The Exact and Approximate Symmetries of Electroweak Interactions

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Unique Low Energy Tests exploiting the special properties of Leptons, Nucleons and Nuclei
Outline of Lectures

- Standard Model of Electroweak Interactions
- Searches for Violations of Discrete Symmetries
- Charged Lepton Flavor Violation and Precision Weak Neutral Current Experiments
- Parity-Violating Electron Scattering Experiments, Electroweak Probes of Hadron Structure & Precision Weak Charged Current Experiments
Review and Perspective

In lecture 1, we introduced the electroweak interaction and its verification using colliders.

In lecture 2, we learned about symmetries and discussed EDM searches to find T-violation.

In lecture 3, we discussed lepton flavor violation and weak neutral current experiments.

Coupled with Vincenzo’s lecture yesterday, we are now quite familiar with the language of BSM searches and the role of symmetries at low energies.
Outline of Lecture #4

- Parity-violating electron scattering as a probe of new flavor diagonal amplitudes at the TeV scale
- Electroweak probes of hadron structure
- Precision charged current experiments
- Muon g-2
Parity-Violating Electron Scattering

Weak Neutral Current (WNC) Interactions at $Q^2 \ll M_Z^2$

Longitudinally Polarized Electron Scattering off Unpolarized Targets

$$\sigma \propto |A_{\gamma} + A_{\text{weak}}|^2$$

$$-A_{\text{LR}} = A_{PV} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \sim \frac{A_{\text{weak}}}{A_{\gamma}} \sim \frac{G_F Q^2}{4 \pi \alpha} (g_A e g_A^T + \beta g_V e g_V^T)$$

$g_V$ and $g_A$ are function of $\sin^2 \theta_W$

$$A_{PV} \sim 10^{-5} \cdot Q^2 \rightarrow 10^{-4} \cdot Q^2$$

Specific choices of kinematics and target nuclei probes different physics:

- In mid 70s, goal was to show $\sin^2 \theta_W$ was the same as in neutrino scattering
- Early 90s: target couplings carry novel information about hadronic structure
- Now: precision measurements with carefully chosen kinematics can probe physics at the multi-TeV scale
Experimental Technique

- Optical pumping of a GaAs wafer
- Rapid helicity reversal: change sign of longitudinal polarization ~ 100 Hz to minimize drifts (like a lockin amplifier)
- Control helicity-correlated beam motion: under sign flip, keep beam stable at the sub-micron level

“Flux Integration”: very high rates direct scattered flux to background-free region

Technical progress over 3 decades has enabled ppb systematic control

Parity-violating electron scattering has become a precision tool:
- Many-body nuclear physics: Neutron skin of $^{208}$Pb
- Nucleon structure: strangeness contribution to form factors
- Valence quark structure: Deep inelastic scattering at high-x
- Search for new TeV physics: Precision electroweak parameters
Purely leptonic reaction

\[ A_{PV} \propto m_e E_{lab} (1 - 4 \sin^2 \theta_W) \]

\[ \frac{\delta(\sin^2 \theta_W)}{\sin^2 \theta_W} \approx 0.05 \frac{\delta(A_{PV})}{A_{PV}} \]

50 GeV at SLAC: \(\sim\) 150 ppb!

45 & 48 GeV Beam
85\% longitudinal polarization

End Station A at the Standard Linear Accelerator Center (SLAC)

\( g-2 \) spin precession
45 GeV: 14.0 revs
48 GeV: 14.5 revs

\[ A_{PV} = (-131 \pm 14 \pm 10) \times 10^{-9} \]
E158 Implications

Limits on “New” Physics

Running of $\sin^2 \theta_W$ established to 6$\sigma$

SLAC E158 Moller

NuTeV ν-DIS

Cesium APV

$0.01 \cdot G_F$

doubly charged scalar exchange

$0.8$ TeV

$1.0$ TeV ($Z'_\chi$)

$16$ TeV

$17$ TeV

$0.01$}

$T - q \sin^2 \theta_W$
Neutrino Deep Inelastic Scattering

Most precise measurement of neutrino-quark coupling

subtle quark physics effects can affect the result

generated great interest in both nuclear and particle phenomenology

\[ R^- = \frac{\sigma_{vN}^{NC} - \sigma_{vN}^{NC}}{\sigma_{vN}^{CC} - \sigma_{vN}^{CC}} \approx \rho^2 \left( \frac{1}{2} - \sin^2 \theta_W \right) \]

\[ \sin^2 \theta_W^{(on-shell)} = 0.2277 \pm 0.0013 \text{(stat.)} \]
\[ \pm 0.0009 \text{(syst.)} \]

Standard Model prediction is 0.2227 (3\sigma deviation)
Lepton-Quark Neutral Current Interactions

Consider \( f_1 \bar{f}_1 \rightarrow f_2 \bar{f}_2 \) or \( f_1 f_2 \rightarrow f_1 f_2 \)

\[
L_{f_1 f_2} = \sum_{i,j=L,R} \frac{4\pi}{\Lambda^2_{ij}} \eta_{ij} f_1 i \gamma_\mu f_1 i f_2 j \gamma^\mu f_2 j
\]

\[
\delta(C_{1q}) \propto (+\eta_{RL}^{eq} + \eta_{RR}^{eq} - \eta_{LL}^{eq} - \eta_{LR}^{eq})
\]

\[
\delta(C_{2q}) \propto (-\eta_{RL}^{eq} + \eta_{RR}^{eq} - \eta_{LL}^{eq} + \eta_{LR}^{eq})
\]

PV elastic e-p scattering, APV
PV deep inelastic scattering

\[
C_{1i} \equiv 2g_A^e g_V^i
\]

\[
C_{2i} \equiv 2g_V^e g_A^i
\]

Qweak at Jefferson Lab
Qweak at Jefferson Laboratory

**$A_{PV}$ in elastic e-p scattering:**

\[
A(Q^2 \to 0) = -\frac{G_F}{4\pi\alpha\sqrt{2}} \left[ Q^2 Q_{\text{weak}}^p + Q^4 B(Q^2) \right]
\]

\[
Q_{\text{weak}}^p = 2C_{1u} + C_{1d} \propto 1 - 4\sin^2 \theta_W
\]

\[
E = 1.165 \text{ GeV}, \theta_{\text{lab}} \sim 9^\circ, \quad Q^2 = 0.026 \text{ GeV}^2
\]

86 scientists from 25 institutions
including U. Manitoba and TRIUMF

- Design and construction over past several years
- Installation nearly complete
- First beam next week!
- Data ~ 2010 thru mid-2012

New, complementary constraints on lepton-quark interactions at the TeV scale

Contains $G_{\gamma E,M}^y$ and $G_{Z E,M}^Z$
Extracted using global fit

86 scientists from 25 institutions
including U. Manitoba and TRIUMF
Atomic Parity Violation

- $6S \rightarrow 7S$ transition in $^{133}\text{Cs}$ is forbidden within QED
- Parity Violation introduces small opposite parity admixtures
- Induce an E1 Stark transition, measure E1-PV interference
- 5 sign reversals to isolate APV signal and suppress systematics
- Signal is $\approx 6$ ppm, measured to 40 ppb

**Boulder Experiment**

- Power build-up cavity ($F=100\ 000$)
- Dye laser beam
- Re-excitation of the depleted HF level
- APV signal: odd in $E$, $\xi_{\text{ex}}, B, B_p, \xi_p$

**Partial Level Structure of Cesium**

- $Q_W \propto C_{1u} + C_{1d}$
- $H_W = \frac{G_F}{2\sqrt{2}} Q_W \gamma_{5} \rho(\vec{r})$

New alkalis being investigated initiatives include TRIUMF
Electroweak and Hadron Physics Interplay

- nuclei and nucleons are special laboratories to test electroweak interactions
- these are bound states ultimately governed by QCD dynamics
- a detailed knowledge of hadron dynamics is often needed to interpret the measurement and probe the TeV scale
- conversely, the experimental techniques being developed lead to new insights on hadron structure. Some classic examples:
  - elastic electron-nucleon and -nuclear scattering
  - nuclear beta decay and muon capture

Very recent: Lamb Shift (1s–2s transition) in muonic hydrogen

Proton charge radius

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Strange Quarks in the Nucleon

Strange quarks carry nucleon momentum: Other external properties affected?

Quark Model \(\rightarrow\) QCD

Late 1980’s

\[Q^2 \sim 0.1 \text{ GeV}^2\]

\[\text{Neutron charge distribution} \rightarrow \text{Proton flavor distribution}\]

\[\text{Neutron} \rightarrow \text{"pion cloud"} \rightarrow \text{Proton} \rightarrow \text{"kaon cloud"}\]

\[\text{Weak Isospin Symmetry} \rightarrow \gamma \rightarrow \gamma \rightarrow \text{Strong Isospin Symmetry}\]

\[\text{SAMPLE} \rightarrow \text{HAPPEX} \rightarrow \text{PVA4} \rightarrow \text{G0}\]

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NNPSS 2010 Lecture 4
A neutron skin is expected: how thick is it?
- The extent of the skin constrains the transition density from solid crust to liquid core in a neutron star
- The density dependence of the symmetry energy constrains the composition of the neutron star core: important implications for rate of neutron star cooling

Both neutron skin and neutron star crust are made out of neutron rich matter at similar densities.

An experimental clean measurement of skin is now viable:
A technically demanding measurement:
- Rate ~ 2 GHz
- Separate excited state at 2.4 MeV
- Stat. Error ~ 15 ppb
- Syst. Error ~ 1 to 2 %

Result highly prized by nuclear structure and nuclear astrophysics communities

Construction and Installation took place over the last 6 months

Physics run April - mid June

expect to have < 3% measurement of neutron radius
MOLLER at Jefferson Laboratory

Measurement of Lepton-Lepton Electroweak Reaction

\[ E_{\text{beam}} = 11 \text{ GeV} \quad 75 \mu \text{A} \quad 80\% \text{ polarized} \quad \Rightarrow \quad \delta(A_{PV}) = 0.73 \text{ ppb} \]

\[ A_{PV} = 35.6 \text{ ppb} \quad \delta(Q_{eW}) = \pm 2.1 \text{ (stat.)} \pm 1.0 \text{ (syst.) \%} \]

\[ \delta(\sin^2 \theta_W) = \pm 0.00026 \text{ (stat.)} \pm 0.00012 \text{ (syst.)} \]

Project design, construction and installation will take 4–5 years

Jefferson Lab 12 GeV Upgrade

\[ L_{e_1 e_2} = \sum_{i,j=L,R} \frac{g_{ij}^2}{2\Lambda^2} \bar{e}_i \gamma_{\mu} e_i \bar{e}_j \gamma^{\mu} e_j \]

\[ A_{PV} \]

\[ \Lambda = \frac{\sqrt{g_{RR}^2 - g_{LL}^2}}{4.4 \text{ TeV} \quad \text{OR} \quad \frac{\Lambda}{g_{RL}} = 5.2 \text{ TeV}} \]

Best current limits on 4-electron contact interactions: LEP II at 200 GeV

(Average of all 4 LEP experiments)

\[ \Lambda = 7.5 \text{ TeV} \]

\[ \chi^2 \text{ d.o.f.} = 11.8 / 5 \]

\[ m_t = 172.7 \pm 2.9 \text{ GeV} \]

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Precision Tests of the Weak Charged Current

\[ L_{CC} = \frac{g}{2\sqrt{2}} W_\mu^+ \left[ \overline{U_i} \gamma^\mu (1 - \gamma^5) V_{ij} D_j + \overline{\nu}_\kappa \gamma^\mu (1 - \gamma^5) l_\kappa \right] + \text{h.c.} \]

\[ M_{CC}^{\text{FC}} \approx \frac{g^2}{8M_W^2} (V - A) \otimes (V - A) \]

Fermi Constants

\[ \mu \text{ decay} \quad \frac{G_F^\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2} \left(1 + \Delta r_\mu\right) \]

\[ \beta \text{ decay} \quad \frac{G_F^\beta}{\sqrt{2}} = \frac{g^2}{8M_W^2} V_{ud} \left(1 + \Delta r_\beta\right) \]

\[ g^2/8M_W^2 \text{ is universal} \]

Universality obscured by \[ G_F^\beta / G_F^\mu = V_{ud} \left(1 + \Delta r_\beta - \Delta r_\mu\right) \]

New physics
Super-Allowed Beta Decays

$0^+ \text{ to } 0^+ \ "\text{Superallowed}"$

\[ Ft = ft(1 + \delta'_R + \delta_{NS})(1 - \delta_C) = \frac{K}{2(G_F^β)^2} \]

Nuclear structure-dependent corrections

\[
\begin{align*}
\chi^2/\nu & = 0.6 \\
\text{J.C. Hardy 2003}
\end{align*}
\]
Beta Decay Correlation Coefficients

\[ \frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} \propto \mathcal{N}(E_e) \left\{ 1 + a \frac{p_e \cdot p_\nu}{E_e E_\nu} + b \frac{\Gamma m_e}{E_e} + \langle J \rangle \cdot \left[ \frac{A}{E_e} + \frac{B}{E_\nu} + \frac{D}{E_e E_\nu} \right] \right\} \]

Jackson, Treiman, Wyld

- Unique sensitivity to S,T operators (via interference terms \( \propto m_e/E_e \))

- Example: limit on \( b \) from \( 0^+ \rightarrow 0^+ \) transitions corresponds to

\[ \frac{\Lambda}{\sqrt{C_S}} \geq 7 \text{ TeV} \]

\[ \Gamma = \sqrt{1 - (Z\alpha)^2} \]

Hardy & Towner 2009

\[ b_F = -0.0022 \pm 0.0026 \]
**The Neutron**

**Major initiatives in Canada, USA, Europe and Japan**

\[
\frac{dw}{dE_e d\Omega_e d\Omega_\nu} \sim k_e E_e (E_0 - E_e)^2 \\
\times \left[ 1 + \frac{\vec{k}_e \cdot \vec{k}_\nu}{E_e E_\nu} + b \frac{m}{E_e} + \langle \vec{\sigma}_n \rangle \cdot \left( \frac{\vec{k}_e}{E_e} + \frac{\vec{k}_\nu}{E_\nu} + \frac{\vec{k}_e \times \vec{k}_\nu}{E_e E_\nu} \right) \right]
\]

**with:**

\[
a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2}
\]

\[
A = -2 \frac{|\lambda|^2 + \text{Re}(\lambda)}{1 + 3|\lambda|^2}
\]

\[
B = 2 \frac{|\lambda|^2 - \text{Re}(\lambda)}{1 + 3|\lambda|^2}
\]

\[
D = 2 \frac{\text{Im}(\lambda)}{1 + 3|\lambda|^2}
\]

\[
\lambda = \frac{G_A}{G_V} \quad (\text{with } \tau_n \Rightarrow \text{CKM } V_{ud})
\]

\[
(D \neq 0 \iff \text{T inv. violation})
\]

**Independent measure of** \(V_{ud}\)

**Other terms sensitive to**

**TeV scale BSM physics**
## Semi-Leptonic Decays

<table>
<thead>
<tr>
<th>$V_{ud}$</th>
<th>$0^+ \rightarrow 0^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\pi^\pm \rightarrow \pi^0 e\nu$)</td>
</tr>
<tr>
<td>$n \rightarrow p e\nu$</td>
<td>$\pi \rightarrow \mu\nu$</td>
</tr>
<tr>
<td>$\tau \rightarrow h_{NS} \nu_\tau$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$V_{us}$</th>
<th>$K \rightarrow \pi \ell\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \rightarrow p e\nu, \ldots$</td>
<td>$K \rightarrow \mu\nu$</td>
</tr>
<tr>
<td>$\tau \rightarrow h_s \nu_\tau$ (inclusive)</td>
<td></td>
</tr>
</tbody>
</table>

### Hadronic Matrix Elements

\[
\Gamma_{ij} = \left[ G_F^{(\mu)} V_{ij} \right]^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{\text{em}}) \times F_{\text{kin}}
\]

### Radiative Corrections

**Radiative corrections**

**Hadronic matrix elements**

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Global Fit to $V_{ud}$ and $V_{us}$

- Remarkable agreement with Cabibbo universality:

- Confirms large EW rad. corr. $(2 \alpha/\pi \log(M_Z/M_P)=+3.6\%)$

- It would naively fit $M_Z = (90 \pm 7) \text{ GeV}$

Fit result

\[
\begin{align*}
V_{ud} &= 0.97425 (22) \\
V_{us} &= 0.2252 (9)
\end{align*}
\]

$\chi^2/\text{dof} = 0.65/1$

$|V_{ud}|^2 + |V_{us}|^2 = 0.9999(6)$

Error equally shared between $V_{ud}$ and $V_{us}$

$\Delta^{\text{CKM}} = -(1 \pm 6) \times 10^{-4}$

Marciano-Sirlin
Muon decay ("Michel") parameters $\rho$, $\eta$, $P_{\mu} \xi$, $\delta$

Muon differential decay rate vs. energy and angle:
\[
\frac{d^2\Gamma}{dx \ d\cos\theta} = \frac{1}{4} m_\mu W_{\mu e}^4 G_F^2 \sqrt{x^2 - x_0^2} \cdot \{\mathcal{F}_{IS}(x, \rho, \eta) + P_{\mu} \cos\theta \cdot \mathcal{F}_{AS}(x, \xi, \delta)\} + R.C.
\]

where
\[
\mathcal{F}_{IS}(x, \rho, \eta) = x(1 - x) + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta x_0 (1 - x)
\]
\[
\mathcal{F}_{AS}(x, \xi, \delta) = \frac{1}{3} \xi \sqrt{x^2 - x_0^2} \left[1 - x + \frac{2}{3} \delta \left(4x - 3 + \left(\sqrt{1 - x_0^2} - 1\right)\right)\right]
\]

and
\[
W_{\mu e} = \frac{m_\mu^2 + m_e^2}{2m_\mu}, \quad x = \frac{E_\mu}{W_{\mu e}}, \quad x_0 = \frac{m_e}{W_{\mu e}}.
\]
TWIST Results

Important new limits on right-handed currents

\[ W_L = W_1 \cos \zeta + W_2 \sin \zeta, \quad W_R = e^{i\omega} (-W_1 \sin \zeta + W_2 \cos \zeta) \]
Muon $g-2$

$$\bar{\mu} = g \frac{e}{2mc} \bar{s}, \quad \bar{s} = \frac{\hbar}{2} \bar{\sigma}$$

- Dirac predicts in $g=2$ in 1928
- 1947: Measurements of Kusch and Foley found $g_e$ deviates from 2
- Schwinger calculated:
  $$g_e = 2(1 + a_e), \quad \text{where} \quad a_e = \frac{(g_e - 2)}{2} = \frac{\alpha}{2\pi} \approx 0.00116$$

- $a_e$ due to corrections from virtual particles appearing in loops (radiative corrections)
- 1 part in 850 effect, huge success for QED!
Radiative Corrections

\[ a_\mu (\text{SM}) = a_\mu (\text{QED}) + a_\mu (\text{Weak}) + a_\mu (\text{Hadronic}) \]

- **EW 1 Loop**
- **EW 2 Loop**
- **Hadronic Leading Order**
- **Higher Order**
- **Light-by-Light**

\[ \Rightarrow a_\mu \text{ gets contributions from all physics - including the unknown} \]
g-2 Experiment at BNL

- Inject polarized muons at 3.094 GeV/c into superferric storage ring, radius = 711.2 cm
- Muon spin precesses in homogeneous 1.45 T field, time dilated lifetime of 64.4 µs, measure for 700 µs

$$\tilde{\omega}_a = \tilde{\omega}_s - \tilde{\omega}_c$$ : difference between spin and cyclotron frequencies

$$\tilde{\omega}_a = -\frac{q}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \Rightarrow \text{at } \gamma = 29.3 \Rightarrow \tilde{\omega}_a = -\frac{q}{mc} \left[ a_\mu \vec{B} \right]$$

⇒ To determine $a_\mu$, need to measure $\omega_a$ and $B$

- Muon spin direction correlated with decay electron direction, and $E_{lab} \approx \gamma E^* (1 + \cos \theta^*)$

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g-2 Data and Plans

![Data and Plans Diagram]

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Summary

- Weak Neutral Current Measurements are an important complement to search for TeV-scale flavor diagonal interactions.
- Electroweak experiments probe novel aspects of hadron structure.
- Charged Current Interactions search for new physics in sectors often not accessible at colliders.
- Muon $g-2$ is a very sensitive indirect search for SUSY.
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Conclusions and Perspectives

I hope you had at least some fraction of the fun I had in preparing these lectures.

Nuclear theory & experiments will continue to explore fundamental symmetries and interpret/complement collider experiments uncovering the underlying theories of nature.

We need you all to develop the next generation of clever ideas that will move this subfield forward into the future.