The Physics of Nuclei – I: Building Nuclei

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Synopsis: Three Lectures

I. Building Nuclei
   Nucleons, NN forces, Effective Forces
   Few-body dynamics, Halo Nuclei

II. Nuclear Structure
   Liquid Drop,
   Shell model,
   Density Functional Descriptions

III. Nuclear Reactions
   Types of Reactions,
   Scattering, Mechanisms
Who am I?

Ian Thompson

- I am a theorist
- Ph.D. in New Zealand; Postdoc. at Daresbury Lab (UK)
- Teaching at Bristol, then Surrey University, up to 2006
- Solving quantum reaction problems (also halo structure)
- Comparisons of ‘good theories’ with experiments.
- 1 year ago: moved to Lawrence Livermore Lab (CA)
- Writing a Book on Reaction Theory
Many students!

- Theory or Experiment?
- Beginning Ph.D. or mid Ph.D., or postdoc? (where in the U.S., or overseas?)
- Which experiments are interesting?
- What do you hope to learn?
We look for how the quarks makes nucleons, which interact to make nuclei.
Using Nuclei

Beams and Targets (or electrons)

Know your target!

Test fundamental symmetries

E.g. by mixing intrinsic nuclear symmetries

Nuclear astrophysics

Nucleosynthesis, supernovae, neutron stars

New structures of exotic nuclei

E.g. near the proton and neutron drip lines
Fermionic Many-Body Systems

Resolution determines level of Dynamical Detail. Entities and Effective Interactions also vary with resolution.
Diffraction and Resolution 1
Diffraction and Resolution 2
Diffraction and Resolution 3
Diffraction and Resolution 4
Principles of Effective Theories 1

\[ \lambda << R \]
If system is probed at low energies, fine details not resolved.
If system is probed at low energies, fine details not resolved
- use low-energy variables for low-energy processes
- short-distance structure can be replaced by something simpler without distorting low-energy observables
Nuclei start when nucleons are resolved

Start from the simplest experiments:

- NN Scattering (nn, np, pp)
  Phase shift analysis:
- Deuteron Bound State:
  Binding 2.224 MeV,
  Quadrupole moment 0.282 fm$^2$. 
Phenomenological NN Potentials

Use meson exchange forms
   Adjust parameters and cutoffs

Reproduce low-energy scattering lengths etc
   \( a_{pp} = -17.3 \pm 0.4 \text{ fm} ; \ a_{nn} = -18.8 \pm 0.3 \text{ fm} ; \ a_{np} = -23.75 \pm 0.1 \text{ fm} ; \)
   Note that \( V_{pp} \neq V_{np} \neq V_{nn} \).

Main features
   Strong tensor force,
   Strong repulsive core at short distances.
Examples of NN Potentials

Argonne potentials

Coulomb + One-pion exchange + intermediate- and short-range

Bonn potential

R. Machleidt, PRC63, 024001 (2001)
Based on meson-exchange, Non-local

Effective field theory

Ordóñez, Ray, van Kolck, PRC 53, 2086 (1996);
Epelbaoum, Glöckle, Meissner, NPA 637, 107 (1998)
Based on Chiral Lagrangians
Expansion in momentum up to cutoff $\sim 1$ GeV
Generally has a soft core
Look at main part – $\pi$-exchange:

Elastic scattering in momentum space

$$V^{\pi NN}_{\text{local}}(q = k' - k) = -\frac{g_{\pi}^2}{4M^2} \frac{\sigma_i \cdot q \sigma_j \cdot q}{q^2 + m_{\pi}^2}$$

or through a Fourier transform, in coordinate space:

$$V_{\pi} = \frac{g_{\pi}^2}{4M^2} \frac{1}{3m_{\pi}} \left[ \sigma_i \cdot \sigma_j + \left( 1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right) (3\sigma_i \cdot \hat{r} \sigma_j \cdot \hat{r} - \sigma_i \cdot \sigma_j) \right] e^{-\mu r}$$

Off-shell component present in the Bonn potentials

$$V^{\pi NN}(k', k) = -\frac{g_{\pi}^2}{4M^2} \frac{(E' + M)(E + M)}{(k' - k)^2 + m_{\pi}^2} \left( \frac{\sigma_i \cdot k'}{E' + M} - \frac{\sigma_2 \cdot k}{E + M} \right) \times \left( \frac{\sigma_2 \cdot k'}{E' + M} - \frac{\sigma_i \cdot k}{E + M} \right)$$

Non-local (depends on initial and final momenta). Plays a role in many-body applications: more binding
Look at main part, **pion-exchange:**

**Needed to Bind $A = 3$ nuclei**
- Two-nucleon interactions under-bind
- Note CD-Bonn has a little more binding due to non-local terms

**Further evidence**
- from by ab initio calculations for $^{10}$B: NN-interactions give the wrong ground-state spin!

**Example: Tucson-Melbourne Force**
- S.A. Coon and M.T. Peña, PRC 48, 2559 (1993)
- Based on two-pion exchange and intermediate $\Delta$s
- The exact form of NNN is not known
Three-Body Dynamics

For two particles we use Schrödinger equation
For three and four, there are Faddeev and Faddeev-Yakubovsky formulations

\[
\Psi = \psi_1 + \psi_2 + \psi_3
\]
\[
\psi_i = \frac{1}{E - H_0} T_i (\psi_j + \psi_k)
\]
\[
H_0 = \sum_i \frac{\vec{p}_i^2}{2m_i}
\]
\[
T_i = V_{jk} + V_{jk} \frac{1}{E - H_0} T_i
\]
\[
(E - H_0 - V_{23}) \psi_1 = V_{23} (P_{12} P_{23} + P_{13} P_{23}) \psi_1
\]

W. Glöckle in Computational Nuclear Physics, Springer-Verlag, Berlin, 1991

Exact methods exist for \( A \leq 4 \).
Effects of Three-Nucleon Force

Binding of Triton ($^3\text{H}$) without and with Tucson-Melbourne Force
More than Four Bodies?

Synopsis of what we can do:

- Cluster Models.
- Liquid-drop Models: see lecture II.
- Greens Function Monte Carlo
  \[ \langle \Psi_{\text{exact}} | \hat{H} | \Psi_{\text{exact}} \rangle = \lim_{\beta \to \infty} \frac{\langle \psi_{\text{trial}} | \hat{H} e^{-\beta \hat{H}} | \psi_{\text{trial}} \rangle}{\langle \psi_{\text{trial}} | e^{-\beta \hat{H}} | \psi_{\text{trial}} \rangle} \]
  \[ \sum_{ijkl} C_{ijkl} a_i^+ a_j^+ a_k a_l + \ldots \]
- Coupled-cluster
  \[ \Psi = e^\Psi_{\text{ref}} \]
- Shell model (lecture II.)
  \[ \phi = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_i(r_1) & \phi_i(r_2) & \ldots & \phi_i(r_A) \\ \phi_j(r_1) & \phi_j(r_2) & \ldots & \phi_j(r_A) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_l(r_1) & \phi_l(r_2) & \ldots & \phi_l(r_A) \end{vmatrix} \]
  \[ = a_i^+ \ldots a_j^+ a_k^+ |0\rangle \]

- Mean-field (energy density functional) methods (lecture II.)
Cluster Models for Halo Nuclei

- **Definition**: Weakly-bound nuclei near drip line that are large
- **Composition**: One or two neutrons (or protons) outside a core nucleus.
- **Interesting**: New physics away from valley of stability
- **Borromean**: Borromean three-body systems bound, even though no pairwise (two-body) bound states:
Examples of Halo Nuclei

One neutron:

\(^{11}\text{Be} \ (S_n = 0.504 \text{ MeV})\)

Two-neutron Borromean:

\(^{6}\text{He} \ (S_{2n} = 0.97 \text{ MeV}), \quad ^{11}\text{Li} \ (S_{2n} = 0.30 \text{ MeV}),\)

One-proton

\(^{8}\text{B} \ (S_p = 0.137 \text{ MeV}),\)

Two-proton Borromean:

\(^{17}\text{Ne} \ (S_{2p} = 0.96 \text{ MeV}),\)
Why Study Haloes?

- Good **few-body** system:
  Continuum is near to bound states, long tails to bound states, so large cross sections & dynamic distortion in reactions.

- See prominent single-particle states

- See pairing outside nuclear surface:
  in two-neutron halo ground states;
  in two-neutron continuum via breakup; and
  in two-proton decay via tunnelling

- See bound states in classically forbidden regions.
First Halo: $^{11}$Be

Strong E1 transition in $^{11}$Be:

$$\tau = 168(17) \text{ fs: } B(E1) = 0.36(3) \text{ W.u.}$$

Millener et al., PRC 28 (1983) 497

“We note that to obtain the $1s_{1/2}p_{1/2}$ matrix element for low binding energies it is necessary to integrate out to large radii”
Borromean Halo: $^6$He

- Two Neutrons and an $\alpha$ particle bound at $S_{2n} = 0.97$ MeV
- n-$\alpha$ unbound, but $p_{3/2}$ resonance at 0.8 MeV
- n-n unbound, but virtual state $a_{nn} = -18.8 \pm 0.3$ fm
Experimental Evidence

Study of halo nuclei (officially) began with measurement of interaction cross sections in Berkeley in 1985.
Two-proton Decay

- Not via point diproton
- Need three-body models with pairing in exterior
- Prediction: pairing acts to correlate the protons to enhance $L = 0$ cluster-nucleus relative motion.
Using Few-Body Methods for More Bodies

Summary:

- Cluster Models.
- Greens Function Monte Carlo

\[
\langle \Psi_{\text{exact}} | \hat{O} | \Psi_{\text{exact}} \rangle = \lim_{\beta \to \infty} \frac{\langle \psi_{\text{trial}} | \hat{O} e^{-\beta \hat{H}} | \psi_{\text{trial}} \rangle}{\langle \psi_{\text{trial}} | e^{-\beta \hat{H}} | \psi_{\text{trial}} \rangle}
\]
Monte Carlo Methods

- For smaller systems (finite or in a box with pbc’s)
  \[ 
  \implies \text{approximate energy and full many-body wave function} 
  \]
- Variational, diffusion, path integral, Green’s function MC
- All use the Metropolis algorithm: random walkers

**Variational Monte Carlo (VMC):** Estimate \( \langle E \rangle = \int d\mathbf{R} \rho(\mathbf{R}) E_L(\mathbf{R}) \)

- local energy \( E_L(\mathbf{R}) = \frac{H\psi_T(\mathbf{R})}{\psi_T(\mathbf{R})} \) with trial wave function \( \psi_T(\mathbf{R}) \)
- probability distribution \( \rho(\mathbf{R}) = \frac{\psi_T^2(\mathbf{R})}{\int d\mathbf{R} \psi_T^2(\mathbf{R})} \)
- accept step to \( \mathbf{R}' \) if \( p = \frac{\psi_T^2(\mathbf{R}')}{\psi_T^2(\mathbf{R})} \geq 1 \),
  else if \( p < 1 \) accept with probability \( p \)
- minimize \( \langle E \rangle \) or variance of \( E_L(\mathbf{R}) \) with respect to variational parameters in \( \psi_T(\mathbf{R}) \)
- gives upper bound to ground state \( E \)

- Requires very good trial wave function for reliable results
Finding the Ground State

- DMC and GFMC exploit S–equation in imaginary time \( \implies \) diffusion!

\[
\begin{align*}
-\hbar \frac{\partial}{\partial \tau} \psi(R, \tau) &= -\frac{\hbar^2}{2M} \nabla_R^2 \psi(R, \tau) + V(R)\psi(R, \tau)
\end{align*}
\]

- Use Metropolis to propagate to large \( \tau \implies \) projects ground state

\[
\psi(R, \tau) = \int dR' \ G(R, R', \tau) \psi(R', \tau)
\]

- Take many steps with small \( \tau \) approximation to \( G \)
- Generates “walker representation” of wave function (a set of \( R_i \)’s) \( \implies \) can only represent a positive density

- Fermion sign problem for diffusion, path integral, GFMC
  - for fermions, even ground-state wavefunction changes sign (anti-symmetric)
  - if trial function provides good representation of nodes, solve in regions with nodal boundary conditions (“fixed node”)
Results for Light Nuclei

Large GFMC computations by Argonne group of Carlson, Pandharipande, Pieper & Wiringa.
Results for Light Nuclei

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Argonne $v_{18}$
With Illinois $V_{ijkl}$
GFMC Calculations