $T_c$ are efficient at producing neutrinos. 
Flowers, Ruderman, Sutherland (1976)

• Fluctuations in the $^3P_2$ superfluid are most efficient: (i) because neutrino’s couple to spin, (ii) conservation laws suppress the emission in the $^1S_0$ channel.

Leinson (2008)
Steiner & Reddy (2009)
Pair Breaking & Formation

- Fluctuations near $T_c$ are efficient at producing neutrinos.
  Flowers, Ruderman, Sutherland (1976)
- Fluctuations in the $^3P_2$ superfluid are most efficient: (i) because neutrino’s couple to spin, (ii) conservation laws suppress the emission in the $^1S_0$ channel.

Leinson (2008)
Steiner & Reddy (2009)
Cooling on a 10 year time scale requires very rapid cooling.

Is a large volume inside the neutron star undergoing a superfluid transition to produce enhanced cooling?

Cooling behavior over the next decade will tell.

Heinke, & Ho (2010)
Page et al. (2011), Shternin et al. (2011)
Cooling on a 10 year time scale requires very rapid cooling.

Is a large volume inside the neutron star undergoing a superfluid transition to produce enhanced cooling?

Cooling behavior over the next decade will tell.

Heinke, & Ho (2010)
Page et al. (2011), Shternin et al. (2011)
Transient Accretion

- Nuclear reactions heat the crust during accretion.
- Crust relaxes during quiescence.
Transient Accretion

- Nuclear reactions heat the crust during accretion.
- Crust relaxes during quiescence.
Crust Relaxation

- Crust relaxes during quiescence.

\[ T_e \] data from MXB 1659
Crust Relaxation

- Crust relaxes during quiescence.


data from MXB 1659
MORE THAN ONE SOURCE!

Cackett et al. 2006

MXB 1659-29

$\tau_{\text{Cool}} = 465 \pm 25$ days

6.6 yr

Cackett et al. 2008

KS 1731-260

$\tau_{\text{Cool}} = 305 \pm 50$ days

4.4 yr
\[ \tau_{\text{Cool}} \approx \frac{C_V}{\kappa} (\Delta R)^2 \]
Crustal Specific Heat

\[ \tau_{\text{Cool}} \sim \frac{C_V}{\kappa} (\Delta R)^2 \]
Crustal Specific Heat

\[ \tau_{\text{Cool}} \approx \frac{C_V}{\kappa} \left( \Delta R \right)^2 \]

Thermal Conductivity
CONNECTING TO CRUST MICROPHYSICS

\[ \tau_{\text{Cool}} \simeq \frac{C_V}{\kappa} \left( \Delta R \right)^2 \]

- Crustal Specific Heat
- Crust Thickness
- Thermal Conductivity
Microscopic Structure of the Crust

Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)
Microscopic Structure of the Crust

Negele & Vautherin (1973)

Baym Pethick & Sutherland (1971)  Negele & Vautherin (1973)
Microscopic Structure of the Crust

- Neutrons
- Protons
- 2\ a
- Theory of electrons and phonons
- Theory of a superfluid + electrons + phonons
- Dripped superfluid neutrons

Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)
Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.
Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

Cirigliano, Reddy & Sharma (2011)
Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

\[ \langle \psi_\uparrow(r)\psi_\downarrow(r) \rangle = |\Delta| \exp(-2i \theta) \]

Collective coordinates:
- Vector Field: \( \xi_i(r, t) \)
- Scalar Field: \( \phi(r, t) \)

Cirigliano, Reddy & Sharma (2011)
Despite strong mixing by entrainment, its variation in the crust for different values of it is predominantly the lattice mode. In these calculations we have neglected the second below can be significant. Away from resonance the eigenmodes contain only small admixtures they cross at for the crustal compositions of catalyzed and accreted matter shown in panels 3A4 and 3B4 of Figure . Velocities of phonons in the inner crust for two chemical compositions from Fig

\[ \omega_{l\text{Ph}}(q) = c_l q \quad \omega_{s\text{Ph}}(q) = \nu_s q \]

\[ \omega_{t\text{Ph}}(q) = c_t q \quad \omega_{\text{electron}} = q \]

\[ A^* : a = 0 \]

\[ \rho \text{ (g/cm}^3\text{)} \]

\[ \text{Velocities (c)} \]
Low Energy Excitations

\[
\begin{align*}
\omega_{l\text{Ph}}(q) &= c_l \, q & \omega_{s\text{Ph}}(q) &= \nu_s \, q \\
\omega_{t\text{Ph}}(q) &= c_t \, q & \omega_{\text{electron}} &= q
\end{align*}
\]

\[\text{A}^*: a = 0.6\]

\[\rho \text{ (g/cm}^3\text{)}\]

\[\text{Velocities (c)}\]

\[\text{0.1}\]

\[\text{0.05}\]

\[\text{0.01}\]

\[\text{0}\]

\[10^{12}\]

\[10^{13}\]

\[10^{14}\]
Crustal Specific Heat

Electrons:

\[ C_v^e = \frac{1}{3} \mu^2_e T \]

Ions:

\[ C_v^{\text{ph}} = \frac{2\pi^2}{15} \left( \frac{T^3}{v_l^3} + \frac{2 T^3}{v_t^3} \right) \]

Neutrons:

\[
\begin{align*}
C_v^{\text{sph}} &= \frac{2\pi^2 T^3}{15 v_\phi^3} \quad (T \ll T_c) \\
C_v^{\text{neutron}} &= \frac{1}{3} m_n k_{F_n} T \quad (T > T_c)
\end{align*}
\]
of normal neutrons above the drip point while models A, and B, predict a thick layer of normal neutrons at the highest densities. Modifications to this simple picture of pairing in uniform neutron matter due to the presence of the nuclei are discussed in this book in the chapter by N. Sandulescu and J. Margueron. Further, we briefly note that like in the case of electrons, coherent Bragg scattering of neutrons by the lattice lead to band structure effects that modify the shape of the Fermi surface. Eq. is an excellent approximation to \( C_v \) in normal phase for reasons described in §.

Elsewhere in the crust where \( T < T_c \), the relevant neutron contribution is from superfluid phonons, i.e., collective instead of single particle excitations, and is given by

\[
C_{\text{sph}} = \frac{\pi^2}{3} T_v \phi^2,
\]

where \( v_\phi = \frac{n_f m_n}{\mu n} \), see §.

\( v_\phi \) is the superfluid phonon velocity, \( n_f \) and \( m_n \) are the number density, chemical potential and mass of the free neutrons, respectively. For weakly coupled systems, \( v_\phi = v_F / \sqrt{\phi} \) where \( v_F \) is the Fermi velocity. In most of the inner crust, \( v_\phi \geq v_t \) see Fig. and hence their contribution to the heat capacity is negligible except perhaps in a sliver where \( T \leq T_c \).
Electron Conduction

\[ \kappa_e = \frac{1}{9} \mu_e^2 T \lambda_e \]

Electron-phonon:

\[
\begin{align*}
\lambda_e^{\text{ph}} &\propto \nu_i^3 / T^2 \quad T \geq T_{\text{um}} \\
\lambda_e^{\text{ph}} &\propto \nu_i^4 / T^3 \quad T \ll T_{\text{um}}
\end{align*}
\]

\[ T_{\text{um}} = \left(4e^3 / 9\pi\right) \nu_t k_{\text{Fe}} \]

Electron-impurity:

\[
\lambda_e^{\text{imp}} = \frac{3\pi \langle Z \rangle}{4e^4 Q_{\text{imp}} k_{\text{Fe}}} \Lambda^{-1}
\]

\[ Q_{\text{imp}} = \frac{1}{n_{\text{ion}}} \sum_i n_i (Z_i - \langle Z \rangle)^2 \]

Impurity scattering is important at low temperature.

Flowers & Itoh (1976)
Electron Conduction

$$\kappa_e = \frac{1}{9} \mu_e^2 T \lambda_e$$

![Graph showing electron conduction](image)

Impurity scattering is important at low temperature.

Flowers & Itoh (1976)
Unraveling thermal relaxation

• Late time signal is sensitive to inner crust thermal and transport properties.
• Impurity parameter can be fixed at earlier times.
• Variations in the pairing gap (changes the fraction of normal neutrons) are discernible!

Brown & Cumming (2009)
Shternin & Yakovlev (2007)
Page & Reddy (2012)
Unraveling thermal relaxation

- Late time signal is sensitive to inner crust thermal and transport properties.
- Impurity parameter can be fixed at earlier times.
- Variations in the pairing gap (changes the fraction of normal neutrons) are discernible!

\[ A: \text{Low } T_c - \text{large normal fraction} \]
\[ B: \text{High } T_c - \text{small normal fraction} \]

Shternin & Yakovlev (2007)
Brown & Cumming (2009)
Page & Reddy (2012)
The gravity wave and EM counterparts are also likely to be sensitive to both the EoS and response functions.
The gravity wave and EM counterparts are also likely to be sensitive to both the EoS and response functions.
Summary & Outlook

• A qualitative understanding of connections between dense matter properties and neutron star observations have emerged in the past decade.

• There is much to do. Pursuing theoretical work to provide a quantitative description of the equation of state and correlations functions of interest will be both challenging and rewarding.

• Multi-messenger probes of the neutron star interior (x-rays, neutrinos, and GWs) contain a wealth of information .. extracting it will require good ideas, theory, and large-scale simulations.