Relativistic Heavy Ions II - Soft physics

RHI Physics
The US National Nuclear Physics Summer School & TRIUMF Summer Institute
Vancouver, Canada
Helen Caines - Yale University
June 2010

Outline:
The Energy Density
The Temperature
Fluid and Flow
Recap of first lecture

- Looking for evidence of a new state of matter → QGP

- Predicted by QCD to occur, due to screening of colour charge, at high T and/or density
  - $T_c \sim 160$ MeV

- Create in laboratory by colliding ultra-relativistic heavy-ions

- Large multi-purpose experiments necessary to sift through all the data produced
The phase transition in the laboratory

Chemical freeze-out ($T_{ch} \leq T_c$): inelastic scattering ceases

Kinetic freeze-out ($T_{fo} \leq T_{ch}$): elastic scattering ceases
The phase transition in the laboratory

Lattice (2-flavor):
\[ T_C \approx 173\pm8 \text{ MeV} \]
\[ \varepsilon_C \approx (6\pm2) T^4 \approx 0.70 \text{ GeV/fm}^3 \]

Remember: cold nuclear matter
\[ \varepsilon_{\text{cold}} \approx u / \frac{4}{3} \pi r_0^3 \approx 0.13 \text{ GeV/fm}^3 \]

Chemical freeze-out \((T_{\text{ch}} \leq T_C)\): inelastic scattering ceases

Kinetic freeze-out \((T_{\text{fo}} \leq T_{\text{ch}})\): elastic scattering ceases
The phase transition in the laboratory

Lattice (2-flavor):
\[ T_C \approx 173\pm8 \text{ MeV} \]
\[ \varepsilon_C \approx (6\pm2) \times T^4 \approx 0.70 \text{ GeV/fm}^3 \]

Remember: cold nuclear matter
\[ \varepsilon_{\text{cold}} \approx u / \frac{4}{3} \pi r_0^3 \approx 0.13 \text{ GeV/fm}^3 \]

Necessary but not sufficient condition
Tevatron (Fermilab)
\[ \varepsilon(\sqrt{s} = 1.8 \text{TeV } pp) >> \]
\[ \varepsilon(\sqrt{s} = 200 \text{GeV } Au+Au \text{ RHIC}) \]

Thermal Equilibrium ⇒
many constituents

Chemical freeze-out \((T_{ch} \leq T_C)\): inelastic scattering ceases

Kinetic freeze-out \((T_{fo} \leq T_{ch})\): elastic scattering ceases

Size matters !!!
**Thermodynamics - phase transitions**

**Phase transition or a crossover?**

Signs of a phase transition:

1st order: **discontinuous in entropy** at $T_c \rightarrow$ Latent heat, a mixed phase

![Graph showing $\Delta S = \frac{L}{T_c}$]

Higher order: **discontinuous in higher derivatives of $\delta^n S/\delta T^n$** $\rightarrow$ no mixed phase - system passed smoothly and uniformly into new state (ferromagnet)

- Temperature $\Leftrightarrow$ transverse momentum $T \propto \langle p_T \rangle$
- Energy density $\Leftrightarrow$ transverse energy $\varepsilon \propto dE_T / dy \equiv \langle m_T \rangle dN / dy$
- Entropy $\Leftrightarrow$ multiplicity $S \propto dN / dy$
The order of the phase transition

“A first-order QCD phase transition that occurred in the early universe would lead to a surprisingly rich cosmological scenario.” Ed Witten, Phys. Rev. D (1984)
"A first-order QCD phase transition that occurred in the early universe would lead to a surprisingly rich cosmological scenario." Ed Witten, Phys. Rev. D (1984)

Apparently it did not! Thus we suspect a smooth cross over or a weak first order transition.
The language of RHI collisions

- Before starting, we need to know some specific terminology used in RHI collisions.

- Relativity:
  - Energy: \( E^2 = p^2 + m^2 \) or \( E = T + m \) or \( E = \gamma m \)
  - where: \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \) and \( \beta = \frac{v}{c} = \frac{p}{E} \)

- Lorentz Transformations:
  - \( E' = \gamma (E + \beta p_z) \)
  - \( p'_z = \gamma (p_z + \beta E) \)

- Kinematics:
  - \( p_L = p_z \)
  - \( p_T = \sqrt{p_x^2 + p_y^2} \)
  - \( m_T = \sqrt{p_T^2 + m^2} \)
  - Transverse mass
  - \( y = \frac{1}{2} \ln \frac{E + p_L}{E - p_L} \)
  - Rapidity
  - \( y' = y + \tanh^{-1} \beta \)
  - \( \eta = \frac{1}{2} \ln \frac{p + p_L}{p - p_L} \)
  - Pseduo-Rapidity
    (no particle id required)
Geometry of a heavy-ion collision

Non-central collision

“peripheral” collision ($b \sim b_{\text{max}}$)

“central” collision ($b \sim 0$)
Geometry of a heavy-ion collision

Number of participants ($N_{\text{part}}$): number of incoming nucleons (participants) in the overlap region

Number of binary collisions ($N_{\text{bin}}$): number of equivalent inelastic nucleon-nucleon collisions

$N_{\text{bin}} \geq N_{\text{part}}$

"peripheral" collision ($b \sim b_{\text{max}}$)

"central" collision ($b \sim 0$)
Quantifying the geometry

p+p: 2 Participants, 1 Binary Collision

Participants: those nucleons that have interacted at least once
Binary collisions: the number of 1+1 collisions
Quantifying the geometry

p+p: 2 Participants, 1 Binary Collision

Participants: those nucleons that have interacted at least once
Binary collisions: the number of 1+1 collisions
Quantifying the geometry

p+A: 8 Participants, 7 Binary Collisions

Generically: $N_{\text{part}} = N_{\text{bin}} + 1$

Participants: those nucleons that have interacted at least once
Binary collisions: the number of 1+1 collisions
Quantifying the geometry

$p+A$: 8 Participants, 7 Binary Collisions

Generically: $N_{\text{part}} = N_{\text{bin}} + 1$

Participants: those nucleons that have interacted at least once

Binary collisions: the number of $1+1$ collisions

Helen Caines - NNPSS-TSI - June 2010
Quantifying the geometry

A+A: 9 Participants, 14 Binary Collisions

A+A: 16 Participants, 14 Binary Collisions

Participants: those nucleons that have interacted at least once
Binary collisions: the number of 1+1 collisions
Quantifying the geometry

A+A: 9 Participants, 14 Binary Collisions

A+A: 16 Participants, 14 Binary Collisions

Participants: those nucleons that have interacted at least once
Binary collisions: the number of 1+1 collisions
Glauber calculations

Use a Glauber calculation to estimate $N_{\text{bin}}$ and $N_{\text{part}}$

• Roy Glauber: Nobel prize in physics 2005 for “his contribution to the quantum theory of optical coherence”

• Application of Glauber theory to heavy ion collisions does not use the full sophistication of these methods. Two simple assumptions:
  • Eikonal: constituents of nuclei proceed in straight-line trajectories  
  • Interactions determined by initial-state shape of overlapping nuclei
Ingredients for Glauber calculations

- Assumptions: superposition of straight-line interactions of colliding nucleons
- Need nucleon-nucleon interaction cross section
  Most use inelastic: 42 mb at $\sqrt{s}=200$ GeV
  Other choices: Non-singly-diffractive, 30 mb at $\sqrt{s} = 200$ GeV
- Need probability density for nucleons:
  `Wood-Saxon’ from electron scattering experiments

M. Miller et al, nucl-ex/0701025
Implementations of Glauber

- **Optical Glauber**
  - Smooth distribution assumed
  - Analytic overlap calculation from integration over nuclear shape functions, weighted with appropriate N-N cross-section

- **Monte Carlo Glauber**
  - Randomly initialize nucleons sampling nuclear shape
  - At randomly selected impact parameter, allow nuclei to interact
  - Randomly sample probability of nucleons to interact from interaction cross-section
    - e.g. if distance $d$ between nucleons is $< \sqrt{\sigma_{int}/\pi}$

Calculate probability that $N_{part}$ or $N_{bin}$ occurs per event
Comparing to data heavy-ion collision

Good agreement between data and calculation

Measured mid-rapidity particle yield can be related to size of overlap region

Helen Caines - NNPSS-LS -June 2010
A peripheral Au-Au collision

Color ☢️ Energy loss in TPC gas
39.4 TeV in central Au-Au collision

- Only charged particles shown
- Neutrals don’t ionise the TPC’s gas so are not “seen” by this detector.

>5000 hadrons and leptons
Only charged particles shown

Neutrals don’t ionise the TPC’s gas so are not “seen” by this detector.

>5000 hadrons and leptons

26 TeV is removed from colliding beams.
The energy is contained in one collision

Central Au+Au Collision:
26 TeV ~ 6 μJoule
The energy is contained in one collision

Central Au+Au Collision:
26 TeV ~ 6 µJoule

Sensitivity of human ear:
10^{-11} \text{erg} = 10^{-18} \text{Joule} = 10^{-12} \text{µJoule}
A Loud “Bang” if $E \Rightarrow \text{Sound}$
The energy is contained in one collision

Central Au+Au Collision:
26 TeV ~ 6 µJoule

Sensitivity of human ear:
10^{-11} \text{erg} = 10^{-18} \text{Joule} = 10^{-12} \mu\text{Joule}
A Loud “Bang” if E ⇒ Sound

Most goes into particle creation
Energy density in central Au-Au collisions

- use calorimeters to measure total energy
Energy density in central Au-Au collisions

- use calorimeters to measure total energy
- estimate volume of collision

Bjorken-Formula for Energy Density:

\[
\varepsilon_{Bj} = \frac{\Delta E_T}{\Delta V} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}
\]

\(R \sim 6.5 \text{ fm}\)

Time it takes to thermalize system
\((t_0 \sim 1 \text{ fm/c})\)

\[dz = \tau_0 dy\]
Energy density in central Au-Au collisions

- use calorimeters to measure total energy
- estimate volume of collision

Bjorken-Formula for Energy Density:

$$\varepsilon_{BJ} = \frac{\Delta E_T}{\Delta V} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

R~6.5 fm

Time it takes to thermalize system ($t_0 \sim 1 \text{ fm/c}$)

$$dz = \tau_0 dy$$

$$\varepsilon_{BJ} \approx 5.0 \text{ GeV/fm}^3$$

~30 times normal nuclear density

~ 5 times > $\varepsilon_{\text{critical}}$ (lattice QCD)
5 GeV/fm$^3$. Is that a lot?

In a year, the U.S. uses ~100 quadrillion BTUs of energy (1 BTU = 1 burnt match):

$$100 \times 10^{15} \text{BTU} \times \frac{1060J}{\text{BTU}} \times \frac{1\text{eV}}{1.6 \times 10^{-19}J} = 6.6 \times 10^{38} \text{eV}$$
5 GeV/fm$^3$. Is that a lot?

In a year, the U.S. uses ~100 quadrillion BTUs of energy (1 BTU = 1 burnt match):

\[
100 \times 10^{15} \text{BTU} \times \frac{1060J}{\text{BTU}} \times \frac{1eV}{1.6 \times 10^{-19}J} = 6.6 \times 10^{38} eV
\]

At 5 GeV/fm$^3$, this would fit in a volume of:

\[
6.6 \times 10^{38} eV \div \frac{5 \times 10^9 eV}{fm^3} = 1.3 \times 10^{29} fm^3
\]
5 GeV/fm$^3$. Is that a lot?

In a year, the U.S. uses ~100 quadrillion BTUs of energy (1 BTU = 1 burnt match):

$$100 \times 10^{15} \text{BTU} \times \frac{1060 \text{J}}{\text{BTU}} \times \frac{1 \text{eV}}{1.6 \times 10^{-19} \text{J}} = 6.6 \times 10^{38} \text{eV}$$

At 5 GeV/fm$^3$, this would fit in a volume of:

$$6.6 \times 10^{38} \text{eV} \div \frac{5 \times 10^9 \text{eV}}{\text{fm}^3} = 1.3 \times 10^{29} \text{fm}^3$$

Or, in other words, in a box of the following dimensions:

$$\sqrt[3]{1.3 \times 10^{29} \text{fm}^3} = 5 \times 10^9 \text{fm} = 5 \mu\text{m}$$
A human hair
What is the temperature of the medium?

- **Statistical Thermal Models:**
  - Assume a system that is thermally (constant $T_{ch}$) and chemically (constant $n_i$) equilibrated
  - System composed of non-interacting hadrons and resonances
  - Obey conservation laws: Baryon Number, Strangeness, Isospin

- **Given $T_{ch}$ and $\mu$'s (+ system size), $n_i$'s can be calculated in a grand canonical ensemble**

$$n_i = \frac{g}{2\pi^2} \int_{0}^{\infty} \frac{p^2 dp}{e^{(E_i(p) - \mu_i)/T} \pm 1}, \ E_i = \sqrt{p^2 + m_i^2}$$
Fitting the particle ratios

Number of particles of a given species related to temperature

\[ dn_i \sim e^{-\left(E-\mu_B\right)/T} d^3p \]

- Assume all particles described by same temperature \( T \) and \( \mu_B \)
- one ratio (e.g., \( \bar{p}/p \) ) determines \( \mu / T \):
  \[
  \frac{\bar{p}}{p} = \frac{e^{-\left(E+\mu_B\right)/T}}{e^{-\left(E-\mu_B\right)/T}} = e^{-2\mu_B/T}
  \]
- A second ratio (e.g., \( K/\pi \) ) provides \( T \rightarrow \mu \)
  \[
  \frac{K}{\pi} = \frac{e^{-E_K/T}}{e^{-E_{\pi}/T}} = e^{-\left(E_K-E_{\pi}\right)/T}
  \]
- Then all other hadronic ratios (and yields) defined
Assume all particles described by same temperature $T$ and $\mu_b$

One ratio (e.g., $\bar{p}/p$) determines $\mu / T$:

$$\frac{\bar{p}}{p} = \frac{e^{-(E+\mu_b)/T}}{e^{-(E-\mu_b)/T}} = e^{-2\mu_b/T}$$

A second ratio (e.g., $K/\pi$) provides $T \rightarrow \mu$

$$\frac{K}{\pi} = \frac{e^{-E_K/T}}{e^{-E_\pi/T}} = e^{-(E_K-E_\pi)/T}$$

Then all other hadronic ratios (and yields) defined
Where RHIC sits on the phase diagram

- **Quark Gluon Plasma**

- **Hadron Gas**

- **Nuclear Matter**

- **Phase Transition (Lattice)**

**Critical Point**

**RHIC**

**SPS**

**AGS**

**SIS**

**Chemical Freeze Out**

- **F. Karsch, hep-ph/0103314**
- **Fodor, Katz, JHEP 0203 (2002), 290**
- **F. Karsch, hep-lat/0401031**
- **Bielefeld-Swansea**
- **Forcrand, Philipsen, NPB 642 (2002), 290**

\[ \langle E \rangle / \langle N \rangle = 1 \text{ GeV} \]

\[ T_{ch} \text{ Data: PBM et al., nucl-th/0304013} \]
Off on a tangent

Take a second look at the anti-proton/proton ratio

\( \overline{p}/p \sim 0.8 \)

There is a net baryon number at mid-rapidity!!

Baryons number is being transported over 6 units of rapidity from the incoming beams to the collision zone!

Consider what impulse that must be

Baryon number not carried by quarks

- baryon junctions postulated
Statistics ≠ thermodynamics

Ensemble of events constitutes a statistical ensemble
T and \( \mu \) are simply Lagrange multipliers
“Phase Space Dominance”

One (1) system is already statistical!
• We can talk about pressure
• T and \( \mu \) are more than Lagrange multipliers
Evidence for thermalization

• Not all processes which lead to multi-particle production are thermal - elementary collisions

• *Any* mechanism for producing hadrons which evenly populates the free particle phase space will mimic a microcanonical ensemble.

• Relative probability to find \( n \) particles is the ratio of the phase-space volumes \( P_n/P_{n'} = \phi_n(E)/\phi_{n'}(E) \Rightarrow \) given by statistics only.

• Difference between MCE and CE vanishes as the size of the system \( N \) increases.

• Such a system is NOT in thermal equilibrium - to thermalize need interactions/re-scattering

Need to look for evidence of collective motion

Helen Caines - NNPSS-TSI - June 2010

Monday, June 28, 2010
Blackbody radiation

Planck distribution describes intensity as a function of the wavelength of the emitted radiation.

\[ S(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \]
Blackbody radiation

Planck distribution describes intensity as a function of the wavelength of the emitted radiation.

"Blackbody" radiation is the spectrum of radiation emitted by an object at temperature $T$.

As $T$ increases curve changes.
Blackbody radiation

Planck distribution describes intensity as a function of the wavelength of the emitted radiation.

“Blackbody” radiation is the spectrum of radiation emitted by an object at temperature $T$.

As $T$ increases curve changes

$\frac{1}{\text{Wavelength}} \propto \text{Frequency} \propto E \propto p$
Determining the temperature

From transverse momentum distribution of pions deduce temperature $\sim120$ MeV

\[ E = \frac{3}{2}kT \]

\[ T = \frac{2E}{3k} \]

\[ = \frac{2 \times 120 \times 10^6}{3 \times 1.4 \times 10^{-23}} \times 1.6 \times 10^{-19} \]

\[ \sim 9 \times 10^{11} K \]
Determining the temperature

From transverse momentum distribution of pions deduce temperature \( \sim 120 \) MeV

\[
E = \frac{3}{2} kT
\]

\[
T = \frac{2E}{3k}
\]

\[
= \frac{2 \times 120 \times 10^6}{3 \times 1.4 \times 10^{-23}} \times 1.6 \times 10^{-19}
\]

\[
\sim 9 \times 10^{11} K
\]

System exist for time in hadronic phase

\[ T_{ch} > T_{fo} \]

Systematic Errors not shown
Strong collective radial expansion

purely thermal source

\[ \frac{1}{m_T} \frac{dN}{dm_T} \]

\[ m_T \]

light

heavy

Central Au+Au @ 200 GeV
RHIC data preliminary

Helen Caines - NNPSS-TSI - June 2010
Strong collective radial expansion

• Different spectral shapes for particles of differing mass → strong collective radial flow

$ m_T = \sqrt{p_T^2 + m^2} $
Strong collective radial expansion

- Different spectral shapes for particles of differing mass → strong collective radial flow

\[ m_T = \left( p_T^2 + m^2 \right)^{1/2} \]

\[ \frac{1}{m_T} \frac{dN}{dm_T} \]

\[ \frac{1}{m_T} \frac{dN}{dm_T} \]

\[ \frac{1}{m_T} \frac{dN}{dm_T} \]

\[ \pi \]

\[ K/10 \]

\[ \text{PHENIX} \]

\[ \text{STAR} \]

\[ \text{PHOBOS} \]

\[ \text{BRAHMS} \]

\[ \text{Hydro} \]

\[ \text{Kolb, Rapp, PRC 67 044903 (2003)} \]

\[ T_{fo} \sim 100 \text{ MeV} \]

\[ \langle \beta_T \rangle \sim 0.55 \text{ c} \]

Good agreement with hydrodynamic prediction for soft EOS (QGP+HG)

Helen Caines - NNPSS-TSI - June 2010
Anisotropic/Elliptic flow

Almond shape overlap region in coordinate space

Interactions/Rescattering

Anisotropy in momentum space

\[ dN/d\phi \sim 1 + 2v_2(p_T)\cos(2\phi) + \ldots \]

\[ \phi = \text{atan}(p_y/p_x) \]

\[ v_2 = \langle \cos 2\phi \rangle \]

\( v_2 \): 2\(^{nd}\) harmonic Fourier coefficient in \( dN/d\phi \) with respect to the reaction plane
**Anisotropic/Elliptic flow**

Almond shape overlap region in coordinate space

Interactions/Rescattering

Anisotropy in momentum space

\[ \frac{dN}{d\phi} \sim 1 + 2v_2(p_T)\cos(2\phi) + \ldots \]

\[ \phi = \arctan(p_y/p_x) \]

\[ v_2 = \langle \cos(2\phi) \rangle \]

\( v_2 \): 2nd harmonic Fourier coefficient in \( \frac{dN}{d\phi} \) with respect to the reaction plane

---

Time

100µs 600µs 1000µs 2000µs

---


---

Helen Caines - NNPSS-TSI - June 2010
Anisotropic/Elliptic flow

Elliptic flow observable sensitive to early evolution of system
Mechanism is self-quenching
Large $v_2$ is an indication of *early* thermalization

---


Helen Caines - NNPSS-TSI - June 2010
Elliptic flow

Distribution of particles with respect to event plane, $\phi - \psi$, $p_t > 2$ GeV; STAR PRL 90 (2003) 032301

- Very strong elliptic flow → early equilibration

Factor 3:1 peak to valley
Elliptic flow

Distribution of particles with respect to event plane, $\phi - \psi$, $p_t > 2$ GeV; STAR PRL 90 (2003) 032301

- Very strong elliptic flow $\rightarrow$ early equilibration

Factor 3:1 peak to valley

- Pure hydrodynamical models including QGP phase describe elliptic and radial flow for many species

QGP $\rightarrow$ almost perfect fluid

Helen Caines - NNPSS-TSI - June 2010
Just a gas of hadrons?
Just a gas of hadrons?

Hadronic transport models (e.g. RQMD, HSD, ...) with hadron formation times ~1 fm/c, fail to describe data.
**Just a gas of hadrons?**

Hadronic transport models (e.g. RQMD, HSD, ...) with hadron formation times $\sim 1$ fm/c, fail to describe data.

Clearly the system is not a hadron gas. Not surprising.
**Just a gas of hadrons?**

Hadronic transport models (e.g. RQMD, HSD, ...) with hadron formation times ~1 fm/c, fail to describe data.

Clearly the system is not a hadron gas. Not surprising.

Hydrodynamical calculations: thermalization time \( t = 0.6 \, \text{fm/c} \)

What interactions can lead to equilibration in < 1 fm/c?
The constituents “flow”

- Elliptic flow is additive.
- If partons are flowing the *complicated* observed flow pattern in $v_2(p_T)$ for hadrons

$$m_T = \sqrt{p_T^2 + m_0^2}$$

$$\frac{d^2N}{dp_T d\phi} \propto 1 + 2v_2(p_T) \cos(2\phi)$$

should become *simple* at the quark level

$p_T \rightarrow p_T / n$

$v_2 \rightarrow v_2 / n$

$n = (2, 3)$ for (meson, baryon)
The constituents “flow”

- Elliptic flow is additive.
- If partons are flowing the *complicated* observed flow pattern in $v_2(p_T)$ for hadrons

\[
\frac{d^2 N}{dp_T d\phi} \propto 1+2 v_2(p_T) \cos(2\phi)
\]

should become *simple* at the quark level

$p_T \rightarrow p_T / n$

$v_2 \rightarrow v_2 / n$

$n = (2, 3)$ for (meson, baryon)

Works for p, π, $K^0_s$, Λ, Ξ..

$v_2^s \sim v_2^{u,d} \sim 7\%$

Constituents of QGP are partons

Helen Caines - NNPSS-TSI - June 2010
Summary of what we learned so far

- Energy density in the collision region is way above that where hadrons can exist

- The initial temperature of collision region is way above that where hadrons can exist

- The medium has quark and gluon degrees of freedom in initial stages

We have created a new state of matter at RHIC - the QGP

- The QGP is flowing like an almost “perfect” liquid