Effective Field Theories
Beyond the Standard Model

Vincenzo Cirigliano
Los Alamos National Laboratory
Plan of the lecture

• Introduction: Search for physics BSM
  ★ direct vs indirect probes (energy vs precision frontiers) and the role of EFT

• BSM EFT prototype: Fermi theory of $\beta$ decay

• General BSM EFT (dim 5 & 6 operators)

• Applications:
  ★ $\beta$ decays: weak universality, non V-A, etc
  ★ Lepton Flavor Violation ($\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion)
Energy vs precision frontiers and the role of EFT
Energy and Precision frontiers

- While the SM successfully describes phenomena from atomic to collider energy scales, a number of open questions (both empirical and theoretical) points to the existence of new degrees of freedom & interactions active at scales $d < 10^{-16}$ cm ($E > 100$ GeV)

- Two complementary strategies to probe BSM physics:

  Energy Frontier
  (direct access to new d.o.f.)
  - EWSB mechanism
  - Discover new particles
  - ...

  Precision Frontier
  (indirect access to new d.o.f through virtual effects)
  - CP violation (w/o flavor)
  - Flavor symmetries (quarks, leptons)
  - L and B violation
  - Gauge universality
  - ....

Both needed to fully describe the “New Standard Model” at the TeV scale and address the outstanding open questions!
• Nuclear Physics plays a central role at the Precision Frontier

• The whole field of “indirect probes” is based on EFT ideas

• EFT provides a model-independent (= that applies to classes of models) framework to analyze and interpret experimental results
How does the precision frontier work?

- Key observation: at low $E$, the presence of heavy particles induces either a renormalization of the coupling constants or new local operators suppressed by powers of the heavy scale.

Example: heavy particle exchange generates new local interaction.

\[
\frac{g^2}{M_W^2 - q^2} \quad \text{and} \quad \frac{g^2}{M_W^2} \sim G_{\text{Fermi}}
\]
- \( L_{\text{eff}} \) is built out of relevant low-E degrees of freedom (SM fields)
- \( L_{\text{eff}} \) reflects symmetries of underlying theory (but not necessarily of SM)
- \( L_{\text{eff}} \) is organized in inverse powers of heavy scale (amplitudes suppressed by powers of \((E/\Lambda))\)
• Dynamics below scale $\Lambda [\sim \text{mass of new particles}]$ described by $\mathcal{L}_{\text{eff}}$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \sum_{d \geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}_n^{(d)} [\phi_{SM}]$$

• Experiments at the precision frontier probe energy scale $\Lambda$ and symmetries of the new interactions ($\leftrightarrow$ coeff. & structure of $\hat{O}_n^{(d)}$)
• $\hat{O}^{(d)}_n$ can be roughly divided in two classes:

(i) Those that **generate corrections to SM “allowed” processes:** probe them with precision measurements (β-decays, muon g-2, $Q_W$, ...).

(ii) Those that **violate (approximate) SM symmetries** and hence mediate rare/forbidden processes (quark and lepton FCNC, LNV, BNV, EDMs).

Figure copyright: David Mack
• No single low-energy probe by itself will uncover the fundamental TeV scale dynamics

• It is the combination of all these efforts (and collider searches) that will ultimately help discriminate among BSM scenarios

- 0νββ
- CPV in ν osc.
- CPV and FCNC in quark sector
- Precision EW tests at Z pole
- Parity Violating electron scattering (Qweak, ...)
- Atomic Parity Violation
- β-decays (universality, non V-A, T violation, ...)
- EDMs
- LFV in μ & τ decays
- muon g-2
BSM EFT prototype: Fermi theory of $\beta$ decay

- Write down $O(\text{GeV})$ scale EFT with given assumptions on symmetries: phenomenology ("bottom-up")
- *Match* electroweak theory (SM) onto GeV-scale EFT ("top-down")
EFT approach to $\beta$ decay

- Neutron beta decay: $n \rightarrow p + e^- + \bar{\nu}_e$

- Simplified picture:
  - "Standard Model" ($E\sim GeV$) ↔ QED + strong interactions (Yukawa):
    $\beta$ decay is forbidden
  - "New physics" mediating weak decay originates at $\Lambda_W \gg 1$ GeV
  - Want to describe the new physics that induces $\beta$ decay through $L_{\text{eff}}$, using a systematic expansion in $E/\Lambda_W$

- Masses:
  - $m_{n,p} \sim 1$ GeV
  - $m_e \sim 0.5$ MeV
  - $m_{\nu} \sim 0$
EFT approach to β decay

- Neutron beta decay: \( n \rightarrow p + e^- + \bar{\nu}_e \)

- Low energy theory (E\(\sim\)GeV): QED + strong interaction (Yukawa) + “new physics” mediating weak decay (originating at \( \Lambda_W \gg 1\ \text{GeV} \))

- Identify ingredients for EFT description:
  - Degrees of freedom (field content): \( n, \ p, \ e, \ (V_e)_{L/R} = (1 \mp \gamma_5)/2 \ \nu_e \)
  - Symmetries: Lorentz, \( U(1)_{\text{EM}} \) gauge invariance, possibly P,C,T?
  - Power counting in \( E/\Lambda_W \): non-derivative 4-fermion interactions

\[ m_{n,p} \sim 1\ \text{GeV} \]
\[ m_e \sim 0.5\ \text{MeV} \]
\[ m_{\nu} \sim 0 \]
• Most general non-derivative effective interaction (Lee-Yang ’57) involves product of fermion bilinears

\[
\mathcal{L}_{\text{eff}} \supset \frac{c_{12}}{\Lambda^2} \bar{\rho} \Gamma_1 n \bar{e} \Gamma_2 \nu_e
\]

- Dimensionless coefficients
- Scale of weak interactions
- Operators of mass dimension 6 (recall \( \Psi = m^{3/2} \)) invariant under \( U(1)_{\text{EM}} \)
- Dirac structures:
  \[
  \Gamma_i = I, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]
  \]
  \[
  S \quad P \quad V \quad A \quad T
  \]

• Problem (discussion): Make sense of dimensional factors in \( L_{\text{eff}} \)
  1. mass dimension of lagrangian density; 2. mass dimension of fields & operators
• Most general non-derivative effective interaction (Lee-Yang ’57) involves product of fermion bilinears

• Impose Lorentz invariance: \( \mathcal{L}_{\text{eff}} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T \)

\[
-\mathcal{L}_{V,A} = \bar{p}\gamma_\mu n \ \bar{\psi} \gamma^\mu (C_V + C'_V \gamma_5) \psi_e + \bar{p}\gamma_\mu \gamma_5 n \ \bar{\psi} \gamma^\mu \gamma_5 (C_A + C'_A \gamma_5) \psi_e
\]

\[
-\mathcal{L}_{S,P} = \bar{p}n \ \bar{\psi} (C_S + C'_S \gamma_5) \psi_e + \bar{p} \gamma_5 n \ \bar{\psi} \gamma_5 (C_P + C'_P \gamma_5) \psi_e + \text{h.c.}
\]

\[
-\mathcal{L}_T = \frac{1}{2} \bar{p} \sigma_{\mu\nu} n \ \bar{\psi} \sigma^{\mu\nu} (C_T + C'_T \gamma_5) \psi_e + \text{h.c.}
\]

• Problem (homework): what happened to the extra tensor term? Hint: use the identity

\[
\sigma_{\mu\nu} \gamma_5 = \frac{i}{2} \varepsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta}
\]
• Most general non-derivative effective interaction (Lee-Yang ’57) involves product of fermion bilinears

• Impose Lorentz invariance: \[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{V,A} + \mathcal{L}_{S,P} + \mathcal{L}_T \]

\[ -\mathcal{L}_{V,A} = \bar{p} \gamma_\mu n \bar{e} \gamma^\mu (C_V + C'_V \gamma_5) \nu_e + \bar{p} \gamma_\mu \gamma_5 n \bar{e} \gamma^\mu \gamma_5 (C_A + C'_A \gamma_5) \nu_e \]

\[ -\mathcal{L}_{S,P} = \bar{p} n \bar{e} (C_S + C'_S \gamma_5) \nu_e + \bar{p} \gamma_5 n \bar{e} \gamma_5 (C_P + C'_P \gamma_5) \nu_e + \text{h.c.} \]

\[ -\mathcal{L}_T = \frac{1}{2} \bar{p} \sigma_{\mu\nu} n \bar{e} \sigma^{\mu\nu} (C_T + C'_T \gamma_5) \nu_e + \text{h.c.} \]

• P-invariance \( \Leftrightarrow \) \( C'_{V,A,S,P,T} = 0 \)

• T-invariance \( \Leftrightarrow \) \( C_i, C'_i \) relatively real
Phenomenology with \( L_{\text{eff}} \)

- Experimental information on \( \beta \)-decays (rates, angular distributions) \( \Rightarrow \)

\[
C_V \equiv \frac{1}{\Lambda_W^2} \quad \Lambda_W \sim 350 \text{ GeV}
\]

\[
C_A \sim 1.25 \, C_V
\]

\[
C_V \sim C'_V \quad C_A \sim C'_A
\]

- Weak decays probe scales of \( O(100 \text{ GeV}) \gg m_{n,p} \)

- Parity maximally violated; chiral nature of the weak couplings

- Information on nature of underlying force mediators (\( \Lambda_{S,T} \geq 2 \text{ TeV} \))

- Important input in developing what we now call Standard Model
Phenomenology with $L_{\text{eff}}$

- Experimental information on $\beta$-decays (rates, angular distributions)

- In applications to BSM physics, one mostly uses the model-independent phenomenological approach described here

- However, if we know the underlying high-energy theory we can calculate the effective couplings in $L_{\text{eff}}$ via a so-called matching calculation

  - Constraints on the $C_{\text{eff}}$ can be converted into constraints on the parameters of any underlying theory

- Next, work out a simple example of matching calculation

  - Important input in developing what we now call Standard Model

($\Lambda_{S,T} \geq 2\text{ TeV}$)
Matching SM onto $L_{\text{eff}}$

- In full underlying theory (=SM), charged current weak processes are mediated by exchange of the $W$ boson.
- $W$ couples to up-down states of weak isospin doublets with strength $g_2$.

$$W \quad \text{---} \quad g_2$$

$$\left( \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \right)_L \quad \left( \begin{array}{c} u \\ d \end{array} \right)_L \quad \left( \begin{array}{c} c \\ s \\ b \end{array} \right)_L$$

$$\Psi_L = (1 - \gamma_5)/2 \quad \Psi$$

- When expressed in terms of quark mass eigenstates, the u-d-W vertex involves unitary matrix $V_{ij}$ (Cabibbo-Kobayashi-Maskawa) describing misalignment of “u” and “d” mass matrices.

$$W \quad \text{---} \quad g_2 \quad V_{ij}$$

$$u_i \quad \text{---} \quad d_j$$

$$V = \left( \begin{array}{ccc} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{array} \right)$$
• Calculate $d \rightarrow u e \nu$ amplitude within the SM

• Exploit hierarchy of scales: $m_{\text{had}} \ll M_{W,Z,t}$

$$A = \frac{g^2}{8} V_{ud} \frac{i}{k^2 - M_W^2} \bar{u} \gamma_\mu (1 - \gamma_5) d \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e$$

• To lowest order in $k^2/M_W^2$, same answer is obtained in a theory with no $W$ and a new local 4-quark operator

$$L^{SL}_{\text{eff}} = - \frac{G_F}{\sqrt{2}} V_{ud} \hat{O} + h.c.$$  \[ G_F \frac{\sqrt{2}}{} = \frac{g^2}{8 M_W^2} \]

$$A = -i \frac{G_F}{\sqrt{2}} V_{ud} \langle \hat{O} \rangle + O \left( \frac{k^2}{M_W^2} \right)$$
• Next step: go from quark-level to nucleon level description

\[
\langle p | \bar{u} \gamma_\mu d | n \rangle = g_V \bar{u}_p \gamma_\mu u_n + O(q) \quad \text{where} \quad q = p_n - p_p
\]

\[
\langle p | \bar{u} \gamma_\mu \gamma_5 d | n \rangle = g_A \bar{u}_p \gamma_\mu \gamma_5 u_n + O(q)
\]

• Final results of matching calculation:

\[
C_V = C'_V = \frac{g^2}{8M_W^2} V_{ud} = \frac{1}{\Lambda_W^2}
\]

\[
C_A = C'_A = -g_A \frac{g^2}{8M_W^2} V_{ud}
\]

\[
C_{S,P,T} = C'_{S,P,T} = 0
\]

• Effective couplings know about masses and coupling constants of the underlying theory.

• Effective scale \( \Lambda_W \) does not coincide in general with mass of new particle (factors of couplings, possibly loops....)
This was a simple example of matching calculation in EFT:

\[ A_{\text{full}} = \sum_i C_i \langle O_i \rangle \equiv A_{\text{EFT}} \]

- “Integrate out” heavy d.o.f (W,Z,t); write \( L_{\text{eff}} \) in terms of local operators built from low-energy d.o.f.

- To a given order in \( E/M_W \), determine effective couplings (Wilson coefficients) from the matching condition \( A_{\text{full}} = A_{\text{EFT}} \) with amplitudes involving “light” states

- We did matching at tree-level, but strong and electroweak higher order corrections can be included

\[
\begin{array}{c}
\text{Full theory} \\
\includegraphics[width=0.2\textwidth]{full_theory_diagram.png}
\end{array} + \begin{array}{c}
\text{Effective theory} \\
\includegraphics[width=0.2\textwidth]{effective_theory_diagram.png}
\end{array} + \ldots = C_i \cdot \left( \includegraphics[width=0.05\textwidth]{tree_diagram.png} + \includegraphics[width=0.05\textwidth]{tree_diagram.png} + \ldots \right)
\]
General BSM EFT
Big picture

- Assume existence of new particles with $M \gg E_{\text{accessible}} \sim G_F^{-1/2}$
- "Integrate out" these particles: describe dynamics below scale $\Lambda [\sim \text{mass of new particles}]$ via $\mathcal{L}_{\text{eff}}$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d \geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}^{(d)}[\phi_{\text{SM}}]$$
• Building $L_{\text{eff}}$ requires specifying:

  ★ **Degrees of freedom**: SM field content
  ★ One Higgs doublet, no light $\nu_R$ and no other light fields

  ★ **Symmetries**: SM gauge group $\text{SU}(3)_c \times \text{SU}(2)_W \times \text{U}(1)_Y$
  ★ Underlying theory respects SM gauge group

  ★ **Power counting** in $E/\Lambda$: organize analysis in terms of operators of increasing dimension (5,6,...)
Lightning review of the SM

- Gauge group: \( \text{SU}(3)_c \times \text{SU}(2)_W \times \text{U}(1)_Y \)

- Notation for gauge group representations:

\[
\psi'(x) = e^{ig_A \alpha_A(x) \frac{\lambda_A}{2}} \psi(x)
\]

\[
\psi'(x) = e^{ig' \gamma(x) Y} \psi(x)
\]

\[
\psi'(x) = e^{ig \beta_a(x) \frac{g_a}{2}} \psi(x)
\]

- (\(\dim[\text{SU}(3)_c]\), \(\dim[\text{SU}(2)_W]\), \(Y\))
Building blocks 1: gauge fields

SU(3)_c \times SU(2)_W \times U(1)_Y

**Representation**

- **Gluons:**
  \[ G^A_\mu, \quad A = 1 \cdots 8, \]
  \[ G^A_{\mu \nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f_{ABC} G^B_\mu G^C_\nu. \] (8,1,0)

- **W Bosons:**
  \[ W^I_\mu, \quad I = 1 \cdots 3, \]
  \[ W^I_{\mu \nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g \epsilon_{IJK} W^J_\mu W^K_\nu. \] (1,3,0)

- **B Boson:**
  \[ B_\mu, \]
  \[ B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \] (1,1,0)

**Gauge Transformation:**

\[
\frac{W^I_{\mu \nu}}{2} \rightarrow V(x) \left[ \frac{W^I_{\mu \nu}}{2} \right] V^\dagger(x)
\]

\[ V(x) = e^{ig \beta_a(x) \frac{\sigma_a}{2}} \]
### Building blocks 2: fermions and Higgs

<table>
<thead>
<tr>
<th>SU(3)_c × SU(2)_W × U(1)_Y representation</th>
<th>SU(2)_W transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = \begin{pmatrix} \nu_L \ e_L \end{pmatrix} )</td>
<td>((1,2,-1/2)) ( l \to V_{SU(2)} l )</td>
</tr>
<tr>
<td>( e = e_R )</td>
<td>((1,1,-1))</td>
</tr>
<tr>
<td>( q^i = \begin{pmatrix} u^i_L \ d^i_L \end{pmatrix} )</td>
<td>((3,2,1/6)) ( q \to V_{SU(2)} q )</td>
</tr>
<tr>
<td>( u^i = u^i_R )</td>
<td>((3,1,2/3))</td>
</tr>
<tr>
<td>( d^i = d^i_R )</td>
<td>((3,1,-1/3))</td>
</tr>
<tr>
<td>( \varphi = \begin{pmatrix} \varphi^+ \ \varphi^0 \end{pmatrix} )</td>
<td>((1,2,1/2)) ( \varphi \to V_{SU(2)} \varphi )</td>
</tr>
<tr>
<td>( \bar{\varphi} = \epsilon \varphi^* = \begin{pmatrix} \varphi_{0*}^0 \ -\varphi_{-}^- \end{pmatrix} )</td>
<td>((1,2,-1/2)) ( \bar{\varphi} \to V_{SU(2)} \bar{\varphi} )</td>
</tr>
</tbody>
</table>

\( \epsilon = i\sigma_2 \)
• **SM Lagrangian:** \[ \mathcal{L}_{SM} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \]

\[ \mathcal{L}_{\text{Gauge}} = -\frac{1}{4} G^{A}_{\mu\nu} G^{A}_{\mu\nu} - \frac{1}{4} W^{I}_{\mu\nu} W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \]

\[ + i \bar{\ell} \mathcal{D} \ell + i \bar{e} \mathcal{D} e + i \bar{q} \mathcal{D} q + i \bar{u} \mathcal{D} u + i \bar{d} \mathcal{D} d \]

\[ \mathcal{L}_{\text{Higgs}} = (D_{\mu} \varphi)^\dagger (D^{\mu} \varphi) - \lambda (\varphi^\dagger \varphi - v^{2})^{2} \]

\[ \langle \varphi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \]

\[ \langle \bar{\varphi} \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix} \]

\[ v = 174 \text{ GeV} \]

\[ \mathcal{L}_{\text{Yukawa}} = Y_{e} \bar{\ell} e \varphi + Y_{d} \bar{q} d \varphi + Y_{u} \bar{u} q \bar{\varphi} + \text{h.c.} \]

• **Covariant derivative**

\[ D_{\mu} = I \partial_{\mu} - ig_{s} \frac{\lambda^{A}}{2} G^{A}_{\mu} - ig \frac{\sigma^{a}}{2} W^{a}_{\mu} - ig' Y B_{\mu} \]
BSM: dimension 5

• Construct all possible dim=5 effective operators in detail: this illustrates the method and leads to a physically interesting result

• Fermions only (and derivatives)? No
  • $\Psi$'s are chiral fermions and $[\Psi]=3/2$, so e.g. $\bar{\psi} D \bar{\psi} \psi = 0$

  \[
  D \bar{\psi} = D_\mu D_\nu g^{\mu\nu} - i D_\mu D_\nu \sigma^{\mu\nu}
  \]

• Scalars only, vectors only? No: use $[\phi] = [V] = 1$ and gauge invariance

• Vectors + Fermions & Vectors + scalars? No

• So, we are lead to consider operators with fermions (2) and scalars (2) and no derivatives
• If scalars are \( \varphi \) and \( \varphi^* \) \( \Rightarrow \)

  • total hypercharge \( Y \) of fermions \( \Psi_1 \) and \( \Psi_2 \) is 0
  • need a multiplet and its charge-conjugate
  • but cannot make non-vanishing Lorentz scalar of dim3 \((\bar{\psi}\psi = 0)\)

• We are left with building blocks \( \varphi, \varphi, \Psi_1, \Psi_2 \)

  • Forming SU(2)\(_W\) invariants: \( \varphi^T \epsilon \varphi = 0 \Rightarrow \Psi_1, \Psi_2 \) must be doublets
    (so we are left with \( \ell \) or q)
    \[
    \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
    \]
  
• Problem (discussion): prove that \( d_1^T \epsilon d_2 \) is SU(2) invariant \((d_{1,2} \) doublets)
• If scalars are $\varphi$ and $\varphi^*$ ⇒

  • total hypercharge $Y$ of fermions $\Psi_1$ and $\Psi_2$ is 0
  • need a multiplet and its charge-conjugate
  • but cannot make non-vanishing Lorentz scalar of dim3 \( \bar{\psi}\psi = 0 \)

• We are left with building blocks $\varphi$, $\varphi^*$, $\Psi_1$, $\Psi_2$

  • Forming $SU(2)_W$ invariants: $\varphi^T \varepsilon \varphi = 0 \Rightarrow \Psi_1, \Psi_2$ must be doublets (so we are left with $\ell$ or $q$)
  • $\ell^T \varepsilon \varphi$ and $\varphi^T \varepsilon \ell$ are $SU(2)_W$ and $U(1)_Y$ invariant
  • Connect them to make Lorentz scalar:

\[
\hat{O}_{\text{dim}=5} = \ell^T C \varepsilon \varphi \varphi^T \varepsilon \ell
\]

\[C = i\gamma_2\gamma_0\]
• Could one replace \( \ell \) with \( q \)? No: invariance under \( SU(3)_c \) and \( U(1)_Y \)

• Conclusion: there is only one dim=5 operator (Weinberg ’79)

\[
\hat{O}_{\text{dim}=5} = \ell^T C \epsilon \varphi^T e \ell \\
C = i \gamma_2 \gamma_0
\]

• it violates total lepton number (\( \ell \rightarrow e^{i\alpha} \ell, \ e \rightarrow e^{i\alpha} e \))

• it generates Majorana mass for L-handed neutrinos (after EWSB)

\[
\frac{1}{\Lambda} \hat{O}_{\text{dim}=5} \xrightarrow{\langle \varphi \rangle = \begin{pmatrix} 0 \\ \nu \end{pmatrix}} \frac{\nu^2}{\Lambda} \nu_L^T C \nu_L
\]

• light neutrino mass scale (\( \lesssim \text{eV} \)) points to high scale of lepton number breaking

\[
m_\nu \sim 1 \text{ eV} \rightarrow \Lambda \sim 10^{13} \text{ GeV}
\]
• Explicit realization of this operator in models with heavy R-handed Majorana neutrinos

\[
\mathcal{L}_5 = g_{\alpha \beta} \ell^T \epsilon \varphi \varphi^T \epsilon \ell \beta
\]
BSM: dimension 6

- Many possible structures, but methodology is the same
- B violating operators: Weinberg’79 & Wilczek-Zee ‘79
- B & L conserving operators: first systematic analysis by Buchmuller-Wyler ’86 (~ 80 operators)
- Here give just a few examples:
  - 4-fermion operators
  - operators involving vectors-fermions-scalars

After EWSB these generate corrections to fermion - gauge boson vertex (vector and dipole)
• Examples of 4-fermion operators (relevant for $\beta$ decay discussion and Lepton Flavor Violation) [Homework: check gauge invariance]

\[
O_{\bar{u}l}^{(1)} = \frac{1}{2} (\bar{l} \gamma^\mu l) (\bar{l} \gamma_\mu l), \quad O_{\bar{u}l}^{(3)} = \frac{1}{2} (\bar{l} \gamma^\mu \sigma^a l) (\bar{l} \gamma_\mu \sigma^a l)
\]

\[
O_{\bar{u}q}^{(1)} = (\bar{l} \gamma^\mu l) (\bar{q} \gamma_\mu q), \quad O_{\bar{u}q}^{(3)} = (\bar{l} \gamma^\mu \sigma^a l) (\bar{q} \gamma_\mu \sigma^a q)
\]

\[
O_{qde} = (\bar{e} e)(\bar{d} q) + \text{h.c.}
\]

\[
O_{lq} = (\bar{\ell}_a e) e^{ab} (\bar{q}_b u) + \text{h.c.}
\]

\[
O_{lq}^t = (\bar{\ell}_a \sigma^{\mu\nu} e) e^{ab} (\bar{q}_b \sigma_{\mu\nu} u) + \text{h.c.}
\]

- Recall:

\[
(t^a)_{ij} (t^a)_{kl} = \frac{1}{2} \left( \delta_{il} \delta_{jk} - \frac{1}{N} \delta_{ij} \delta_{kl} \right)
\]

- These operators contribute to both charged-current and neutral current transitions
• Examples of vectors-fermions-scalars operators (relevant for $\beta$ decay and Lepton Flavor Violation) [\textbf{Homework: check gauge invariance}]

\[ O_{\varphi l}^{(1)} = i(\varphi^\dagger D^\mu \varphi)(\bar{l}\gamma_\mu l) + \text{h.c.}, \quad O_{\varphi l}^{(3)} = i(\varphi^\dagger \sigma^a D^\mu \varphi)(\bar{l}\gamma_\mu \sigma^a l) + \text{h.c.} \]

\[ O_{\varphi q}^{(1)} = i(\varphi^\dagger D^\mu \varphi)(\bar{q}\gamma_\mu q) + \text{h.c.}, \quad O_{\varphi q}^{(3)} = i(\varphi^\dagger \sigma^a D^\mu \varphi)(\bar{q}\gamma_\mu \sigma^a q) + \text{h.c.} \]

\[ O_{eW} = (\bar{l}\sigma^{\mu\nu} \sigma^a e) \varphi W_{\mu\nu}^a, \quad O_{eB} = (\bar{l}\sigma^{\mu\nu} e) \varphi B_{\mu\nu} \]
Applications

- $\beta$ decays: weak universality, non V-A, etc
- Lepton Flavor Violation: discriminating power of $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion
“β-decays”
Semi-leptonic CC decays

- Mediated by $W$ exchange in the SM
- Only V-A structure
- Universality relations
  \[ \frac{[G_F]_e}{[G_F]_\mu} = 1 + \Delta_{e/\mu} \]
  \[ |V_{ud}|^2 + |V_{us}|^2 + |\bar{V}_{ub}|^2 = 1 + \Delta_{CKM} \]

- Sensitivity to BSM scale: \( \Delta \sim \frac{c_n}{g^2} \frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \leftrightarrow \Lambda \sim 1-10 \text{ TeV} \)

Lepton universality
Cabibbo universality

- Mediated by $W$ exchange in the SM
- Only V-A structure
- Universality relations
  \[ \frac{[G_F]_e}{[G_F]_\mu} = 1 + \Delta_{e/\mu} \]
  \[ |V_{ud}|^2 + |V_{us}|^2 + |\bar{V}_{ub}|^2 = 1 + \Delta_{CKM} \]

- Sensitivity to BSM scale: \( \Delta \sim \frac{c_n}{g^2} \frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \leftrightarrow \Lambda \sim 1-10 \text{ TeV} \)
# Paths to $V_{ud}$ and $V_{us}$

| $V_{ud}$ | $0^+ \rightarrow 0^+$  
$\pi^\pm \rightarrow \pi^0 e^+\nu$ | $n \rightarrow p e^−\nu$ | $\pi \rightarrow \mu\nu$ | $\tau \rightarrow h_{NS} \nu_\tau$ |
|---|---|---|---|---|
| $V_{us}$ | $K \rightarrow \pi\ell\nu$ | $\Lambda \rightarrow p e^−\nu,...$ | $K \rightarrow \mu\nu$ | $\tau \rightarrow h_s \nu_\tau$  
(inclusive) |

\[
\Gamma_{ij} = \left[ G_F^{(\mu)} V_{ij} \right]^2 \times |M_{had}|^2 \times (1 + \delta_{em}) \times F_{kin}
\]

- **Hadronic matrix elements**
- **Radiative corrections**
Global Fit to $V_{ud}$ and $V_{us}$

**Fit result**

$V_{ud} = 0.97425 (22)$

$V_{us} = 0.2252 (9)$

$\chi^2$/dof = 0.65/1

$|V_{ud}|^2 + |V_{us}|^2 = 0.9999(6)$

Error equally shared between $V_{ud}$ and $V_{us}$

- Remarkable agreement with Cabibbo universality: $\Delta_{CKM} = -(1 \pm 6) \times 10^{-4}$
- Confirms large EW rad. corr. $\left(2 \alpha/\pi \log(M_Z/M_p) = +3.6\%\right)$
- It would naively fit $M_Z = (90 \pm 7)$ GeV
Implications for BSM physics

- Extraction of $V_{ij}$ uses Fermi constant from muon decay

\[ \Gamma_{ij} = \left[ G_F^{(\mu)} V_{ij} \right]^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{\text{em}}) \times F_{\text{kin}} \]

- In SM extensions, Fermi constant in muon decay and semi-leptonic transitions may differ (vertex corrections and boxes)

- $\Delta_{\text{CKM}}$ is sensitive to these apparent violations of weak universality from TeV extensions of the SM:

\[ \frac{[G_F^{(\beta)}]^2}{[G_F^{(\mu)}]^2} = 1 + \Delta_{\text{CKM}} \]
EFT analysis

- Explore in a **model-independent** way:
  
  1. significance of $\Delta_{\text{CKM}}$ constraint vs other precision measurements.
  2. correlations between potential universality deviations and other low- and high-energy observables

- Setup: parameterize BSM interactions via SU(2)$\times$U(1) gauge-invariant higher-dim operators built out of SM fields

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_X \frac{1}{\Lambda_X^2} O_X \]

Buchmuller-Wyler 1986, … … Han-Skiba 2004

- Flavor properties: include only $U(3)^5$-invariant operators $\Rightarrow$ no problems with FCNC.
**Δ_{CKM} is sensitive to four operators:**

$$Δ_{CKM} = 4 \left( \hat{\alpha}^{(3)}_{ll} - \hat{\alpha}^{(3)}_{lq} - \hat{\alpha}^{(3)}_{\varphi l} + \hat{\alpha}^{(3)}_{\varphi q} \right)$$

Vertex corrections

\[ \hat{\alpha}_X = \frac{v^2}{\Lambda_X^2} \]

\[ v \sim 200 \text{ GeV} \]
• Δ_{CKM} is sensitive to four operators:

\[ Δ_{CKM} = 4 \left( \hat{A}_{ll}^{(3)} - \hat{A}_{lq}^{(3)} - \hat{A}_{\varphi l}^{(3)} + \hat{A}_{\varphi q}^{(3)} \right) \]

\[ \hat{A}_X = \frac{v^2}{\Lambda_X^2} \]

\[ O_{ll}^{(3)} = \frac{1}{2} (\bar{\ell}_{\mu} \Gamma_{\alpha} l)(\bar{\ell}_{\mu} \Gamma_{\alpha} l) \]

\[ O_{lq}^{(3)} = (\bar{u}_{\mu} \Gamma_{\alpha} q)(\bar{q}_{\mu} \Gamma_{\alpha} q) \]

Gauge invariance

\[ l^i = \left( \begin{array}{c} \nu^i_L \\ e^i_L \end{array} \right) \quad \varphi = \left( \begin{array}{c} \varphi^+ \\ \varphi^0 \end{array} \right) \quad q^i = \left( \begin{array}{c} u^i_L \\ d^i_L \end{array} \right) \]

• Relevant operators affect other precision EW observables! Assess significance of Δ_{CKM} vs other EWPT
Question (1): What is the range of $\Delta_{\text{CKM}}$ allowed by precision EW tests?

- Global fit and covariance matrix from Han-Skiba 04

\[ -9.5 \times 10^{-3} \leq \Delta_{\text{CKM}} \leq 0.1 \times 10^{-3} \]

90% C.L.

- Direct constraint implies $|\Delta_{\text{CKM}}| \leq 1.0 \times 10^{-3} @ 90\% \text{ CL}$

EW precision data alone would leave room for large $\Delta_{\text{CKM}}$!
Question (2): What is the strength of $\Delta_{\text{CKM}}$ constraint?

Same level or better than Z-pole obs.: $\Lambda > 11$ TeV @ 90% CL
Question (2): What is the strength of \( \Delta_{\text{CKM}} \) constraint?

Same level or better than Z-pole obs.: \( \Lambda > 11 \text{ TeV} @ 90\% \text{ CL} \)

Deviations as large as \( \Delta_{\text{CKM}} \sim -0.01 \) at 90\% CL could be blamed on \( O_{lq}^{(3)} \) without conflicting with LEP2 data on hadronic cross section.

Dramatic improvement (one order of magnitude) over LEP2 and APV.
Muons and Lepton Flavor Violation: an EFT perspective
Evidence of $\nu$ oscillations implies that individual lepton family numbers $(L_e, \mu, \tau)$ are not conserved.

In SM + massive $\nu$, charged LFV rates are negligible (GIM-suppression).

Great discovery channels. Extremely clean probe of BSM physics.
• Experimental status (90% CL): muons

<table>
<thead>
<tr>
<th>Process</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\mu \to e\gamma}$</td>
<td>$&lt; 1.2 \times 10^{-11}$</td>
</tr>
<tr>
<td>$B_{\mu \to 3e}$</td>
<td>$&lt; 1.0 \times 10^{-12}$</td>
</tr>
<tr>
<td>$B_{\mu \to e}^{Ti}$</td>
<td>$&lt; 4.3 \times 10^{-12}$</td>
</tr>
<tr>
<td>$B_{\mu \to e}^{Au}$</td>
<td>$&lt; 8 \times 10^{-13}$</td>
</tr>
<tr>
<td>$B_{\mu \to e}^{Pb}$</td>
<td>$&lt; 4.6 \times 10^{-11}$</td>
</tr>
</tbody>
</table>

$\rightarrow 10^{-13/14}$ (MEG at PSI, now running)

$\rightarrow 10^{-16/17}$ (Mu2e, COMET)

• $\mu$-to-e conversion rate is normalized to total muon capture rate

$$B_{\mu \to e} = \frac{\Gamma(\mu^- + (Z, A) \to e^- + (Z, A))}{\Gamma(\mu^- + (Z, A) \to \nu_{\mu} + (Z - 1, A))}$$
BSM, several dim-6 operators contribute to LFV processes

- Dominant in SUSY-GUT and SUSY seesaw scenarios
- Enhanced in Left-Right symmetric models
- Dominant in RPV SUSY and RPC SUSY for large $\tan(\beta)$ and low $m_A$

\[ \mathcal{L}_{\text{eff}} \supset \frac{[\alpha_D]^{ij}}{\Lambda^2} \varphi^i \bar{\ell}_R \sigma_{\mu\nu} \ell_L^j F^{\mu\nu} + \frac{[\alpha_S]^{ij}}{\Lambda^2} \bar{\ell}_R^i \ell_L^j \bar{q}_L d_R^j \]

\[ + \frac{[\alpha_{V(z)}]^{ij}}{\Lambda^2} \bar{\ell}_L^i \gamma_{\mu} \ell_L^j \varphi^i D^{\mu} \varphi + \frac{[\alpha_{V(\gamma)}]^{ij} e_q}{\Lambda^2} \bar{\ell}_L^i \gamma_{\mu} \ell_L^j \varphi^i D^{\mu} \varphi + \ldots \]

- Generated by Z-penguin
- Enhanced in Left-Right symmetric models
• BSM, several dim-6 operators contribute to LFV processes

Key Questions for LFV dynamics in LHC era

0 - What is the overall size of LFV effects?

Current limit from $\mu \rightarrow e\gamma$ implies

$$\frac{\Lambda}{\sqrt{\alpha_D} e^\mu} > 2 \times 10^4 \text{ TeV}$$

- In TeV extensions of the SM, flavor symmetry is broken in a non-generic way (small mixing!)
- New physics at TeV (and reasonable mixing pattern) $\Leftrightarrow$ LFV signals are within reach of planned searches

Be optimistic: assume that BSM physics produces observable rates. Ask questions that probe more deeply LFV dynamics and help discriminating underlying SM extensions
Key Questions for LFV dynamics in LHC era

1 - What is the relative strength of various operators (\(\alpha_D\) vs \(\alpha_S\) ...)?
   - Can be addressed experimentally through analysis of \(\mu \to e\gamma\) and \(\mu \to e\) conversion in different target nuclei
     

2 - What is the flavor structure of the couplings (\([\alpha_D]_{e\mu}\) vs \([\alpha_D]^{\tau\mu}\)...)?
   - Many possible scenarios
   - Question can in part be addressed experimentally, by testing the predicted pattern of \(\mu \to e\gamma\) vs \(\tau \to \mu\gamma\) rates
   - For a simple and predictive scheme (Minimal Flavor Violation)
     see references below

**Discriminating power of $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion**

- $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion probe different combinations of operators

\[
\begin{align*}
\mu \rightarrow e\gamma & \quad \text{Transition dipole moment:} \quad (im_\mu \sigma_{\mu\nu} q^\nu) \\
\mu q \rightarrow eq & \quad \text{Transition charge radius:} \quad (q^2 \gamma_\mu - \bar{q} q_\mu)
\end{align*}
\]

- Conversion amplitude has non-trivial dependence on target nucleus, that distinguishes D,S,V underlying operators

\[
\langle e^-; A, Z | \hat{O}_\ell \hat{O}_q | \mu^-; A, Z \rangle \sim \int d^3x \bar{\psi}_e O_\ell \psi_\mu \langle A, Z | \hat{O}_q | A, Z \rangle
\]

Relativistic components of muon wave-function give different contributions to D,S,V overlap integrals
- Models in which a single operator dominates can be tested with one double ratio (two LFV measurements):

\[ \frac{B(\mu \rightarrow e, Z)}{B(\mu \rightarrow e, \gamma)} \]

Deviation from this pattern indicates presence of scalar and/or vector contributions.

- Essentially free of theory uncertainty (largely cancels in ratios)
- Discrimination: need 5% measure of Ti/Al or 20% measure of Pb/Al
- Ideal world: use Al and a large Z-target (D,V,S have largest separation)
Models in which two operators dominate can be tested with two double ratios (three LFV measurements!).

Consider $S$ and $D$: realized in SUSY via competition between dipole and scalar operator (mediated by Higgs exchange)

- Uncertainty from strange form factor largely reduced by lattice QCD

\[
y = \frac{2 \langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \quad \in \quad [0, 0.4] \quad \rightarrow \quad [0, 0.05]
\]

Relative sign: +
• Models in which two operators dominate can be tested with two double ratios (three LFV measurements!).

• Consider $S$ and $D$: realized in SUSY via competition between dipole and scalar operator (mediated by Higgs exchange)

- Uncertainty from strange form factor largely reduced by lattice QCD

$$y = \frac{2 \langle p | \bar{s} s | p \rangle}{\langle p | \bar{u} u + \bar{d} d | p \rangle} \in [0, 0.4] \rightarrow [0, 0.05]$$

fat error band

thin error band

JLQCD 2008
• Models in which two operators dominate can be tested with two double ratios (three LFV measurements!).
• Consider $S$ and $D$: realized in SUSY via competition between dipole and scalar operator (mediated by Higgs exchange).

In summary:

- Theoretical hadronic uncertainties under control (OK for 1-operator dominance, need Lattice QCD for 2-operator models)
- Realistic model discrimination requires measuring $T_{i}/A_{i}$ at <5% or $P_{b}/A_{i}$ at <20%; challenge for future experiments
- Uncertainty from strange form factor largely reduced by lattice QCD

$$y = \frac{2 \langle p | \bar{s} s | p \rangle}{\langle p | \bar{u} u + \bar{d} d | p \rangle} \in [0, 0.4] \rightarrow [0, 0.05]$$

fat error band \hspace{2cm} thin error band

JLQCD 2008