Electromagnetic structure of light nuclei

and the current operator

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Outline

1. Nuclear forces: a brief overview
2. Preliminary results and conclusions
3. Nucleon-nucleon interaction: the potential
4. Meson exchange potentials and construction of the effective propagator
5. Nucleon-nucleon interaction: the $g_8$ potential
6. The nuclear electromagnetic currents
Introduction nuclear models are "far" from a quantitative understanding of the $NN$ force from this point of view due to the difficulties of solving QCD in the low-energy regime. We completely determined by the underlying quark-gluon dynamics on this basis, the nucleon-nucleon ($NN$) interaction is Quantum Chromodynamics (QCD) is the fundamental theory of strong

Nuclear Forces
Therefore the quark-gluon dynamics is included in the parametrization

Realistic models of \( \omega, \pi, \Lambda \) are based on experimental data hinting

and \( \omega, \pi, \Lambda \) are the 2- and 3-nucleon interaction operators:

\[
\ldots + \gamma \Lambda \sum + \gamma \omega \sum + \left( \frac{i\gamma \mu}{2} \mathbf{d} + \gamma \mu \right) \sum_{\mathcal{V}} = H
\]

by

Composite system of A interacting nucleons, the Hamiltonian is given

non-relativistic treatment hence, assuming that the nucleus is a

nucleon typical velocity \( v \approx 0.05 \\
(\Lambda + W) \approx \frac{1}{m_{\nu}} \cdot Y \) below the threshold

nuclear degrees of freedom: nucleons

Basic Model
Fortunately, there is a strong experimental support to the theoretical expectation that the long range part of the potential is due to pion exchange processes, therefore only their short range parts need to be modeled. All realistic models contain the OPEP meson-exchange idea goes back to Yukawa who, in 1935, stated that the meson was then observed in 1947, the pion-p'p−p′−p

\text{One-Pion-exchange potential (OPEP)}
the interaction of the pion field with a Pauli nucleon is described by the Hamiltonian:

\[ H = \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O} \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \]

\[ + \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \]

\[ = \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \cdot \mathcal{O}\left(\mu + \nu\right) \]

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The configurations space is obtained from its Fourier transform:

$$\rho = \frac{NN^\mu f}{N N^\mu f}$$

The value obtained in 1993 from scattering data is:

$$NN^\mu f = 0.075$$

The description of the long-range part depends on only one parameter, i.e., the pion-nucleon coupling constant. The Nijmegen group fitted the scattering data assuming that the long-range part is given by the exchange of a pseudoscalar-isovector.

The tensor operator, and are functions of $T$.

$$S^\mu_2 T^\nu L = \rho \cdot \rho - \rho \cdot \rho \left[ (x)^{\mu u} \frac{\rho}{N N^\mu f} - \rho \lambda \right] + \rho \cdot \rho \frac{\rho}{N N^\mu f} - \rho \mu$$

The configuration space is obtained from its Fourier transform.
we can separate the NN potential as follows:

Realistic model
\[(\mathbb{I} q)^\ell \cdot \mathbb{I} S \cdot \mathbb{I} = \mathbb{I} = dO \]

and the two iso scalar spin-orbit operators are:

\[(\mathbb{I} \mathbb{I} q)^\ell \cdot \mathbb{I} \mathbb{I} S \cdot \mathbb{I} = \mathbb{I} \mathbb{I} = dO \]

where the static operator are:

\[(\mathbb{I} \mathbb{I} q)^\ell \cdot \mathbb{I} \mathbb{I} S \cdot \mathbb{I} = \mathbb{I} \mathbb{I} = dO \]

\[(\mathbb{I} \mathbb{I} q)^\ell \cdot \mathbb{I} \mathbb{I} S \cdot \mathbb{I} = \mathbb{I} \mathbb{I} = dO \]

The strong interaction part of the iso scalar can be expressed as a sum of:

\[\mathbb{I} \mathbb{I} = dO \]

The \( \mathbb{I} \mathbb{I} \) potential.
\[ \text{potential in the momentum space} \]
The idea is to evaluate the invariant amplitude induced by the one-boson-exchange (OBE) Lagrangians: the invariant amplitude is related to the non-relativistic (NR) potential through the following relation:

\[ \mathcal{L} = \mathcal{E} \text{ and } \frac{2}{\mu w + a w} = w \]

and subsequently make a link between the so obtained NN and the one.

The meson exchange potential

\[ \nu \mathcal{E} \]

\[ \nu \alpha \]

\[ \nu \mathcal{E} \]

\[ \nu \alpha \]
The One Boson Exchange (OBE) Lagrangians
The are obtained by multiplying by multiplicity $J' \Lambda d' S' = \alpha''$. The $\alpha''$ propagator is

$$\left[ b \cdot \mathcal{O} b \cdot \mathcal{O} \frac{\mathcal{O} \not{\nu} \mathcal{O}}{\not{\nu}} \frac{0 J' \not{\nu} + \not{\nu} b}{\not{\nu} b} \right] + \left[ d \times \not{d} \cdot (\mathcal{O} + \mathcal{O}) \frac{\mathcal{O} \not{\nu} \mathcal{O}}{\not{\nu}} \frac{0 J' \not{\nu} + \not{\nu} b}{\not{\nu} b} \right] = 0 J' \not{\nu}$$

$$+ b \cdot \mathcal{O} b \cdot \mathcal{O} \frac{\mathcal{O} \not{\nu} \mathcal{O}}{\not{\nu}} \frac{0 J' \not{\nu} + \not{\nu} b}{\not{\nu} b} - \frac{\mathcal{O} \not{\nu} \beta}{\not{\nu} b} - \frac{\mathcal{O} \not{\nu} \beta}{\not{\nu} b} - 1 \right] \frac{0 J' \not{\nu} + \not{\nu} b}{\not{\nu} b} = 0 J' \not{\nu}$$

$$b \cdot \mathcal{O} b \cdot \mathcal{O} \frac{\mathcal{O} \not{\nu} \mathcal{O}}{\not{\nu}} \frac{0 J' \not{\nu} + \not{\nu} b}{\not{\nu} b} = 0 S d' \not{\nu}$$

$$\left[ d \times \not{d} \cdot (\mathcal{O} + \mathcal{O}) \frac{\mathcal{O} \not{\nu} \mathcal{O}}{\not{\nu}} \frac{0 S \not{\nu} + \not{\nu} b}{\not{\nu} b} - \frac{\mathcal{O} \not{\nu} \beta}{\not{\nu} b} + 1 \right] \frac{0 S \not{\nu} + \not{\nu} b}{\not{\nu} b} = 0 S \not{\nu}$$
\[ q \cdot q \cdot \mathcal{L} = \mathcal{L} \quad \text{same for} \quad b \cdot \mathcal{L} \]

\[
\begin{aligned}
[(u)_{q_{\omega}}] \mathcal{L} &= (b)_{q_{\omega}} \\
[(u)_{q_{\omega}}] \mathcal{L} \mathcal{E} &= (b)_{q_{\omega}} \\
[(u)_{q_{\omega}}] \mathcal{L} + [(u)_{q_{\omega}}] \mathcal{L} &= (b)_{q_{\omega}} \\
[(u)_{q_{\omega}}] \mathcal{L} &= (b)_{q_{\omega}}
\end{aligned}
\]

\[
\ell \cdot \ell [\cdots] + (d \times \beta d) \cdot (\ell \cdot \mathcal{O} + \mathcal{O}) \frac{\mathcal{O}}{\mathcal{O}} (b)_{q_{\omega}} +
\]

\[
\begin{aligned}
b \cdot \ell \cdot \mathcal{O} (b)_{q_{\omega}} + \ell \cdot \mathcal{O} (b)_{q_{\omega}} b + (b)_{q_{\omega}} &= \mathcal{O}
\end{aligned}
\]

The potential in the momentum space (bis)
The effective propagators

\[ q \frac{\partial^2}{\partial \mu^2} - \alpha \partial_\mu \nabla - \alpha \frac{\partial}{\partial \mu} = (b)_0 \Delta \]

\[ \alpha \frac{\partial}{\partial \mu} + q \frac{\partial}{\partial \mu} = (b)_0 \Lambda \]

\[ q \alpha \partial_m \nabla - \alpha \partial_m \nabla^2 = (b)_0 \Sigma \]

\[ \alpha \partial_m - q \frac{\partial}{\partial \mu} = (b)_0 \Sigma \]

\[ \frac{g^a_\mu + \alpha b}{g^a} = g^a \Delta \]

\[ 1 = g^a \Lambda \Sigma \Delta \]

\[ S \]

hence we are left with the following relations:

\[ 0 = g^a \Lambda \Sigma \Delta \]

\[ S = \alpha \]

\[ \frac{\partial}{\partial \mu} + \alpha b \]

\[ g^a \]

\[ = g^a \Delta \]

\[ \text{define} \]

\[ \text{the potential with the} \]

\[ \text{now we are able to construct the effective propagators by comparing} \]

\[ \Delta \]

The effective propagators
Electromagnetic current in order to study and predict the behavior of our system in presence of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe, the model must contain currents operators of an external probe. 

These operators are expanded as a sum of nucleon operators, have the form: the charge density \((b)(n)\) and the current density \((b)d\) of the nucleons.

In order to study and predict the behavior of our system in presence of an external probe, the model must contain currents operators of an external probe.
One body current operators

The one body operators describe the current of a single nucleon:

\[
(b \times \vec{\mathcal{L}}) (\vec{b} \cdot \vec{\theta}) = (b) \cdot \vec{\mathcal{D}}
\]

Interaction with the field is:

The one body charge operator due to the nucleon magnetic moment is:

\[
(i \Delta \vec{b} \cdot \vec{\theta}) \frac{\omega b Z}{I} = (b) \cdot \vec{\mathcal{D}}
\]

The one body convolution current operator is given by:

\[
\vec{d}(\vec{b} \cdot \vec{\theta}) \frac{\omega b Z}{I} = (b) \cdot \vec{\mathcal{D}}
\]

Moments of nucleons in nuclei absorb photons via their charges and magnetic moments.

One body current operators
By the nucleons

meson currents: The photon is absorbed by the current of a charged meson being exchanged virtual and has to be reabsorbed by another nucleon meson currents: The photon produces a meson by hitting the nucleon, the meson is

photo-meson currents: The photon produces a meson by hitting the nucleon, the meson is

Two body current operators
and meson propagators

Marcucci et al., Phys. Rev. C72, 014001, 2005 to obtain the effective $\tau$

The present method generalizes the procedure adopted in L. E.

The bare propagators are then replaced by the effective propagators $NN$ potential

The diagrams induced by the OBE Lagrangians
evaluating the invariant amplitudes associated to the Feynman

The photon-meson and meson-exchange currents are derived by
The current operator in the momentum space

\[ \mathcal{J} = \mathcal{J} \mathcal{D} - \mathcal{J} \mathcal{D} = \mathcal{J} \mathcal{D} \]

\[ \mathcal{J} = \mathcal{J} + (\mathcal{J} \mathcal{B} \cdot \mathcal{J} \mathcal{D}) \mathcal{J} \mathcal{D} \left( \mathcal{b} \times \mathcal{b} \right) \frac{\mathcal{L} \mathcal{z} \mathcal{w} \mathcal{v}}{\mathcal{L} \mathcal{z}} = (\mathcal{J} \mathcal{B}, \mathcal{J} \mathcal{B}) \mathcal{I} \mathcal{S} \mathcal{D} \]
The current operators in the configuration space are obtained from:

\[ I' \ 0 = \mathcal{J} \ \Lambda \ S_d \ S = \Theta \]

with

\[ (\ell \mathbf{b} \cdot \mathbf{b})_{g^x} \int (\mathbf{b} - \ell \mathbf{b} + \mathbf{b}) \mathcal{E}_x(\nu \mathbf{Z}) \times \]

\[ \ell \mathbf{b} \cdot \mathbf{b} \mathcal{E}_x(\nu \mathbf{Z}) \mathcal{E}_y(\nu \mathbf{Z}) \int = \ (\ell \mathbf{b} \cdot \mathbf{b})_{g^x} \int \]

The configuration-space expressions of the currents are obtained from:
The magnetic form factor is then defined as:

\[ \frac{b}{(b)_{\mathcal{H}}} \frac{n}{\mu} \mathcal{Z}^\Lambda = (b)_{\mathcal{H}} \]

where \( n \) is the magnetic moment of the nucleus in nuclear magnetons.

\[ n \frac{\mu}{b} \mathcal{Z}^\Lambda \approx (b)_{\mathcal{H}} \]

The elastic form factor becomes like:

\[ \langle 0 | (x b)^{\mathcal{H}} | \mathcal{H} 0 \rangle \mathcal{Z}^\Lambda = (b)_{\mathcal{H}} \]

The elastic form factor is then:

\[ \mathcal{M} \]

where \( \mathcal{M} \) denotes the ground state recoiling with momentum \( \mathcal{Z}^\Lambda \).
we constructed the two-body electromagnetic currents starting from the meson-exchange mechanism theoretical insight. The systematic use of the effective propagator in the evaluation of the non-local terms have been included in the evaluation of the currents operators (with the exception of the \( J_1 \) non-local contribution) and the fixed effective propagators such to reproduce the \( n\bar{g} \) structure will lead to a good agreement with the experimental data. We fixed the effective propagators to reproduce the \( n\bar{g} \) structure of the currents operators. A good agreement with the experimental data will lead to a systematic use of the effective propagator in the evaluation of the currents operators.
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CDBonn Potential

\{ \Omega \} \nu

\text{P1 channel}

bare propagator

effective propagator

[b_{\text{cm}}]
evaluated in the configuration-space yet

\( J^{\mathcal{CN}}_{\Lambda} \)
is the non local piece of the vector current which has not been

\[
J \rightleftharpoons \{ \left[ (\ell_d + \ell_{\mathcal{L}}) - \ell_b \times (\ell_{\mathcal{O}} + \ell_{\mathcal{L}}) \right] \left( \mathcal{Z} \mathcal{L}_{\Lambda} \mathcal{I}_{\mathcal{H}} + \ell_{\mathcal{L}} \cdot \ell_{\mathcal{L}} \right) \mathcal{I}_{\mathcal{H}} + \mathcal{L} \mathcal{I}_{\mathcal{H}} \} \\
+ \left[ (\ell_b \times \ell_{\mathcal{O}}) \times \ell_{\mathcal{L}} + (\ell_{\mathcal{L}} + \ell_{\mathcal{L}}) \times \ell_{\mathcal{L}} + \ell_{\mathcal{L}} \right] \mathcal{I}_{\mathcal{H}} \mathcal{L} \mathcal{I}_{\mathcal{H}}

\cdot (\ell_{\mathcal{B}})_{\mathcal{Z}} \mathcal{I}_{\mathcal{H}} \frac{\mathcal{Z} \mathcal{L}_{\Lambda} \mathcal{I}_{\mathcal{H}}}{\mathcal{L}} = (\ell_{\mathcal{B}} \mathcal{L} \mathcal{B})_{\mathcal{Z}} \mathcal{I}_{\mathcal{H}}}

The current operator in the momentum space