Superallowed Fermi $\beta$ Decay and CKM Unitarity

NNPSS Student Talk
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CKM Matrix

For quarks,

mass eigenstates ≠ weak eigenstates

That means there's a mixing matrix that connects the two bases. This is called the CKM (Cabibbo-Kobayashi-Maskawa) matrix

\[
V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

N. Cabibbo (1963), M. Kobayashi and T. Maskawa (1973)
What effect does the CKM matrix have?

Weak charged current is

\[ J_{cc} = \sum_{m=1}^{3} \left[ \bar{\nu}_m \gamma^\mu (1 - \gamma_5) l_m + \sum_{n=1}^{3} V_{mn} \bar{u}_m \gamma^\mu (1 - \gamma_5) d_n \right] \]

where m and n are generation indices
What effect does the CKM matrix have?

Compared to muon decays, nuclear/neutron β decay amplitudes are suppressed by $V_{ud}$. 

\[ v_e \xrightarrow{\text{w}} e \propto e_W \]

\[ u \xrightarrow{\text{w}} d \propto V_{ud} e_W \]
Why should we care about CKM unitarity?

• Self-consistency check for Standard Model
  If there are just 3 generations for quarks, 3x3 CKM matrix should be unitary. Each row and column should satisfy relations like this
  \[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \]

• Constraints on physics beyond SM
  4th generation of quarks
  Exotic muon decay

  I'll focus on extracting $V_{ud}$ from nuclear $\beta$ decay
2 Types of $\beta$ Decay

In $q\to0$ (allowed) limit

- **Fermi**
  
  Vector current, $g_V$

  \[ M_F = \sum_k \tau_+(k) \]

  \( \Delta J = 0 \)

  \( \Delta T = 0 \)

  \( \Delta T_3 = 1 \)

  same parity

- **Gamow-Teller**

  Axial current, $g_A$

  \[ \vec{M}_{G-T} = \sum_k \tau_+(k) \vec{\sigma}(k) \]

  \( \Delta J = \pm 1, 0 \) (except $0\to0$)

  \( \Delta T = \pm 1, 0 \) (except $0\to0$)

  \( \Delta T_3 = 1 \)

  same parity
Can we get rid of one of them?

Yes!

For $J_i = J_f = 0$, we'll have no Gamow-Teller (Sorry GWU)

And if the initial and final nuclei are in the same isospin multiplet, then it's called a “superallowed” Fermi $\beta$ decay

There are many isotopes you (not I) can measure:

$^{14}$O, $^{26}$Al$^m$, $^{34}$Cl, $^{38}$K$^m$, $^{42}$Sc, $^{46}$V, $^{50}$Mn, $^{54}$Co...
Now everything's simple, right?

- From vector current conservation (recall the nucleon structure lectures), $g_V = 1$

- So for all $J=0, T=1$ decay nuclei, the leading-order decay amplitudes are identical

- Just factor out the phase space dependence, and the normalized lifetimes ($ft$ value) will be identical as well

$$ft \equiv \left[ m_e^{-5} \int_{m_e}^{Q} d\epsilon \sqrt{\epsilon^2 - m_e^2} \epsilon (Q - \epsilon)^2 \right] \frac{\ln 2}{\Gamma}$$

$$= \frac{2\pi^3 \ln 2}{m_e^5 G_F^2 |V_{ud}|^2 |M_F|^2}$$
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Uh...not quite
Those $f_t$ values don't look so identical
There's more to $\beta$ decay than just the leading order
Theoretical Corrections

We need to calculate how higher-order processes affect the amplitude, e.g.

Some diagrams are nucleus-independent (left), but others depend on the nucleus (right)
Almost all the important terms have been calculated, and...

Corrections appear to be working...
Nucleus Dependence

But because of the origin of this problem in particle physics, nuclear structure often has been taken lightly.

This 2-nucleon diagram has been considered, but the nucleons are treated as being almost free.
Nucleus Dependence

• Simple dependence on Z
  Coulomb correction, QED-type corrections, etc.
  Well-established

• Isospin symmetry breaking
  Isospin mixing and radial overlap
    (I.S. Towner and J.C. Hardy arXiv:0710.3181,
    G.A. Miller and A. Schwenk arXiv:0805.0603)
What I'm trying to do

Calculate the contribution to the amplitude from excited intermediate states

There is some overlap with earlier calculations, but this has never been done
What I'm trying to do

From non-relativistic P.T.

\[ T_{fi} = \sum_n \frac{\langle f | H'_{em} | n \rangle \langle n | H_W | i \rangle}{E_n - E_i} \]

Subtlety here is that I need to subtract out

\[ \langle f | H_{em} | f \rangle \]

from the EM Hamiltonian.

Otherwise the T matrix above can't be derived
What I'm trying to do

- Multipole expansion of the Hamiltonians
  Formalism available from electron-nucleus scattering

- Looking at limits
  High loop momentum: 2 momentum transfers are nearly back-to-back
  Low loop momentum: expansion in powers of loop momentum
More explicitly

\[
< f | H'_{em} | n > < n | H_W | i > 
= - \frac{e^2 G_F}{q^2} [4\pi \begin{array}{c} \sum_{J=1}^{\infty} 2\pi (2J + 1) (-1)^J \left\{ < f | \left[ \sum_{\lambda' = \pm 1} (j_{em, -\lambda'} T_{J, \lambda'}^{el}) + \sqrt{2} j_{em, 0'} M_{J0'} \right] \right\} | n > \\
\times < n | \left[ \sum_{\lambda = \pm 1} j_{W, \lambda} (\lambda T_{J, -\lambda}^{mag} + T_{J, -\lambda}^{el}) + \sqrt{2} (j_{W, 0} M_{J0} - j_{W, 3} L_{J0}) \right] | i > \\
+ < f | \sum_{\lambda' = \pm 1} j_{em, \lambda' 0'} \lambda' T_{J, -\lambda'}^{mag} | n > \\
\times < n | \left[ \sum_{\lambda = \pm 1} j_{W, \lambda} (\lambda T_{J, -\lambda}^{mag} + T_{J, -\lambda}^{el5}) + \sqrt{2} (j_{W, 0} M_{J0}^5 - j_{W, 3} L_{J0}^5) \right] | i > \end{array} ]
\]
Outlook

- Nuclear-structure dependence calculations are coming along (both ISB and our calculation)
  \[ V_{ud} = 0.97378(27) \] from nuclear decays, and uncertainties are dominated by theory

- Neutron decay rate measurements might yield more precise values in the near future
  No nuclear structure here, so we know the corrections better

- Current top-row unitarity test
  \[ 0.9483(5) + 0.0509(9) + 0.0000(1) = 0.9992(11) \]
  It's satisfied...for now
Good Reads

Reviews:
I.S. Towner and J.C. Hardy, nucl-th/0412056
“The CKM Quark-Mixing Matrix” and “Vud, Vus, The Cabibbo Angle, and CKM Unitarity” from Particle Data Group (pdg.lbl.gov)

Nucleus-dependent Corrections:
I.S. Towner and J.C. Hardy nucl-th/0209014