Halo Nuclei: Theory and Precision Experiments

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Joint 2010 National Nuclear Physics Summer School (NNPS) and 2010 TRIUMF Summer Institute (TSI)

June 21 - July 2, 2010 Vancouver, BC, Canada
• What are halo nuclei?

• Why are halo nuclei interesting?

• Summary on experimental advances

• Theory: Different approaches to the potentials
  Different ab-initio approaches to the many-body problem

• Towards halo nuclei from EFT: $^6$He and $^8$He

• Summary and Outlook
Halo Nuclei

- Exotic nuclei with an interesting structure

- Neutron halos: Large n/p ratio (neutron-rich)

<table>
<thead>
<tr>
<th>Halo</th>
<th>n/p</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{He}$</td>
<td>2</td>
</tr>
<tr>
<td>$^8\text{He}$</td>
<td>3</td>
</tr>
<tr>
<td>$^{11}\text{Li}$</td>
<td>2.66</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>1</td>
</tr>
</tbody>
</table>
Halo Nuclei

- Large size

Nuclear radius for stable nuclei: $R_N \sim r_0 A^{1/3}$ with $r_0 \sim 1.2$ fm

from: I. Tanihata

June 24 2010 @ NNPSS-TSI

Sonia Bacca
Halo Nuclei

- Small nucleon(s) separation energies

\[ S_{2n} = BE(Z, N) - BE(Z, N - 2) \]

Neutron-rich

Proton-rich

Drip line

Long tail in the w.f.

\[ \psi \sim e^{-\frac{\sqrt{2mS_{2n}}}{\hbar} r} \]
Problem #1

Why does the w.f. have that exponential fall off?

\[ \psi(r) \propto e^{-\rho r} \]

\[ \rho = \sqrt{\frac{2\mu S_{2n}}{\hbar^2}} \]
Why are they interesting?

- Their behavior deviates from nuclei in the stability line: we want to understand why?
- Enormous progress from the experimental point of view: new precision era!
- Test our understanding of their exotic structure by comparing theory-experiment
- For the very light halo we can challenge ab-initio methods: test our knowledge on nuclear forces
The helium isotope chain

Even if they are exotic short lived nuclei, they can be investigated experimentally. From a comparison of theoretical predictions with experiment we can test our knowledge on nuclear forces in the neutron rich region.
Halo Nuclei - Experiment
New Era of Precision Measurements for masses and radii

- Masses (and thus binding energies) are measured with Penning traps
  - TITAN
  - TRIUMF
  - Can reach a relative precision of $10^{-8}$

- Charge radii are measured with Laser Spectroscopy
  - ARGONNE
  - GANIL
  - ISOLDE
Production of rare isotopes at TRIUMF

Halo Nuclei - Experiment
New Era of Precision Measurements for masses and radii
Penning Trap  
Superposition of strong homogeneous magnetic field and a weak electrostatic quadrupole electric field

ion eigenmotions are known

\[ \nu_c = \frac{1}{2\pi} \frac{qB}{m} \]

\[ \frac{\delta m}{m} = \frac{\delta \nu_c}{\nu_c} = \frac{\delta \nu_c 2\pi m}{qB} \]

\[ \delta \nu_c \sim \frac{1}{T} \frac{1}{\sqrt{N}} \]

\[ \frac{\delta m}{m} = \frac{m}{TB\sqrt{N}} \]

known from the beam
Halo Nuclei - Experiment
New Era of Precision Measurements for masses and radii

Laser Spectroscopy for radii

\[ \delta \nu_{A,A'} = \nu_{A'} - \nu_{A} \]

Theory: from precise atomic structure calculations

\[ = \delta \nu_{A,A'}^{mass} + K \delta \langle r^2 \rangle_{AA'}^{ch} \]

- Mass shift dominates for light nuclei
- Nuclear masses are input for calculations of \( K \) can be the largest source of systematic errors if not known precisely
- Precise mass measurements are key for a better determination of radii
Masses and radii of helium isotopes are important challenges for theory!

\[ \langle r_p^2 \rangle = \langle r_{ch}^2 \rangle - \langle R_p^2 \rangle - \frac{3}{4M_p^2} - \frac{N}{Z} \langle R_n^2 \rangle \]
Halo Nuclei - Theory

Why are halo nuclei a challenge to theory?

- It is difficult to describe the long extended wave function
- They test nuclear forces at the extremes, where less is known

Ab-initio calculations: treat the nucleus as an A-body problem

Full antisymmetrization of the w.f.

Use modern Hamiltonians to predict halo properties

\[ H = T + V_{NN} + V_{3N} + \ldots \]

Methods: GFMC, NCSM, CC, HH, ...
1935 Yukawa: one pion exchange (OPE) model

\[ \begin{align*}
\text{electromagnetic force:} & \quad \text{infinite range} \\
& \quad \text{exchange of massless particle}
\end{align*} \]

\[ \text{NN force:} \quad \text{finite range} \]
\[ \text{exchange of massive particle} \]

\[ r \sim \frac{1}{m} \]

**REALISTIC NN POTENTIALS:** fit to NN scattering data

- 1960s one boson exchange model

- 1970s: two-pion exchange added

- 1980s: OPE + phenomenology
  40 parameters to fit phase shifts \text{AV18}

- 1990-today: *new vision of effective field theory*

\[ \omega, \sigma, \rho \]

\[ r \sim \frac{1}{m} \]

\[ ? \]

\[ \text{long range} \]

\[ \text{short range phenomenology} \]
Effective Field Theory: Bridges the non-perturbative low-energy regime of QCD with forces among nucleons

\[
\frac{1}{\lambda} = Q \ll \Lambda_b = \frac{1}{R}
\]

Use effective degrees of freedom: p, n, pions

Construct the most general Hamiltonian which is consistent with the chiral symmetry of QCD

Have a systematic expansion of the Hamiltonian in terms of diagrams

\[
\mathcal{L} = \sum_k c_k \left( \frac{Q}{\Lambda_b} \right)^k
\]

Power counting

\[
k = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2)
\]

Fix the short range couplings on experiment
Problem #2

Given the power counting:

\[ k = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2) \]

- # of nucleons
- # of loops
- # of derivatives, momenta or pion mass
- # of nucleon lines
- # of nucleon vertices

• Calculate the order \( k \) of this three-body diagram

• What is the difference between these two diagrams?

\[
\begin{align*}
\text{Diagram 1: } & \quad k = 0 \\
\text{Diagram 2: } & \quad k = 2
\end{align*}
\]
Effective field theory potentials and low-momentum evolution

Evolution of 2N forces: phase-shift equivalent

Low-momentum interactions: Bogner, Kuo, Schwenk (2003) need smaller basis

Like acting with a unitary transformation $U^{-1}VU$ still preserve phase-shifts and properties of 2N systems

$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$

Variation of the cutoff provides a tool to estimate the effect of 3N forces

Can evolve consistently

3N forces: Jurgenson, Navratil, Furnstahl, (2009)
Ab-initio Calculations for Halo Nuclei

GFMC - Quantum Monte Carlo Method, Uses local two- and three-nucleon forces

Argonne v_{18}
With Illinois-2
GFMC Calculations
6 November 2002

Pieper et al. (2002)

N.B.: parameters of the IL2 force are obtained from a fit of 17 states of A<9 including the binding energy of $^6\text{He}$ and $^8\text{He}$
Ab-initio Calculations for Halo Nuclei

- GFMC estimation of the proton radius -

Ab-initio Calculations for Halo Nuclei

NCSM

Diagonalization Method using Harmonic Oscillator Basis
Can use non-local two- and three-nucleon forces

so far not for halo nuclei in large spaces

Helium Isotopes

Caurier and Navratil, PRC 73, 021302(R) (2006)

Slow convergence and HO parameter dependence in radius

CD-Bonn $\rightarrow$ meson exchange theory
short range phenomenology

\[ \psi_{nl}(r) \sim e^{-\nu r^2} L_{n+1/2}^{1+2}(2\nu r^2) \quad \nu = \frac{m \omega/2}{\hbar} \]

\[ E_B [\text{MeV}] \quad \text{Expt.} \quad \text{CD-Bonn 2000} \]

\begin{array}{ccc}
4^\text{He} & 28.296 & 26.16 (6) \\
6^\text{He} & 29.269 & 26.9 (3) \\
8^\text{He} & 31.408 (7) & 26.0 (4) \\
\end{array}

OPE $\rightarrow$ long range

short range phenomenology
Towards Halo Nuclei from EFT

Ideally we want:

1. To use methods that enable to incorporate the correct asymptotic of the w.f. for loosely bound systems

2. To obtain convergent calculations, with no dependence on the model space parameters

3. To systematically study the cutoff (in)dependence of predicted observables with two- and three-body forces

Two methods that enable us to achieve point 1. and 2.:

- Hyper-spherical Harmonics Expansion for $^6$He

- Cluster Cluster Theory for $^8$He
Hyper-spherical Harmonics

- Few-body method - uses relative coordinates

\[ |\psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})\rangle \]

Recursive definition of hyper-spherical coordinates

\[ \rho, \Omega \]

\[ \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2 \]

\[ \eta_0 = \sqrt{A \vec{R}_{CM}} \quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1} \]

A=3

\[ \left\{ \begin{array}{l}
\vec{\eta}_1 = \{ \eta_1, \theta_1, \phi_1 \} \\
\vec{\eta}_2 = \{ \eta_2, \theta_2, \phi_2 \}
\end{array} \right. \]

\[ \rho = \sqrt{\eta_1^2 + \eta_2^2} \]

\[ \sin \alpha_2 = \frac{\eta_2}{\rho} \]

A=4

\[ \left\{ \begin{array}{l}
\vec{\eta}_1 = \{ \eta_1, \theta_1, \phi_1 \} \\
\vec{\eta}_2 = \{ \eta_2, \theta_2, \phi_2 \} \\
\vec{\eta}_3 = \{ \eta_3, \theta_3, \phi_3 \}
\end{array} \right. \]

\[ \rho = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2} \]

\[ \sin \alpha_2 = \frac{\eta_2}{\rho} \]

\[ \sin \alpha_3 = \frac{\eta_3}{\rho} \]
Hyper-spherical Harmonics

- Few-body method - uses relative coordinates
  $|\psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})\rangle$

Recursive definition of hyper-spherical coordinates

$\rho, \Omega$

$\rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$

$H(\rho, \Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$

$\Psi_{K_{max}, \nu_{max}} = \sum_{[K, \nu]} c_{\nu} e^{-\rho/2b} \rho^{n/2} L_n^\nu(\frac{\rho}{b}) [Y_{[K]}^\nu(\Omega)] \chi_{ST}^\mu jT$

Asymptotic

$e^{-a\rho}$

$\rho \to \infty$

Model space truncation $K \leq K_{max}$, Matrix Diagonalization

$\langle \psi | H_{(2)} | \psi \rangle = \frac{A(A-1)}{2} \langle \psi | H_{(A,A-1)} | \psi \rangle$

Can use non-local interactions

Most applications in few-body; challenge in $A>4$  

Coupled Cluster Theory

\[ |\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle \]

\[ T = \sum T_{(A)} \]

CCSD

CCSDT

Asymptotic
\[ \phi_i \sim e^{-k_i r_i} \quad r \to \infty \]

Model space truncation
\[ N \leq N_{max} \]

Can use non-local interactions

Applicable to medium-mass nuclei

Use it for \(^8\text{He},\) closed shell nucleus
Benchmark on $^4$He

HH-CC-FY

$V_{\text{low } k} \text{ NN from } N^3\text{LO (500 MeV)}$

$E_{0} \text{ [MeV]}$

<table>
<thead>
<tr>
<th>Method</th>
<th>$\Lambda = 2.0 \text{ fm}^{-1}$</th>
<th>$E_{0}(^4\text{He}) \text{ [MeV]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faddeev-Yakubovsky (FY)</td>
<td></td>
<td>-28.65(5)</td>
</tr>
<tr>
<td>Hyperspherical harmonics (HH)</td>
<td></td>
<td>-28.65(2)</td>
</tr>
<tr>
<td>CCSD level coupled-cluster theory (CC)</td>
<td></td>
<td>-28.44</td>
</tr>
<tr>
<td>Lambda-CCSD(T) (CC with triples corrections)</td>
<td></td>
<td>-28.63</td>
</tr>
</tbody>
</table>

$E_{\text{exp}} = -28.296 \text{ MeV}$
Helium Halo Nuclei


Virtually no model space dependence: can improve $K_{\text{max}}$ convergence by exponentially extrapolate

$$E(K_{\text{max}}) = E_{\infty} + Ae^{-BK_{\text{max}}}$$

Virtually no model space dependence: can improve by adding more correlations

$\Lambda = 1.8 \text{ fm}^{-1}$
$\Lambda = 2.0 \text{ fm}^{-1}$
$\Lambda = 2.4 \text{ fm}^{-1}$
Hilbert space: 15 major shell

Values in MeV

<table>
<thead>
<tr>
<th>Λ</th>
<th>E[CCSD]</th>
<th>E[Lambda-CCSD(T)]</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>-30.33</td>
<td>-31.21</td>
<td>0.88</td>
</tr>
<tr>
<td>2.0</td>
<td>-28.72</td>
<td>-29.84</td>
<td>1.12</td>
</tr>
<tr>
<td>2.4</td>
<td>-25.88</td>
<td>-27.54</td>
<td>1.66</td>
</tr>
</tbody>
</table>

- Triangles corrections are larger for larger cutoff
- Their relative effect goes from 3 to 6%

Q: Why do we gain more energy for larger cutoffs?
Binding Energy Summary

$E_0 [\text{MeV}]$

$^6\text{He}$

$^8\text{He}$

NCSM

GFMC

HH

previous

Lambda-CCSD(T)

previous

Our estimated error in neglected short range 3NFs

Experimental data
Radii of Halo Nuclei

**rms matter radius**

\[ \langle r^2 \rangle = \frac{1}{A} \sum_i r_i^2 \]

knows about where all nucleons are

**one-body operator**

\[ \hat{r}^2 = \frac{1}{A} \sum_i \hat{r}_i^2 \]

In HH for \(^6\)He

\[ \hat{r}^2 = \frac{1}{A} \hat{\rho}^2 \]

In CC for \(^8\)He: work with lab coordinates

\[ \hat{r}^2 = \frac{1}{A} \sum_i (\hat{r}_i - \hat{R}_{CM})^2 \]

not translationally invariant

**rms point proton radius**

\[ \langle \hat{r}_p^2 \rangle = \frac{1}{A} \sum_i \hat{r}_i^2 \left( 1 + \frac{\tau_i^z}{2} \right) \]

knows about where protons are
Problem #3

- Derive the expression for the translational invariant matter radius as a two-body operator

\[ \hat{r}^2 = \frac{1}{A} \sum_i (\hat{r}_i - \hat{R}_{CM})^2 \quad \hat{R}_{CM} = \frac{1}{A} \sum_j \hat{r}_j \]

- Try to do the same with the point proton radius
Radii of Halo Nuclei

\[ V_{\text{lowk NN from } N^3LO} \quad \Lambda=2.0 \text{ fm}^{-1} \]

- **matter radius**
- **point-proton radius**

- Point proton radius converges better and are smaller than matter radii

Q: Why is that? Can we understand it?
Matter radii Summary

- Benchmark on $^4$He, CC using translational invariant operators
  \[ \Lambda = 2.0 \text{ fm}^{-1} \]
  \[ \text{HH} \ 1.434 \text{ fm} \]
  \[ \Lambda - \text{CCSD(T)} \ 1.429 \text{ fm} \]

\[
\begin{array}{cccc}
\Lambda = 1.8 \text{ fm}^{-1} & \Lambda = 2.0 \text{ fm}^{-1} & \Lambda = 2.4 \text{ fm}^{-1} \\
4^\text{He} & 6^\text{He} & 8^\text{He} \\
\text{EIHH} & \text{EIHH} & \text{EIHH} \\
\text{NCSM} & \text{GFMC} & \text{NCSM} \\
\text{GFMC} & \text{GFMC} & \text{GFMC} \\
\text{EXP BAND} & \text{EXP BAND} & \text{EXP BAND} \\
\end{array}
\]
Proton radii Summary

- From laser spectroscopy using binding energy as input

- The fact that for some “choice” of the NN force one gets correct radii and wrong energies (or vice-versa) shows that halo nuclei provide important tests of the different aspects of nuclear forces, which includes 3NF
• We provide a description of helium halo nuclei from evolved EFT interactions with the correct asymptotic in the wave function

• We estimate the effect of short range three-nucleon forces on binding energies and radii by varying the cutoff of the evolved interaction

• Our matter radii agree with experiment whereas our point-proton radii under-predict experiment

Future:

• Include three-nucleon forces

• Extend coupled cluster theory with 3NF calculations to heavier neutron rich nuclei, e.g. lithium or oxygen isotope chain
Effect of Three Nucleon Forces

Nuclear low energy spectra

Navratil et al. (2008)

From EFT

Hadronic Reactions

From semi-phenomenological potentials

Nollett et al. (2007)
Effect of Three Nucleon Forces


First results with 3NF
(effective 2NF)

3NF fits to $E(^3\text{H})$ and $^4\text{He}$ rms

Future:
Neutron-rich isotopes with Coupled Cluster Theory (beyond core-approximation) and 3NF

any SM calculation with realistic 2NF predicts bound $^{25-28}\text{O}$ in contrast with experimental observation