Sunday

Preliminaries
Nuclear binding and masses
→ Indicator of shell structure
How to measure a mass

Monday

Thursday

Friday
Goal: Establish physical properties of rare isotopes and their interactions to gain predictive power

Experiments: Measure observables

Observables: May or may not need interpretation to relate to physical properties

- e.g., half-life and mass connect directly to physical properties
- e.g., cross sections for reaction processes usually need interpretation to connect to physical properties (model dependencies are introduced)
Theories and models can relate observables to physical properties – often, experiments are motivated by theoretical predictions that need validation.

But: Theories and models have their own realm of applicability that everybody involved in the experiment/data analysis/interpretation should be aware of!

Predictions or systematics come with a warning: Might lead to expectations that can influence the implementation of an experiment and ultimately limit the scope of discovery.
Implementation of experiments can limit the scope of discovery.

Examples:
- Lifetime of a nucleus
- Excitation energy

Analysis and “selection” of data (cuts, gates, using subsets, …) can influence the result.

Example: $e^+e^-$ resonances in heavy-ion collisions at GSI.

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A. FRANKLIN

“Selectivity and the Production of Experimental Results”


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First half of the data analyzed

Identical analysis applied to other half of data set
Excitation energies in molecules and nuclei

- Molecular excitations:
  \[ E_{\text{rot}} \ll E_{\text{vib}} \ll E_{\text{el}} \]  
  \( \mu\text{eV} \ll \text{meV} \ll \text{eV} \)

As a consequence, these different motions can be treated separately and the wavefunction ends up as a product of terms.

- In nuclei, the energy scales are much closer:
  \[ E_{\text{rot}} \sim E_{\text{vib}} \sim E_{\text{sp}} \]  
  \( \text{MeV} \)

Collective and single-particle excitation can be separated but interact strongly.
The nucleus is a bound collection of N neutrons and Z protons mass number A=Z+N

\[ ^A_Z \text{N} \]

- **Isotopes**: Nuclei with the same Z but different N – e.g. \( ^9\text{C} \), \( ^{10}\text{C} \), \( ^{11}\text{C} \), \( ^{12}\text{C} \)

- **Isotones**: Nuclei with the same N but different Z – e.g. \( ^9\text{C} \), \( ^8\text{B} \), \( ^7\text{Be} \), \( ^6\text{Li} \)

- **Isobars**: Nuclei with the same mass number – e.g. \( ^9\text{C} \), \( ^9\text{B} \), \( ^9\text{Be} \), \( ^9\text{Li} \)
About 3000 isotopes have been made in laboratories.
6000-8000 isotopes might be out there
Exotic nuclei

Normal Nucleus:

- 6 neutrons
- 6 protons (carbon)
- $^{12}\text{C}$
- Stable, found in nature

Exotic Nucleus:

- 16 neutrons
- 6 protons (carbon)
- $^{22}\text{C}$
- Radioactive, at the limit of nuclear binding

Characteristics of exotic nuclei: Excess of neutrons or protons, short half-life, neutron or proton dominated surface, low binding
What binds nucleons into nuclei?

The neutrons and protons are bound together by the **strong or color force**

The strong force between the quarks in one proton and the quarks in another proton is strong enough to overcome the electromagnetic repulsion

Two ways of thinking about the strong force:
As a residual color interaction or as the exchange of mesons
Binding energy, mass and mass excess

Mass $M(N, Z)$ of the neutral atom

Mass excess: $\Delta(N, Z) \equiv M(N, Z) - uA$

Atomic mass unit $u$:

$$u = \frac{M(^{12}\text{C})}{12} = 931.49386 \text{ MeV}/c^2$$

Equivalent to $\Delta(^{12}\text{C})=0$

Binding energy:

$$B(N, Z) = Z \Delta_H c^2 + N \Delta_n c^2 - M(N, Z) c^2$$

$$\Delta_H c^2 = 7.2890 \text{ MeV} \quad \Delta_n c^2 = 8.0713 \text{ MeV}$$

$$B(N, Z) = Z \Delta_H c^2 + N \Delta_n c^2 - \Delta(N, Z) c^2$$
Average binding energy of nuclei

Binding energy per nucleon / MeV

Number of nucleons  $A$

Number of nucleons

Fusion

Fission
A semi-empirical description of nuclear binding

- $B(Z,A) = a_V A + a_S A^{2/3} + a_C Z^2/A^{1/3} + a_A (N-Z)^2/A - a_P/A^{3/4}$

  - Volume term
  - Surface energy term
  - Coulomb term
  - Asymmetry term
  - Pairing term

\[ R \sim A^{1/3} \]

- $a_V = -15.68 \text{ MeV}$
- $a_S = 18.56 \text{ MeV}$
- $a_C = 0.717 \text{ MeV}$
- $a_A = 28.1 \text{ MeV}$
- $a_P = 34.0 \text{ MeV}$ for even-even, $-34.0 \text{ MeV}$ for odd-odd, 0 for even-odd
The different contributions to nuclear binding

![Graph showing the different contributions to nuclear binding.](image)
Semi-empirical mass formula versus experimental reality

\[ B(N, Z) = Z M_H c^2 + N M_n c^2 - M(N, Z) c^2 \]
Q values and nucleon separation energies

Q value of a process \((^A Z_i \rightarrow ^A Z_f)\):

\[
Q = \sum_i M(N_i, Z_i)c^2 - \sum_f M(N_f, Z_f)c^2 = \sum_f B(N_f, Z_f) - \sum_i B(N_i, Z_i)
\]

Nucleon separation energies:

\[
S_n = B(N, Z) - B(N - 1, Z),
\]

\[
S_p = B(N, Z) - B(N, Z - 1),
\]

\[
S_{2n} = B(N, Z) - B(N - 2, Z),
\]

\[
S_{2p} = B(N, Z) - B(N, Z - 2).
\]
Nucleon separation energies

\[ S_n = B(N, Z) - B(N - 1, Z) \]

Huge changes in the separation energy at neutron numbers 8, 20, 28, 50, 82, 126 \implies \text{Those nuclei are particularly stable}
Seen that before ... compare to atomic shell structure!

Bohr, Mottelson
Shell structure – magic numbers

Maria Goeppert-Mayer 1963
Hans D. Jensen 1963

Proton number

Neutron number

0 20 40 60 80 100 120 140 160 180
0 20 40 60 80 100 120 140 160 180

Single-particle levels in nuclei
The single-particle levels of this fermionic system are grouped. Large, stabilizing gaps between groups of single-particle states occur at certain occupation numbers of the orbits with a “magic number” of protons and neutrons.

Magic numbers
Numbers of neutrons and protons in nuclei which correspond to particularly stable structures (2, 8, 20, 28, 50, 82, 126)

\[ \ell = 0, 1, 2, 3, \ldots \quad j = \ell \pm 1/2 \]

Max. occupancy: \(2j+1\)

Experimental signatures of nuclear shells
- low capture cross sections
- little collectivity
- more tightly bound than neighboring nuclei

\[ H = H_0 + H_{\text{res}} = \sum_{i=1}^{A} \left[ \frac{p_i^2}{2m_i} + U_i(r) \right] + H_{\text{res}} \]
Shell structure – magic numbers

Nuclear Shell Structure

Near stability

- $p_{1/2}$
- $f_{5/2}$
- $i_{13/2}$
- $p_{3/2}$
- $h_{9/2}$
- $f_{7/2}$
- $d_{3/2}$
- $h_{11/2}$
- $s_{1/2}$
- $g_{7/2}$
- $d_{5/2}$
- $g_{9/2}$

For $N >> Z$

- $h_{9/2}$
- $f_{5/2}$
- $p_{1/2}$
- $p_{3/2}$
- $f_{7/2}$
- $h_{11/2}$
- $g_{7/2}$
- $d_{3/2}$
- $s_{1/2}$
- $d_{5/2}$
- $g_{9/2}$

- Mean field near stability
- Strong spin-orbit term

- Mean field for $N >> Z$?
- Reduced spin-orbit
- Diffuse density
- Tensor force

Towards neutron-rich nuclei

Shell gap larger than expected
Shell gap less than expected
An indicator for changes in nuclear structure

- Neutron number
- Magic number
- Deformation

Diagram showing the relationship between $S_{2n}$ (MeV) and Neutron number, with a focus on the magic number and deformation regions.
Masses – what are they good for?

Nuclear structure

![Graph showing nuclear structure with axes labeled $S_{2n}$ (MeV) and $N$.]
Observables

\[ S_{2n} = B(N,Z) - B(N-2,Z) \]
\[ = M(N-2,Z) + 2M_N - M(N,Z) \]

Measure nuclear masses of exotic nuclei!

… but first we have to produce them!
Production of exotic nuclei

- Transfer reactions
- Fusion-evaporation
- Fission
- Fragmentation
  - Target fragmentation (HRIBF, TRIUMF, SPIRAL, ISOLDE)
  - Projectile fragmentation (NSCL, GSI, RIKEN, GANIL)
Masses

Indirect

- Decay measurements and kinematics in two-body reactions

Direct

- Conventional mass spectrometry
  - Cern PS, Chalk River
- Time-of-flight
  - spectrometer (SPEG, TOFI, S800)
  - Multi-turn (cyclotrons, storage rings)
- Frequency measurements
  - Penning traps
  - Storage rings

\[ A(a,b)B \]
\[ Q = M_A + M_a - M_b - M_B \]
\[ Q_\alpha = M_B - M_A \]
TOF mass measurement
– Cyclotrons at GANIL

A~100 nuclei: $^{50}\text{Cr}^{58}\text{Ni}$ at 250 MeV

- Heavy-ion primary beam delivered by CSS1 with a few MeV/nucleon
- The fusion-evaporation products formed in the reaction with the production target are injected into the CSS2 and accelerated
- Detected in a silicon-detector telescope

$\delta m/m = \delta t/t$

Mass Measurement of $^{100}\text{Sn}$

M.E.($^{100}\text{Cd}$) = $-74.180 \pm 0.200$ (syst) MeV,
M.E.($^{100}\text{In}$) = $-64.650 \pm 0.300$ (syst)
$\pm 0.100$ (stat) MeV,
M.E.($^{100}\text{Sn}$) = $-57.770 \pm 0.300$ (syst)
$\pm 0.900$ (stat) MeV.

$\frac{B}{\omega/h} = \gamma \frac{m}{q} = \frac{B\rho}{v}$, \quad $\omega_c = \omega / h$

$\frac{\delta T_{\text{turn}}}{T_{\text{turn}}} = \frac{\delta m/q}{m/q}$ \quad $h = \# \text{rf periods/turn}$
TOF mass measurements on neutron-rich isotopes

goal: $\delta m = 0.2$ MeV for $A \sim 70$  
$\Rightarrow \delta m/m = 2 \times 10^{-6}$

A. Estrade, in preparation (NSCL)

$B_\rho = \gamma m/q \ (dx/dt)$

Measure $B_\rho$ and TOF

- Measure many masses simultaneously
- Mass accuracy: $\Delta m/m \sim 10^{-6}$
- Beam rate: particles/min (e.g., 10000 particles total for $\delta m \sim 200$ keV for $A \sim 100$)
Mass measurements in the ESR storage ring at GSI

\[ \Delta f \frac{v}{f} = -\frac{1}{\gamma_t^2} \frac{\Delta (m/q)}{m/q} + \frac{\Delta v}{v} \left(1 - \frac{\gamma_t^2}{\gamma_t^2}\right) \]

\( \gamma_t \): relative change in path length by turn relative to change in Bρ

\[ T_{1/2} > 1 \text{ s} \]

\[ T_{1/2} > 10 \mu\text{s} \]
Mass measurements in the storage ring at GSI

I. Schottky mass spectrometry

- Schottky spectrometry in storage ring (GSI), e.g. $^{184}$Pt

Mass excess for $^{184}$Pt as determined in several runs using different reference isotopes and in different ionic charge states $q$. $(dm/m=5 \times 10^{-7})$

T. Radon et al., PRL 78, 4701 (1997)
Mass measurements in the storage ring at GSI
II. Isochronous mass spectrometry

- Mass measurement of short-lived $^{44}$V, $^{48}$Mn, $^{41}$Ti and $^{45}$Cr (X-ray burst models)

Accuracy of $\delta m = 100$-500 keV was achieved (lifetimes $\sim 100$ ms)
Mass measurements with Penning traps

Mass measurement via determination of cyclotron frequency

\[ f_c = \frac{1}{2\pi} \cdot \frac{q}{m} \cdot B \]

from characteristic motion of stored ions

**PENNING trap**

- Strong homogeneous magnetic field of known strength \( B \) provides radial confinement
- Weak electric 3D quadrupole field provides axial confinement
Mass measurements with Penning traps

Motion of an ion is the superposition of three characteristic harmonic motions:
- axial motion (frequency $f_z$)
- magnetron motion (frequency $f_-$)
- modified cyclotron motion (frequency $f_+$)

The frequencies of the radial motions obey the relation:
$$f_+ + f_- = f_c = \frac{1}{2\pi} \cdot \frac{q}{m} \cdot B$$

Typical frequencies:
- $q = e$, $m = 100\ u$, $B = 6\ T$
- $f_- \approx 1\ kHz$
- $f_+ \approx 1\ MHz$

Excite the cyclotron motion with multipolar RF (Goal: excite the cyclotron motion to resonance)

Transform radial to axial energy (gradient $\frac{dB}{dz}$) and eject ions

Measure time of flight (TOF) - the shorter TOF, the closer is the excitation frequency to the resonance

Adapted from K. Blaum
Mass measurements with Penning traps

Eject thermalized ions from buncher
Capture ions in Penning trap
Perform RF excitation
Eject ions and measure TOF

\[ \text{38Ca}^{2+} \]

\[ v_{RF} = 7595522 \text{ (Hz)} \]

\[ \text{ME} = -22058.53(28) \text{ keV} \]

\[ \delta m = 280 \text{ eV} \]

G. Bollen et al., PRL 96, 152501 (2006)
ISOLTRAP at CERN
GSI/Munich/NSCL/Mainz/Greifswald/CSNSM/CERN/McGill/Jyvaskyla

The Triple Trap
Mass Spectrometer

ISOLTRAP

• e.g. $^{123,124,126}$Ba

F. Ames et al., NPA 651, 3 (1999)

isoltrap.web.cern.ch/isoltrap/

Many other traps around the world, see e.g. EMIS 14
www.triumf.ca/emis14abs/abstracts.html#Topic06
Trap measurements – Overview

- **SHIPTRAP @ GSI:**
  - masses of rp nuclei
  - Proton drip-line nuclei

- **LEBIT @ MSU:**
  - $^{38}\text{Ca}$, $^{70m}\text{Br}$, $^{68}\text{Se}$
  - $^{44}\text{S}$, n-rich $^{65}\text{Fe}$ and $^{66}\text{Co}$

- **CPT @ Argonne:**
  - $^{46}\text{V}$, $^{64}\text{Ge}$
  - heavy fission products

- **ISOLTRAP @ CERN:**
  - ~300 isotopes measured
  - $^{22}\text{Mg}$, $^{32}\text{Ar}$, $^{72}\text{Kr}$, $^{74}\text{Rb}$, $^{81}\text{Zn}$, $^{133}\text{Sn}$

- **JYFLTRAP @ Jyväskylä:**
  - ~200 isotopes measured
  - $^{23}\text{Al}$, $^{62}\text{Ga}$, $^{92}\text{Rh}$, $^{108}\text{Te}$
  - fission products; $^{83}\text{Ga}$, $^{110}\text{Mo}$

- **TITAN @ TRIUMF**
  - $^{9,11}\text{Li}$, $^{8}\text{He}$

J. Aysto (Trento, spring 2008)
Masses – what are they good for?

- **Structure information**
  - Shell closures and deformation from separation energies ($\delta m/m < 10^{-5}$)
- **Astrophysics (Nucleosynthesis)**
  - r process ($\delta m/m < 10^{-5}, \delta m < 10$ keV)
  - rp process ($\delta m/m \sim 10^{-7}$)
- **Fundamental interactions and symmetries ($\delta m/m < 10^{-8}$)**
  - CVC
  - CKM
**Theoretical description of masses**

**Algebraic:** Garvey Kelson (**GK**), sum and difference relations between masses:

\[
M(N+2,Z-2) - M(N,Z) + M(N,Z-1) - M(N+1,Z-2) + M(N+1,Z) - M(N+2,Z-1) = 0
\]

**Microscopic-macroscopic:** For example the finite-range droplet model **FRDM** (31 parameters, largely fit to known masses), bulk part from liquid drop model (macroscopic) + shell and pairing corrections (microscopic) (**extrapolation is dangerous**)

**Microscopic:** Relativistic mean-field (**RMF**) and Hartree-Fock Bogoliubov (**HFB**), RMF is based on meson/photon exchange Lagrangian, HFB uses Skyrme or Gogny effective nucleon-nucleon interactions (**computationally demanding**)

http://www.nuclearmasses.org/resources.html
Masses – what are they good for?

Constrain theory

Model difference (MeV/c²)

N (Z = 37)

Measured masses

Groote et al., 1976
Janecke & Masson, 1988
Tachibana et al., 1988
Comay et al., 1988
Moeller et al., 1995
Duflo & Zuker, 1996
Masson & Janecke, 1988
Aboussir et al., 1992

Needed for r-process
Masses – what are they good for?

Nuclear astrophysics

Difference due to shell quenching for neutron-rich nuclei, or a problem with astrophysical model?
Masses – what are they good for?

Fundamental interactions/symmetries

Physics beyond the Standard Model
(required precision: as good as possible, at least: $\delta m/m < 10^{-8}$)

- Conserved vector current (CVC) hypothesis
- Unitarity of the Cabbibo-Kobayashi-Maskawa (CKM) matrix

Details: lecture by Tim Chupp
Takeaway

• Nuclear masses and resulting nucleon separation energies are an indicator of nuclear structure and indicate changes in the shell structure in the exotic regime

• Exotic nuclei can be produced with different methods and reactions

• Masses of short-lived nuclei can be measured in different ways
  • Time of flight mass measurements
  • Storage rings
  • Penning traps

• Masses are important input for nuclear astrophysics and the study of fundamental symmetries
Related review articles

**Masses**

- Mass measurements of short-lived nuclides with ion traps, G. Bollen, NPA 693, 3 (2001)