Lecture 3: Partonic Structure of the Nucleon
The nucleon is an excitation of 3 quarks in the QCD vacuum. Understanding the vacuum structure and its properties, such as color confinement, is essential.

Light quarks (up and down) are nearly massless, responsible for a small fraction of nucleon mass. Chiral symmetry and its spontaneous breaking is essential to understand many of its properties.

Understanding the role of the gluons is a key. The gluons excited from the presence of the valence quarks generate many of the fundamental properties of the nucleon, including spin and mass.

Solving the nucleon structure is at least as difficult as solving $^{208}\text{Pb}$!!!
How to make progress?

❖ Make the most realistic lattice QCD calculation possible with best available algorithms, the limited computation resource, and the graduate students one can support
  o convince the funding agency that lattice QCD is about to make an important impact (or already has).

❖ Find ideas connecting data, the fundamental theory, and “intuition”
  o Wave function is not the right language.
Some Good Theoretical Ideas

- Strange quarks can be directly probed! (Kaplan & Manohar, …)
- Pion cloud physics can be studied in a systematic expansion (Weinberg, Gasser & Leutwyler, Bernard, Meissner, Holstein, …)
- $N_c = 3$ is a large number! (‘t Hooft, Witten, Dashen, Jenkins, Manohar, Goity, Lebed, …)
- Generalized parton distributions (Ji, Radyushkin, Muller, Burkardt, Diehl, Vanderhargen, …)
Transverse momentum dependent parton distributions (Sivers, Collins, Mulders, Brodsky, Ji,...)

Hard exclusive processes can be factorized & soft and collinear effective field theory (Brodsky, Lepage, Radyushkin, Bauer, Fleming, Steward,...)

Two-photon exchange can be important in elastic electron-nucleon scattering (Carlson, Vanderhaeghen, ...)

...
The Set-up: the nucleon moving at the speed of light (infinite momentum frame, IFM)

- The spatial profile collapses into 2D
- The interactions between the underlying constituents are time-dilated and appears to be free: partons!
Parton’s longitudinal momentum $k_z$ can be described by the momentum fraction $x = k_z/P_z$

Parton distributions in $x$ can be measured in deep-inelastic scattering & other hard processes
The overall scale set by $\Lambda_{\text{QCD}}$

$$M_{\text{planck}} \exp(-8\pi^2 / g^2 \beta_0)$$

A perfectly natural mass scale! (F. Wilczek)

- However, gluons are massless and light quarks have small masses
  
  1 / Einstein: energy generates mass!

  $$M = E/c^2$$

- Quark and gluon energy generates the mass
One can calculate the proton mass through the expectation value of the QCD hamiltonian,

\[ H_{QCD} = H_q + H_m + H_g + H_a. \]

\[ H_q = \int d^3 \vec{x} \ \bar{\psi}(-iD \cdot \alpha)\psi, \]

\[ H_m = \int d^3 \vec{x} \ \bar{\psi}m\psi, \]

\[ H_g = \int d^3 \vec{x} \ \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \]

\[ H_a = \int d^3 \vec{x} \ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2). \]
The proton mass budget

Vacuum energy

- Trace Anomaly 20%
- Gluon Energy 34%
- Quark Energy 29%
- Quark Mass 17%

Partons can be localized in the transverse plane (D. Soper, 1972)

Parton densities in the transverse plane are related to the Fourier transformation of the Dirac form factor $F_1$ measured in elastic scattering (Burkardt, 2002)
Measurement in form factors

Progress in the last decade (since 1997)

proton

neutron
Partons in transverse plane

\[ \rho \text{ [fm}^{-2}] \]

\[ b \text{ [fm]} \]

G. Miller, 2007
In its simplest incarnation, it is a distribution of partons joint in longitudinal momentum x and transverse coordinate b!

- Give a more detailed description of partons than Feynman distributions and form factors alone.

A key to understand the spin structure of the proton.

- The spin of the proton in the simple quark model comes entirely from spin of the un-paired quark.
- The EMC experiment shows that the actual situation is much more complicated.
Polarized muon + proton deep-inelastic scattering

It measures the spin density of the quarks

\[ g(x) \sim \sum e^2(q_+(x) - q_-(x)) \]
\[ \Delta q(x) \]
The result


\[ \langle S_z \rangle_{\text{quarks}} = \frac{1}{2} \left( \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} \right) \]

\[ = \frac{1}{2} \sqrt{\frac{3}{2}} a_0 = +0.060 \pm 0.047 \pm 0.069. \]

No. of citations: 1500!
Impressive follow-ups
- SLAC Exp: E142, E143, E155, E156
- SMC
- HERMES
- COMPASS

The EMC result is correct!
Global fits of data with NLO QCD formalism yield

\[ \langle S_z \rangle_{\text{quarks}} = 0.25 \pm 0.10 \]

At \( Q = 1 \) GeV
The spin of the proton must be generated from the angular momentum of the internal quarks and gluons!

- The quarks have both spin $S$ and orbital angular momentum (OAM) $L$, giving a total $J_q$
- The gluons must also carry angular momentum $J_g$

$J = 1/2 = J_q(\mu) + J_g(\mu)$
The quark orbital motion, \( r \times p \), is suspected to be significant in proton

- Quarks are essentially massless, and therefore are ultra relativistic. For relativistic particles in a bound state, Dirac equation yields a substantial lower component wave function, which is a p-wave
- Large anomalous magnetic moment of the proton also point to the large orbital motion.

But how to measure it?

We must know some information about momentum and position simultaneously!
For 1-D quantum system, it is defined as

\[
W(x, p) = \int \psi^*(x - \eta/2)\psi(x + \eta/2)e^{ip\eta}d\eta ,
\]

- When integrated over \( x \) (\( p \)), one gets the momentum (probability) density.
- Not positive definite in general (not strict density), but is in classical limit!
- Any dynamical variable can be calculated as

\[
\langle O(x, p) \rangle = \int dx dp O(x, p)W(x, p)
\]
Harmonic oscillator & squeezed light

Wigner distribution of squeezed light!

Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

D. T. Smithey, M. Beck, and M. G. Raymer
Department of Physics and Chemical Physics Institute, University of Oregon, Eugene, Oregon 97403
A. Faridani
Department of Mathematics, Oregon State University, Corvallis, Oregon 97333
(Received 16 November 1992)
Knowing GPD, one can reconstruct the angular momentum of partons (Ji, Phys.Rev.Lett. 78: 610–613, 1997)

\[ J_q = \lim_{t \to 0} \frac{1}{2} \int dx x [H_q(x, t, \xi) + E_q(x, t, \xi)] \]

- x: parton momentum fraction
- \(\xi\): skewed parton momentum fraction
- t: t-channel momentum transfer
A process in which the proton is probed like the elastic scattering. However, the partons are probed more exclusively
Early measurements were made at ZEUS & H1

More extensive measurements at HERMES and JLab

Extensive and exciting program at Jlab 12 GeV upgrade,

One of the important motivations for a future electron-ion collider (EIC, recommend in LRP)
Example of data
Up and down quark AM in the proton
Thought to be large because of the possible role of axial anomaly $-(\alpha_s/2\pi)\Delta g$ (Altarelli & Ross, 1988)

- 2-4 units of hbar!

Of course, the gluon contribute the proton spin directly.

$$\frac{1}{2} = \Delta g + \ldots$$

One of the main motivations for RHIC spin and COMPASS expts.

Surprisingly - rapid progress, but the error bars remain large.
Q-evolution in inclusive spin structure function $g_1(x)$

Two leading-hadron production in semi-inclusive DIS

$\Delta G/G$ is small or has a node around $x_g \approx 0.1$

SMC HEMRES COMPASS
π production in polarized PP collision at RHIC

Two jet production in polarized PP collision at RHIC
Many efforts in the past have been made

- Gluck, Reya, Stratmann, Vogelsang (2001)
- Blumlein and Bottcher (2003)
- Leader, Sidorov, Stamenov (2006)
- Hirai, Kumano, Saito (2006)
- ..... 

One of the most recent is the NLO fit by de Florian, Sassot, Stratmann and Vogelsang (hep-ph/0804.0422) in which pp collision jet data are first included. *(Technically challenging!)*
Polarized sea distributions

RHIC spin asymmetries
The gluon pol. is small, but the uncertainty is large. Future data will improve this
Gluon polarization and chi-squared

(a) \( \chi^2 \) for all data sets in the x-range: 0.05-0.2

(b) Comparison of different experiments: PHENIX, STAR, SIDIS, DIS
The gluon helicity contribution to the proton spin is not overwhelming!

However, it can make a significant contribution compared to $\frac{1}{2}$, the data is becoming more constraining.

Need better data..

How to measure the total gluon contribution!
RHIC
- Direct photon production
- Higher precision in jet and pion

Electron-ion Collider!
Only way we know how to solve QCD non-perturbatively with controllable precision.

As computer power improves and algorithms get better, lattice computation finally becomes the benchmark for hadronic physics.

What can one calculate
- Mass/spectroscopy
- Form factors, couplings,
- Parton distributions/GPDs
- Low-energy constants, polarizabilities...

Let the professionals do the work.