Fundamental Symmetry Tests and Precision Measurements

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Lectures at the National Nuclear Physics Summer School, George Washington University, June, 2008
The quarks we know vary widely in mass...

See QuarkDance.org for more!

In this lecture we take their masses as given.

We wish to interpret the pattern of hadron masses which result in terms of the symmetries of the underlying theory.
The splittings of the $0^-$ and $1^-$ states decreases substantially with increasing quark mass and become degenerate in the $m_f \to \infty$ limit.

The $0^-$ states are particularly low-lying only for states with $u, d, s$ quark content — and there is one outlier.

Parity doubling does not occur! This also holds for the low-lying spectra of baryons and nuclei.
The Pattern of Low-Lying Meson Masses

Masses of states which differ only in $(u, d)$ are nearly degenerate.

There are eight low-lying $0^-$ states — $\pi^\pm$, $\pi^0$, $K^0$, $\bar{K}^0$, $K^\pm$, and $\eta$ — the $\eta'$ is much heavier.

We can explain this pattern by invoking symmetries which are, in turn, approximate, spontaneously broken, and anomalous.
Spontaneous Symmetry Breaking

Here’s a class of potentials which can be used to describe the spontaneous breaking of a continuous symmetry...

A “Mexican Hat” Potential
Last time, taking the quark masses as given, we interpreted the pattern of low-lying meson spectra in terms of the global symmetries of the strong interaction. Now we turn to how the quark masses themselves might be generated.

Mass is also key to explaining the relative strength of the weak and electromagnetic interactions.

<table>
<thead>
<tr>
<th>Unified Electroweak spin = 1</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td></td>
<td></td>
</tr>
<tr>
<td>photon</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$W^-$</td>
<td>80.39</td>
<td>-1</td>
</tr>
<tr>
<td>$W^+$</td>
<td>80.39</td>
<td>+1</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>91.188</td>
<td>0</td>
</tr>
</tbody>
</table>

What is the origin of mass?
The Higgs Mechanism

A continuous, local symmetry can be spontaneously broken without yielding Goldstone bosons; rather, the gauge bosons gain mass. Our by now-familiar potential is that of the Higgs scalar field.

For now, we set aside the question of why the $W^\pm$ and $Z$ gauge bosons have the masses that they do; this mechanism provides no explanation for this, nor for the pattern of fermion masses.

A “Mexican Hat” Potential
Symmetries sculpt the Standard Model of particle interactions.

- The observation of parity violation in Nature (a NIST first!) mandates that the weak gauge bosons couple to left- and right-handed fermions differently.
- The fermions thus have fundamental axial couplings, but the specific charge assignments, with equal numbers of lepton and quark generations, make the theory anomaly free!

The violation of matter-antimatter symmetry (CP) also appears in a highly restricted way.

CP violation is relegated to a single phase ($\delta$) in the quark weak mixing matrix and to a single phase ($\bar{\theta}$) in the strong interactions.

Marvellously, Nature demands both $\delta \sim \mathcal{O}(1)$ and $\bar{\theta} \ll 10^{-9}$!
There is much theoretical “evidence” that the Standard Model is incomplete — it leaves many questions unanswered. Here are a few.

- It does not explain the number of generations nor the pattern of fermion masses and mixings.
- It does not explain why the $W^\pm$ and $Z$ have the masses that they do.
- It does not explain dark matter, dark energy.
- It cannot explain the baryon asymmetry of the Universe.
- It does not include gravity (by design).

Most notably, the Standard Model only explains 5% of the known Universe. There is much observational evidence for dark matter.

[Clowe et al., astro-ph/0608407]
The Standard Model is theoretically consistent up to \( \Lambda \sim 10^{19} \) GeV (the Planck scale), though we must accommodate neutrino masses.

**Why do we think there is new physics at \( \Lambda \sim 1 \) TeV?**

Suppose the Standard Model is valid for scales \( E \leq \Lambda \), where \( \Lambda \sim \mathcal{O}(1 \text{TeV}) \). At one-loop level, we find large corrections to the tree-level Higgs mass \( m_{\text{tree}} \).

All contributions must sum to \( m_h^2 \sim (200 \text{GeV})^2 \), but each one \( \sim \Lambda^2 \)!

At \( \Lambda = 10 \) TeV, \( m_{\text{tree}} \) must be tuned to one part in 100!

[Schmaltz, hep-ph/0210415]

New TeV-scale physics can make the cancellations “natural.”
“Fine-Tuning” does exist in Nature

[Hoyle, 1953; Cook, Fowler, Lauritsen, Lauritsen, 1957]
Symmetries sculpt the Standard Model of particle interactions.

$P$- and $CP$-violation play a special role.

- The pattern of predicted effects, be it in neutron (or nuclear) decay correlations or in $CP$-asymmetries in B-meson decays, is falsifiable.

- Other observables, such as the permanent electric dipole moment of the neutron, are so small in the Standard Model that their measurement serve as “null tests” at current levels of sensitivity.

- Individual, non-zero measurements can also limit deviations from known Standard Model parameters. Such studies are possible in electron scattering from atoms, protons, and nuclei, in neutron and muon decay, and in ....

Such limits directly constrain possible, new $P$- and $CP$-violating effects at the Lagrangian level and may give hints as to emergent symmetries at the TeV scale.
**Polarized Neutron $\beta$-decay in a V-A Theory**

\[ d^3\Gamma = \frac{1}{(2\pi)^32m_B} \left( \frac{d^3p_p}{2E_p} \frac{d^3p_e}{2E_e} \frac{d^3p_\nu}{2E_\nu} \right) \delta^4(p_n - p_p - p_e - p_\nu) \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \]

\[ \mathcal{M} = \frac{G_F}{\sqrt{2}} \langle p(p_p)|J^\mu(0)|\vec{n}(p_n, P)\rangle [\bar{u}_e(p_e)\gamma_\mu(1 - \gamma_5)u_\nu(p_\nu)] \]

\[ \langle p(p_p)|J^\mu(0)|\vec{n}(p_n, P)\rangle = \bar{u}_p(p_p)(f_1 \gamma^\mu - i \frac{f_2}{M_n} \sigma^{\mu\nu}q_\nu + \frac{f_3}{M_n} q^\mu) \]

\[ -g_1 \gamma^\mu\gamma_5 + i \frac{g_2}{M_n} \sigma^{\mu\nu}\gamma_5q_\nu - \frac{g_3}{M_n} \gamma_5 q^\mu)u_\vec{n}(p_n, P) \]

Note \( q = p_n - p_p \) and for baryons with polarization \( P \), \( u_\vec{n}(p_n, P) \equiv (\frac{1+\gamma_5P}{2})u_n(p_n) \)

\[ f_1 \ (g_V) \quad \text{Fermi or Vector} \quad g_1 \ (g_A) \quad \text{Gamow-Teller or Axial Vector} \]
\[ f_2 \ (g_M) \quad \text{Weak Magnetism} \quad g_2 \ (g_T) \quad \text{Induced Tensor or Weak Electricity} \]
\[ f_3 \ (g_S) \quad \text{Induced Scalar} \quad g_3 \ (g_P) \quad \text{Induced Pseudoscalar} \]

Since \((M_n - M_p)/M_n \ll 1\), a “recoil” expansion is efficacious.
To see how, consider the observables....
\[ d^3 \Gamma \propto E_e |p_e| (E_e^\text{max} - E_e)^2 \times \]

\[
[1 + \alpha \frac{p_e \cdot p_\nu}{E_e E_\nu} + P \cdot (A \frac{p_e}{E_e} + B \frac{p_\nu}{E_\nu} + D \frac{p_e \times p_\nu}{E_e E_\nu})] dE_e d\Omega_e d\Omega_\nu
\]

A and B are \( P \) odd, \( T \) even, whereas D is (pseudo)\( T \) odd, \( P \) even.
\[ \lambda \equiv |g_1/f_1| > 0 \]

and predictions:
\[ a = \frac{1 - \lambda^2}{1 + 3 \lambda^2} \quad A = 2 \frac{\lambda(1 - \lambda)}{1 + 3 \lambda^2} \quad B = 2 \frac{\lambda(1 + \lambda)}{1 + 3 \lambda^2} \quad [+O(R)] \]

implying \( 1 + A - B - a = 0 \) and \( aB - A - A^2 = 0 \), testing the V-A structure of the SM to recoil order, \( O(R) \), \( R \sim E_e^\text{max} / M_n \sim 0.0014 \).

Currently
\[ a = -0.102 \pm 0.005 \quad A = -0.1162 \pm 0.0013 \quad B = 0.983 \pm 0.004 \]

so that the relations are satisfied.
With \( \tau_n = 885.7 \pm 0.8 \text{ sec} \) and \( \tau_n \propto f_1^2 + 3g_1^2 \) more tests are possible.

RPP, Particle Data Group, 2002.
Beyond “V-A” in Neutron $\beta$-Decay

The search for non-V-A interactions continues...

$$H_{int} = (\bar{\psi}_p \psi_n)(C_S \bar{\psi}_e \psi_\nu + C'_S \bar{\psi}_e \gamma_5 \psi_\nu) + (\bar{\psi}_p \gamma_\mu \psi_n)(C_V \bar{\psi}_e \gamma^\mu \psi_\nu + C'_V \bar{\psi}_e \gamma^\mu \gamma_5 \psi_\nu)$$

$$- (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \bar{\psi}_e \gamma^\mu \gamma_5 \psi_\nu + C'_A \bar{\psi}_e \gamma^\mu \psi_\nu) + (\bar{\psi}_p \gamma_5 \gamma_\mu \psi_n)(C_P \bar{\psi}_e \gamma_5 \psi_\nu + C'_P \bar{\psi}_e \psi_\nu)$$

$$+ \frac{1}{2}(\bar{\psi}_p \gamma_\lambda \gamma_\mu \psi_n)(C_T \bar{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C'_T \bar{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu) + h.c.$$  

[Lee and Yang, 1956; note also Gamow and Teller, 1936]

$C'_X$ denote parity-nonconserving interactions.

In polarized neutron (nuclear) $\beta$-decay one more correlation appears: $b$

$$d^3\Gamma = \frac{1}{(2\pi)^5} \xi E_e |p_e| (E_e^{max} - E_e)^2 \times$$

$$[1 + a \frac{p_e \cdot p_\nu}{E_e E_\nu} + b \frac{m}{E_e} + P \cdot (A \frac{p_e}{E_e} + B \frac{p_\nu}{E_\nu} + D \frac{p_e \times p_\nu}{E_e E_\nu})]dE_e d\Omega_e d\Omega_\nu$$

[Jackson, Treiman, and Wyld, Phys. Rev. 106, 517 (1957)]

Note, e.g.,

$$b\xi = \pm 2\text{Re}[C_S C'_V^* + C'_S C_V^* + 3(C_T C_A^* + C'_T C'_A^*)]$$

If the electron polarization is also detected, more correlations enter.
Recent limits on $b$ come from nuclear $\beta$-decay:

$$b = -0.0027 \pm 0.0029$$

from survey of $0^+ \rightarrow 0^+$ (“superallowed” Fermi) transitions in nuclei


$$\tilde{a} \equiv a/(1 + bm_e/\langle E_e \rangle) = 0.9981 \pm 0.0030 \pm 0.0037$$

from $0^+ \rightarrow 0^+$ pure Fermi decay of $^{38}\text{mK}$

[A. Gorelov et al. PRL 94, 142501 (2005)]

Both limits are consistent with the Standard Model.

A new neutron “$a$” experiment, aCORN, is under construction at NIST!
**Transformations of Lorentz Bilinears under P, T, and C**

\[
\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad ; \quad \sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]
\]

<table>
<thead>
<tr>
<th></th>
<th>$\bar{\psi}\psi$</th>
<th>$i\bar{\psi}\gamma_5\psi$</th>
<th>$\bar{\psi}\gamma^\mu\psi$</th>
<th>$\bar{\psi}\gamma^\mu\gamma_5\psi$</th>
<th>$\bar{\psi}\sigma^{\mu\nu}\psi$</th>
<th>$\partial_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong></td>
<td>+1</td>
<td>−1</td>
<td>$(-1)^\mu$</td>
<td>$(-1)^\mu$</td>
<td>$(-1)^\mu(-1)^\nu$</td>
<td>$(-1)^\mu$</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>+1</td>
<td>−1</td>
<td>$(-1)^\mu$</td>
<td>$(-1)^\mu$</td>
<td>$(-1)^\mu(-1)^\nu$</td>
<td>$(-1)^\mu$</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>+1</td>
<td>+1</td>
<td>−1</td>
<td>+1</td>
<td>−1</td>
<td>+1</td>
</tr>
<tr>
<td><strong>CPT</strong></td>
<td>+1</td>
<td>+1</td>
<td>−1</td>
<td>−1</td>
<td>+1</td>
<td>−1</td>
</tr>
</tbody>
</table>

- **S** is for Scalar
- **P** is for Pseudoscalar
- **V** is for Vector
- **A** is for Axial-Vector
- **T** is for Tensor

All scalar fermion bilinears are invariant under CPT.
**Discrete Symmetries — P, T, and C**

**Parity P:**
Parity reverses the momentum of a particle without flipping its spin.

\[
P a_p^s P^\dagger = a_{-p}^s, \quad P b_p^s P^\dagger = -b_{-p}^s \quad \Rightarrow \quad P \psi(t, x) P^\dagger = \gamma^0 \psi(t, -x)
\]

**Time-Reversal T:**
Time-reversal reverses the momentum of a particle and flips its spin.

It is also antiunitary; note \([x, p] = i\hbar\).

\[
T a_p^s T^\dagger = a_{-p}^{-s}, \quad T b_p^s T^\dagger = b_{-p}^{-s} \quad \Rightarrow \quad T \psi(t, x) T^\dagger = -\gamma^1 \gamma^3 \psi(-t, x)
\]

**Charge-Conjugation C:**
Charge conjugation converts a fermion with a given spin into an antifermion with the same spin.

\[
C a_p^s C^\dagger = b_p^s, \quad C b_p^s C^\dagger = a_p^s \quad \Rightarrow \quad C \psi(t, x) C^\dagger = -i \gamma^2 \psi^*(t, x)
\]