Studying the Exact and Approximate Symmetries of Electroweak Interactions

Krishna Kumar, UMass Amherst
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Unique Low Energy Probes of the Early Universe exploiting special properties of Leptons, Nucleons and Nuclei
Outline of Lectures

- Standard Model of Electroweak Interactions
- Searches for Violations of Discrete Symmetries
- Charged Lepton Flavor Violation and Precision Weak Neutral Current experiments
- Precision Weak Charged Current Experiments & Electroweak Probes of Hadron Structure
Scattering of longitudinal vector bosons \((m=0)\)

- \(e e Z\) couplings depend on \(\sin^2 \theta_W\)
  \[
  \frac{m_W}{m_Z} = \cos \theta_W
  \]

\(e^+ e^- \rightarrow W^+ W^-\)

**Scattering to longitudinal vector bosons \((m=0)\)**

For good high energy behaviour, Higgs must couple to \(m_e\).

Why? Hint: if massless, helicity must be conserved
Outline of Lecture #2

- Symmetries and Conservation Laws
- Discrete and Continuous Symmetries
- Discoveries of P and CP violation
- T Violation and EDMs
Symmetries and Conservation Laws

Noether’s Theorem: If Euler-Lagrange equation is invariant under any coordinate transformation, \( \exists \) an integral of motion

Not just space-time symmetries: Invariance of Lagrangian/Hamiltonian

\[ [Q, H] = 0 \rightarrow \frac{d < Q >}{dt} = 0 \]

Conserved Quantities/Quantum Numbers
Symmetries and Groups

Symmetry operations:

- Finite Group
- Infinite Group

Group of all operations: display closure &
Associativity and have identity and inverse

Continuous Symmetry

Discrete Symmetry

In Physics, group operations can be represented by matrices

SO(n): n-D rotations
SO(3) ↔ SU(2)

Invariance under SU(2): Angular Momentum Conservation
Continuous Symmetries

Dirac free particle
Lagrangian

\[ \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \]

U(1) Invariance: conserved current \( \partial_\mu J^\mu = 0 \)

Local U(1) Invariance: \( A_\mu J^\mu \)

Electromagnetic Interactions

Rotation in “Isospin Space”

\[
\begin{pmatrix}
    p \\
    n
\end{pmatrix}
\]

nucleon-nucleon interaction Hamiltonian invariant under SU(2) transformations in Isospin Space

\[
\begin{pmatrix}
    \nu_e \\
    e^-
\end{pmatrix}_L
\]

the “massless” left-handed electron and electron-neutrino are part of a similar “weak isospin” doublet

SU(2) invariance yields 3 independent conserved currents

(there are 3 independent 2x2 Pauli spin matrices)
Symmetries of the Electroweak Lagrangian

$SU(2)_L \times U(1)_Y$ 

4 conserved currents
local gauge invariance yields 4 bosons: $W^+, W^-, W^0, B^0$

After spontaneous symmetry breaking via Higgs Mechanism:

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

two weak charged currents
$W^\pm$

electromagnetic current
$\gamma$

weak neutral current
$Z^0$

SU(3)$_c$ and gluons ↔ Quantum Chromodynamics

Exact symmetries of nature: fully manifest in the early universe

Unbroken exact symmetries: massless mediator & infinite range force
Additive Conservation Laws

Are there other symmetries at low energies? Are they exact?

Electric Charge is conserved

\[ p \rightarrow e^+ \pi^0 \]

We know baryons are made of 3 quarks

Proton Decay? Highly suppressed (lucky us!)

Charge is quantized

sum of initial charges = sum of final charges

Conserved quark current:

consequence of SU(3)_c

quark (baryon) # violations suppressed on general grounds based on the symmetries of the electroweak Lagrangian
**Baryon & Lepton Number**

Introduce Baryon B and Lepton number L: opposite sign for anti-particles

\[ n \to p e^- \bar{\nu}_e \]
\[ \bar{\nu}_e p \to e^+ n \quad \text{observed} \]
\[ \bar{\nu}_e n \to e^- p \quad \text{not observed} \]

Are these exact conservation laws?

No fundamental reason: Certainly not as unbroken exact symmetries of nature

Neutrino Mass leads to the speculation that neutrinos are their own anti-particles

Neutrino-less Double Beta Decay

If a process is not forbidden, it will occur!
Discrete Symmetries
C, P & T

Parity P
\[ x, y, z \rightarrow -x, -y, -z \]
\[ P\psi(\vec{r}) = \psi(-\vec{r}) \]
\[ P^2 = I \] Group has 2 elements, P and I
\[ [H, P] = 0 \]
\[ H\psi = E\psi \quad \& \quad P\psi = \pi\psi \]
\[ \pi = \pm 1 \]

If hamiltonian is invariant under parity transformations, then \( \pi \) is conserved and observable

Charge Conjugation C
\[ C|p> = |\bar{p}> \]
All quantum numbers flip sign except mass and spin
particles that are its own anti-particles are eigenstates of C
\[ C|\gamma> = -|\gamma> \quad \pi^0 \rightarrow \gamma\gamma \]
\[ C|\pi^0> = +|\pi^0> \]
\[ \pi^0 \rightarrow \gamma\gamma\gamma \] forbidden

Time Reversal T
\[ T\psi(t) = \psi^*(-t) \]
reactions are reversible in principle if T is conserved
Discovery of Parity Violation

Particle Classification \( S^\pi \)

e.g. pions: \( 0^+ \) pseudoscalar mesons

Tau-theta puzzle (1956)

\[ \theta^+ \rightarrow \pi^+ \pi^0 \text{ (P=+1)} \]

\[ \tau^+ \rightarrow \pi^+ \pi^0 \pi^0 \text{ (P=-1)} \]

same mass but different parities! Lee and Yang propose:

The SAME particle is produced in strong interactions, but decays via weak interactions;
P conserved in strong interactions, but not in weak interactions

C.S. Wu et al: Beta’s in decays of \( ^{60}\)Co nuclei aligned in a magnetic field showed anisotropy

Classic example: Puzzle in accelerator result; theorists propose a solution; test on a different process (table-top)
Neutral Kaon System

\[ \tau^+ = \theta^+ = K^+ \equiv \bar{s}u \quad \text{Also } K^-: \text{Opposite "strangeness" quantum numbers} \]

\[ K^0 \equiv \bar{s}d \quad \& \quad \bar{K}^0 \equiv \bar{s}d \quad \text{Also opposite strangeness: are they distinct?} \]

Gell-Mann and Pais propose a test assuming CP conservation

\[ CP|\nu_{eL} > = |\bar{\nu}_{eR} > \]

\[
\begin{align*}
|K_1 > &= \frac{1}{\sqrt{2}}(|K^0 > + |\bar{K}^0 >) \\
|K_2 > &= \frac{1}{\sqrt{2}}(|K^0 > - |\bar{K}^0 >)
\end{align*}
\]

\[
\begin{align*}
CP|K^0 > &= |K^0 > \\
CP|\bar{K}^0 > &= |\bar{K}^0 >
\end{align*}
\]

\[
\begin{align*}
CP|K_1 > &= +|K_1 > \\
CP|\bar{K}_1 > &= +|\bar{K}_1 > \\
CP|\pi\pi > &= +|\pi\pi > \\
CP|\pi\pi\pi > &= +|\pi\pi\pi >
\end{align*}
\]

If CP is conserved:

\[
\begin{align*}
K_1 &\to \pi\pi \quad K_2 \to \pi\pi\pi \quad \text{allowed} \\
K_1 &\to \pi\pi\pi \quad K_2 \to \pi\pi \quad \text{forbidden}
\end{align*}
\]

Elegant Prediction: Existence of \( K_2 \)

two pion decay lifetime much shorter than three pion case

start with \( K_0 \)'s; have near and far detectors; 2 pions in near detector, 3 pions in far detector
Discovery of CP Violation

Christensen, Cronin, Fitch and Turlay

\[ |K^0 > = \frac{1}{\sqrt{2}}(|K_1 > + |K_2 >) \]
\[ |\bar{K}^0 > = \frac{1}{\sqrt{2}}(|K_1 > - |K_2 >) \]

start with K0’s:
contains K1’s and K2’s

drift region
(vacuum)

K2’s only:
antiK0’s!

Anti-K0’s have much larger cross-section to scatter off nuclei

K2’s only:
antiK0’s!

material

K1’s again:
2 pion decays!

Startling observation: take material away and some residual 2 pion decays remain!!!!

\[ |K_L > = \frac{1}{\sqrt{1 + \epsilon^2}}(|K_2 > + \epsilon|K_1 >) \]
\[ |K_S > = \frac{1}{\sqrt{1 + \epsilon^2}}(|K_1 > - \epsilon|K_2 >) \]

Impressive experimental challenges overcome:
only careful, methodical and confident experimentalists need apply!
Matter-Antimatter Asymmetry

Sakharov Criteria
• $B$ violation
• $C$ & $CP$ violation
• Nonequilibrium

dynamics
Sakharov, 1967

Anomalous $B$-violating processes

$\Gamma(A + B \rightarrow C) \neq \Gamma(\bar{A} + \bar{B} \rightarrow \bar{C})$

Prevent washout by inverse processes

CP violation in the weak interactions requires 3 generations of quarks:
Ensures quark mixing matrix has complex phase
However, electroweak CP phase explains Kaons; insufficient for consideration above
CPT Theorem and T Violation

The renormalizable field theories such as the ones that describe strong and electroweak interactions conserve CPT: e.g. masses and lifetimes of particle and anti-particle

CP violation therefore implies T violation

added impetus for new sources of CP & T violation: observed matter-anti-matter asymmetry

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\left( \frac{\hbar^2}{2m} \right) \frac{\partial^2 \Psi}{\partial x^2} + V\Psi \]

If V is real then T is a good symmetry \( \Psi(t) \) & \( \Psi^*(-t) \) are solutions

If V is complex, then T is violated; quantified by a complex phase
Electric Dipole Moments

Most practical way to find T violation:
establish permanent electric dipole moment for a fundamental particle

Charge q displaced from -q by a distance r creates an EDM

\[ \vec{d} = q \vec{r} \]

If T is conserved and d is non-zero: degenerate particle states

raw sensitivity:

\[ d \sim (m \times e)/\Lambda^2 \]

\[ d \sim 10^{-27}: \Lambda \sim 100 \text{ TeV} \]
EDM Approaches

Nuclear
- Neutron
- Diamagnetic Atoms: Hg, Xe, Rn
- Paramagnetic Atoms: Tl, Cs, Fr

Atomic
- Atomic Theory
- Nuclear Theory
- Quark Chromo-EDM
- Electron EDM

Molecular
- Molecules: PbO, YbF, TlF

Fundamental Theory – Supersymmetry, Strings

Experiments
- $10^{-24}$ eV
- 1 eV
- 1 MeV
- 1 GeV
- 1 TeV

M. Romalis
Experimental Concept

- Measure spin-precession frequencies

\[ \omega_1 = \frac{2 \mu B + 2dE}{\hbar} \]

\[ \omega_2 = \frac{2\mu B - 2dE}{\hbar} \]

\[ \omega_1 - \omega_2 = \frac{4dE}{\hbar} \]

- Statistical Sensitivity:

Single atom with coherence time τ:

\[ \delta \omega = \frac{1}{\tau} \]

N uncorrelated atoms measured for time T >> τ:

\[ \delta d = \frac{\hbar}{2E} \frac{1}{\sqrt{2\pi TN}} \]

\[ H = -\vec{\mu} \times \vec{B} - \vec{d} \times \vec{E} \]
Hg EDM Experiment

High purity non-magnetic vessel

All materials tested with SQUID

Solid-state Quadrupled UV laser

100,000 hours of operation

Hg Vapor cells

Spin coherence time: 300 sec
Electrical Resistance: $2 \times 10^{16} \, \Omega$
Interpretation of Hg EDM

- No atomic EDM due to EDM of the nucleus – Schiff’s Theorem
  ⇒ Electrons screen applied electric field

- $d(\text{Hg})$ is due to finite nuclear size
  ⇒ nuclear Schiff moment $S$ – Difference between mean square radius of the charge distribution and electric dipole moment distribution

\[
\bar{S} = \frac{2\pi}{5} \int d^3 \rho(x) \left( x^2 \bar{x} - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \bar{x} \right)
\]

⇒ Schiff moment induces parity mixing of atomic states, giving an atomic EDM:

\[
d_a = R_A S
\]

⇒ $R_A$ - from atomic wavefunction calculations, uncertainty 20%
**Ra-225 Experiment**

**EDM of $^{225}$Ra enhanced:**
- Large intrinsic Schiff moment due to octupole deformation;
- Closely spaced parity doublet;
- Relativistic atomic structure.

**Future:**
Improve sensitivity by 2 orders of magnitude

**Statistical uncertainty:**

$$\delta d = \frac{\hbar}{2E\sqrt{\tau N \varepsilon T}}$$

- $100 \text{ kV/cm}$
- $10^4$ s
- $10\%$
- $\delta d = 3 \times 10^{-26} \text{ e cm}$

- $\text{Ra / Hg Enhancement factor } \sim 10^2 - 10^3$
- $\delta d^{^{199}\text{Hg}} = 1.5 \times 10^{-29} \text{ e cm}$

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Z-T Lu, ANL
Atom Trap Program

Krishna Kumar
Neutron EDM

- Superthermal production in superfluid $^4$He
  $\Rightarrow$ N increased by 100 – 10000
- He-4 good isolator, low temperature
  $\Rightarrow$ E increased by 5
- Superconducting magnetic shields
- SQUID magnetometers

ILL, Grenoble, France

New Concept: $^3$He co-magnetometer
US nEDM experiment

SNS

stay tuned → 2016....
Summary of EDM Experiments

Area of Intense Activity
Both theory and experiment
Atomic experiments, cryogenic experiments, storage rings.....

<table>
<thead>
<tr>
<th>Particle/Atom</th>
<th>SM value [e·cm]</th>
<th>Current EDM Limit</th>
<th>Future Goal</th>
<th>$d_n$ equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutron</td>
<td>$10^{-32} - 10^{-31}$</td>
<td>$&lt; 2.9 \times 10^{-26}$</td>
<td>$10^{-28}$</td>
<td>$10^{-28}$</td>
</tr>
<tr>
<td>$^{199}$Hg</td>
<td>$10^{-32} - 10^{-31}$</td>
<td>$&lt; 3.1 \times 10^{-29}$</td>
<td>$10^{-29}$</td>
<td>$2 \times 10^{-26}$</td>
</tr>
<tr>
<td>$^{129}$Xe</td>
<td>$10^{-32} - 10^{-30}$</td>
<td>$&lt; 6 \times 10^{-27}$</td>
<td>$10^{-30} - 10^{-33}$</td>
<td>$10^{-26} - 10^{-29}$</td>
</tr>
<tr>
<td>Proton</td>
<td>$\approx 10^{-40}$</td>
<td>$&lt; 7.9 \times 10^{-25}$</td>
<td>$10^{-29}$</td>
<td>$10^{-29}$</td>
</tr>
<tr>
<td>Deuteron</td>
<td>$10^{-40}$</td>
<td>$&lt; 1.6 \times 10^{-27}$</td>
<td>$10^{-29} - 10^{-31}$</td>
<td>$3 \times 10^{-29} - 5 \times 10^{-31}$</td>
</tr>
<tr>
<td>Electron</td>
<td>$10^{-40}$</td>
<td>$&lt; 1.6 \times 10^{-27}$</td>
<td>$10^{-29} - 10^{-31}$</td>
<td>$3 \times 10^{-29} - 5 \times 10^{-31}$</td>
</tr>
</tbody>
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Summary

- Symmetries have played and continue to play a profoundly important role in shaping theoretical and experimental research in the search for physics beyond the standard model.

- A very important complementary experimental approach that involve nuclear theorists and experimentalists is the search for a permanent electric dipole moment of an elementary particle.