Nuclear & Particle Physics of Compact Stars

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National Nuclear Physics Summer School
July 24-28, 2006, Bloomington, Indiana
The Role of the Equation of State in Binary Mergers

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&

PALS
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James M. Lattimer (Stony Brook University)
Our Thoughts on this Subject

1. M. Prakash,  

2. S. Ratkovic, M. Prakash & J. M. Lattimer,  

3. S. Ratkovic, M. Prakash & J. M. Lattimer,  
astro-ph/0512133; 0512136  
The Binary Merger Experience

The Ultimate Heavy-Ion Collision

- \( M_1 \leq M_2 \)
- radial separation: \( a(t) \)
- \( M_1 - NS \) or \( SQM \)
- \( M_2 - BH, NS, \ldots \)
- GW emission \( \Rightarrow \)

\[
L_{GW} = \frac{1}{5} \frac{G}{c^5} \langle \tilde{F}_{jk} \tilde{F}_{jk} \rangle
= \frac{32}{5} \frac{G^4}{c^5} \frac{M^3 \mu^2}{a^6}
\]

orbit shrinks
- Mass transfer
Einstein’s General Relativity

\[ G^{\alpha\beta} [g, \partial g, \partial^2 g] = 8\pi \ T^{\alpha\beta} [g] \]

- \( G^{\alpha\beta} \): 2\textsuperscript{nd}-order nonlinear differential operator acting on \( g_{\alpha\beta} \)
- \( T^{\alpha\beta} \): Stress-energy tensor of matter fields

**Parametrized Post-Newtonian (PPN) Formulation**

In weak field limit,

\[ g^{PPN}_{\mu\nu} = \eta_{\mu\nu} + h^{1PN}_{\mu\nu} (M) + h^{2PN}_{\mu\nu} (M) + h^{3PN}_{\mu\nu} (M) + \cdots \]

- \( \eta_{\mu\nu} \): flat-space Minkowski metric
- \( M \): incorporates dependence on matter fields
- \( 1PN, 2PN, \cdots \Rightarrow [O(v^2/c^2)]^\epsilon \) with \( \epsilon = 1, 2, \cdots \)

For vacuum gravitational fields (in transverse traceless gauge),

\[ \left( -\frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{x/+} = 0 \]
GW’s have two transverse polarizations, $h_+$ & $h_\times$. 
Laser Interferometer GW Detector

For a readable account, see K. Thorne, arXiv:gr-qc/9506084
Gravitational Wave Detection

GW Strain: \( h(t) = F_x h_x(t) + F_+ h_+(t) \)

- \( F_{x,+} \) : Constants of order unity

- \( h_{x,+} \sim \frac{\delta L}{L_0} \sim \frac{1}{c^2} \frac{4G(E_{\text{kin}}^{\text{ns}}/c^2)}{r} \) : Gravitational waveforms
  - \( L_0 \): Unperturbed length of detector arm
  - \( \delta L \): Relative change in length
  - \( E_{\text{kin}}^{\text{ns}} \): Nonspherical part of the internal kinetic energy

- ELF: \( 10^{-15} - 10^{-18} \) Hz
- VLF: \( 10^{-7} - 10^{-9} \) Hz
- LFB: \( 10^{-4} \) Hz - 1 Hz, HFB: \( 1 \) Hz - \( 10^4 \) Hz

Astrophysical Sources Radiating GW’s in the HFB

- Supernovae at 10 Mpc \( h \geq 10^{-25} \)
- Supernovae Milky Way \( h \sim 10^{-18} \)
- 1.4M⊙ NS Binaries at 10 Mpc \( h \sim 10^{-20} \)
- 10M⊙ BH Binaries at 150 Mpc \( h \sim 10^{-20} \)
Inspiral Waveform

Chirp signal:

\[
\begin{align*}
    h_+ & \propto \frac{M^{5/3}}{r} f^{2/3} \cos(2\pi ft) \\
    h_\times & \propto \frac{M^{5/3}}{r} f^{2/3} \sin(2\pi ft) \\
    f & = K_0 \mathcal{M}^{-5/8} (t_c - t)^{-3/8}
\end{align*}
\]

with the “chirp mass”:

\[
\mathcal{M} = (M_1 M_2)^{3/5} M^{-1/5}
\]

and the constant:

\[
K_0 = \frac{5^{3/8}}{8\pi} \left(\frac{c^3}{G}\right)^{5/8}
\]
- Binary pulsar PSR 1913+16
- Period: 7 h 45 min
- $M_{NS} = 1.4408 \pm 0.0003 \, M_\odot$
- $M_c = 1.3873 \pm 0.0003 \, M_\odot$
- Distance: 7.13 kpc
- Merger in 300 Myr
# Merger Rates of Binary Systems

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Information</th>
<th>Type</th>
<th>Merger Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phinney (1991)</td>
<td>pulsar lifetimes, distributions</td>
<td>cons.</td>
<td>$5 \times 10^{-8}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bguess</td>
<td>$7 \times 10^{-6}$</td>
</tr>
<tr>
<td>Van den Heuval &amp; Lorimari (1996)</td>
<td>pulsar detectability, distribution</td>
<td>cons.</td>
<td>$3 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bguess</td>
<td>$8 \times 10^{-6}$</td>
</tr>
<tr>
<td>Bailes (1996)</td>
<td>galactic pulsar birth rates</td>
<td>lbound</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ubound</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>Potegies Zwart &amp; Yungelson (1998)</td>
<td>“scenario machine” w/ supernova kicks</td>
<td></td>
<td>$0.2 - 3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Bethe &amp; Brown (1998)</td>
<td>common envelope hypercritical accretion</td>
<td>ubound</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>

Rates in $yr^{-1} \ Mpc^{-3}$ \[ 1 \ pc = 3 \times 10^{18} \ cm. \]
### Discovery of Double-Pulsar System

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>PSR J0737-3039A</th>
<th>PSR J0737-3039B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pulse Period</strong> $P$ (ms)</td>
<td>$22.69937855615(6)$</td>
<td>$2773.4607474(4)$</td>
</tr>
<tr>
<td><strong>Period derivative</strong> $\dot{P}$</td>
<td>$1.74(5) \times 10^{-18}$</td>
<td>$0.88(13) \times 10^{-15}$</td>
</tr>
<tr>
<td><strong>Orbital period</strong> $P_b$ (day)</td>
<td>$0.102251563(1)$</td>
<td>—</td>
</tr>
<tr>
<td><strong>Eccentricity</strong> $e$</td>
<td>$0.087779(5)$</td>
<td>—</td>
</tr>
<tr>
<td><strong>Characteristic age</strong> (My)</td>
<td>$210$</td>
<td>$50$</td>
</tr>
<tr>
<td><strong>Magnetic field</strong> $B_s$</td>
<td>$6.3 \times 10^9$</td>
<td>$1.6 \times 10^{12}$</td>
</tr>
<tr>
<td><strong>Spin-down luminosity</strong> $\dot{E}$ (erg/s)</td>
<td>$5.8 \times 10^{33}$</td>
<td>$1.6 \times 10^{30}$</td>
</tr>
<tr>
<td><strong>Distance</strong> (kpc)</td>
<td>$\sim 0.6$</td>
<td>—</td>
</tr>
<tr>
<td><strong>Stellar mass</strong></td>
<td>$1.337(5)$</td>
<td>$1.250(5)$</td>
</tr>
</tbody>
</table>

Merger expected in 85 Myr, a factor 3.5 shorter than PSR 1913+16


Kalogera et al. (2004): Revisions w/ PSR J037-3039 imply 1 event per 1.5 yr for initial LIGO (for advanced LIGO, 20-1000 events per yr).
PSR J0737 3039 and LIGO

- Merger rate: \( R \propto N/\tau \)

- Binary pulsar lifetime: \( \tau = \tau_{BIRTH} + \tau_{COAL} \)
  \[
  \frac{\tau_{1913}}{\tau_{0737}} = \frac{365 \text{ Myr}}{185 \text{ Myr}} \approx 2
  \]

- Scaling factor: \( N \propto L_{400}^{-1} \)
  \[
  \frac{N_{0737}}{N_{1913}} = \frac{L_{1913}}{L_{0737}} = \frac{200 \text{ mJy kpc}^2}{30 \text{ mJy kpc}^2} \approx 6
  \]

\[ 2 \times 6 = 12 \Rightarrow \text{an order of magnitude increase of merger rates!} \]
GW Detectors & Expected Gains

- Ground-Based Laser Interferometers
  - LIGO, VIRGO, GEO, TAMA, ...
- The Laser Interferometer Space Antenna (LISA)

- GW’s provide valuable new information “orthogonal” to electromagnetic observations
  - First direct test of GR
  - Precise ($\pm$ a few %) determination of Hubble’s constant $H_0$
  - Calibration of distance measurements
  - Masses of NS, BH (large scale structure formation)
  - ......

Objectives

- Explore EOS dependence of GW signals from mergers.
  - Specifically, look at differences between “normal” stars and “self-bound” (e.g., SQM) stars.
    - EOS parameter: \( \alpha(M_1) \equiv d\ln(R_1)/d\ln(M_1) \)
    - \( \alpha_{NS} \leq 0 \), while \( \alpha_{SQM} \geq 0 \ (\approx 1/3) \)

- Incorporate analysis to include GR (2PN, ...) orbital dynamics.
  - Extend the Roche lobe analysis from Newtonian to 2PN, ...
    - GR makes stable mass transfer easier.
  - Utilize pseudo-GR potential to account for innermost circular orbit changes as a function of mass ratio. Study effects on results for existence of stable mass transfer.

- Explore astrophysical consequences of differences in \( \alpha(M_1) \) in (1) merger time scales and (2) GW signals.
\[ M \sim (1 - 2) M_\odot \approx 2 \times 10^{33} \text{ g.} \]
\[ R \sim (8 - 16) \text{ km} \]
\[ \rho > 10^{15} \text{ g cm}^{-3} \]
\[ B_s = 10^9 - 10^{15} \text{ G.} \]
\[ \text{Tallest mountain:} \]
\[ \sim \frac{E_{\text{liq}}}{A m_p g_s} \sim 1\text{ cm} \]
\[ \text{Atmospheric height:} \]
\[ \sim \frac{RT}{\mu g_s} \sim 1\text{ cm} \]

Equation of State: $\alpha(M)$

\[ \alpha = \frac{d[\ln R]}{d[\ln M]} \]

\[ I_{NS} = (\approx 1/3) \]

\[ I_{SQM} \geq 0 \]

\[ \alpha_{NS} \leq 0 \]
Roche Lobe Overflow

- **Energy Loss**

\[ L_{GW} = \frac{1}{5} \langle \dddot{F}_{jk} \dddot{F}_{jk} \rangle = \frac{32}{5} a^4 \mu^2 \omega^6 \]

- **Angular Momentum Loss**

\[ \left( \dot{J}_{GW} \right)_i = \frac{2}{5} \epsilon_{ijk} \langle \dddot{F}_{jm} \dddot{F}_{km} \rangle = \frac{32}{5} a^4 \mu^2 \omega^5 \]

- **a(t) and V_{Roche} shrink!**

- **R_1 = r_{Roche}**

\[ \Rightarrow \text{Mass transfer begins!} \]

- **To merge or not to merge?**
Pseudo-GR Potentials

- Paczyński-Wiita (accretion disks)
  \[ \phi_N(r) = - \frac{M}{r} \quad \rightarrow \quad \phi_{PW}(r) = - \frac{M}{r - r_G} \]

- Innermost Circular Orbit (ICO) at \( r_{ICO} = 3r_G \); \( r_G = 2M \)

- Post-Newtonian (PN) : \( r_{ICO} < 3r_G \) for \( q \neq 0 \)

- Pseudo-GR or Hybrid Potential :
  \[ \phi_H(r) = - \frac{M}{r - \zeta(q)r_G} ; \quad q = \frac{M_1}{M_2} \]

- \( \zeta(q) \) - Mimics 2PN, 3PN Corrections to ICO
Test Particle Effective Potentials and ICO

L/M = 4.0

Newton

GR

pseudo-GR

L/M = 4.5

Circular Orbits

$r_{ICO} = 6M (= 3r_G)$
Roche Lobes: PW vs. 2PN

Approximation valid for \( R_{01} \ll R_{12} \)
Effective Roche Lobe Radii

Ratković, Prakash, & Lattimer (2005)

\[
\frac{r_{\text{Roche}}}{a} = Q(q) \ C(q,z)
\]

\[
Q(q) = \frac{0.49q^{2/3}}{0.57q^{2/3} + \ln(1+q^{1/3})}
\]

\[
C(q,z) = 1 + z(2.54q^{1/5} - 2.32)
\]

\[
Q(q) = \frac{0.49q^{2/3}}{0.57q^{2/3} + \ln(1+q^{1/3})}
\]

\[
C(q,z) = 1 + z(2.08q^{1/5} - 1.87)
\]
Orbital Evolution

Angular Momentum Loss:

\[
\frac{1 - q}{1 + q} + \frac{r_G q \zeta'(q)}{a - \zeta(q) r_G} \frac{\dot{q}}{q} + \frac{a - 3 \zeta(q) r_G}{2(a - \zeta(q) r_G) a} \frac{\dot{a}}{a} = - \frac{\dot{J}_{GW}}{J_{BS}} = - \frac{32}{5} a^2 \mu \omega^4
\]

Roche Lobe:

\[
\frac{\dot{q}}{q} = \frac{1 - \frac{\partial \ln C(q, z)}{\partial \ln z}}{\alpha(M_1) - \frac{\partial \ln Q(q) C'(q, z)}{\partial \ln q}} \times \frac{\dot{a}}{a}
\]

Connection to the dense matter EOS through

\[
\alpha(M_1) \equiv \frac{d \ln (R_1)}{d \ln (M_1)}
\]
Regions of stable mass transfer

Dashed curves: Lower mass limit to $M_2$ for stable mass transfer. Solid curve: Upper boundary for transfer beginning outside the ISCO.
Evolution: Orbit Separation $\alpha$

![Graphs showing orbit separation evolution for different models: AP4, GS1, MS0, PAL6, SQM1, SQM3. The graphs compare 2PN, N, PW, and Hyb. models over time.](image-url)
Evolution: Mass Ratio $q$

2PN, N, PW, Hyb.

AP4, GS1, MS0

PAL6, SQM1, SQM3

$t [M_\odot]$
Evolution: Angular Frequency $\omega$

- **2PN**
- **N**
- **PW**
- **Hyb.**

The plots show the evolution of angular frequency $\omega$ for different models and time scales $t$. Each graph compares the predicted angular frequency against time, highlighting the behavior of each model over a range of time periods.

- **AP4**
- **GS1**
- **MS0**
- **PAL6**
- **SQM1**
- **SQM3**

The graphs use different line styles and colors to distinguish between the models, providing a visual comparison of their predictions.
Evolution: Distance × Gravitational Amplitude $r h_+$

- 2PN
- N
- PW
- Hyb.

Graphs showing the evolution of $r h_+$ for different models (AP4, GS1, MS0, PAL6, SQM1, SQM3) over time, with time $t$ in units of $M_\odot$. The graphs compare the predictions of 2PN, N, PW, and Hyb. models.
Evolution: Normal Star \((APR)\)

\[
h_+ = \frac{4}{r} \omega^2 a^2 \mu \cos(2\omega t)
\]

- \(M = 4M_\odot, \ q_{ini} = 1/3\)
- GR speeds up evolution
- \(a(t)\) increases after "touchdown"
- \(\omega(t)\) stabilizes at long times
- Little variation among EOS’s of normal stars.
- \(M_1\) approaches the NS minimum mass; subsequent plunge (timescale \(\sim\) a few minutes) yields a second spike in the GW signal!
Evolution: $\textit{SQM}$ Star

$M = 4M_\odot, \quad q_{ini} = 1/3$

$\omega(t)$: “hovers” after “touchdown”

$\omega(t)$: relaxes to $\gg \omega_{\text{initial}}$

$h_+/x(t) \& q(t)$: exponential decay unlike for a NS

$M_{1,\text{final}} \rightarrow M^{SQM}_{\text{nugget}}$

unlike for a normal star; time to tiny $M_{1,\text{final}}$ is very long!

$$h_+ = \frac{4}{r} \omega^2 a^2 \mu \cos(2\omega t)$$
Major Results

- Incorporating GR into orbital dynamics leads to an evolution that is faster than the Newtonian evolution.

- Large differences exist between mergers of “normal” and “self-bound (SQM)” stars.
  - SQM stars penetrate to smaller orbital radii; stable mass transfer is more difficult than for normal stars.
  - For stable mass transfer, \( q = \frac{M_1}{M_2} \) and \( M = M_1 + M_2 \) limits on SQM stars are more restrictive than for normal stars.
  - The SQM case has exponentially decaying signal and mass, while normal star evolution is slower.
Future Tasks

- Evolution of normal & self-bound star-black hole mergers including the effects of
  - non-conservative mass transfer,
  - tidal synchronization,
  - the presence accretion disk, etc.

- Calculation of templates of expected GW signals
That's All Folks!