Nuclear & Particle Physics of Compact Stars

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Thermal Evolution of a Neutron Star
(Spherical, non-rotating & non-magnetic)

\[
\frac{dM}{dr} = 4\pi r^2 \epsilon; \quad \frac{dP}{dr} = -\frac{GM \epsilon}{c^2 r^2} \left[ 1 + \frac{P}{\epsilon} \right] \left[ 1 + \frac{4\pi r^3 P}{Mc^2} \right] e^{2\Lambda}
\]

\[
\frac{d}{dr} \left( T e^{\Phi/c^2} \right) = -\frac{3}{16\sigma} \frac{\kappa \rho}{T^3} \frac{L_d e^{\Phi/c^2} e^\Lambda}{4\pi r^2}
\]

\[
\frac{d\Phi}{dr} = \frac{G (M + 4\pi r^3 P/c^2)}{r^2} e^{2\Lambda}
\]

\[
\frac{d}{dr} \left( L_\nu e^{2\Phi/c^2} \right) = \epsilon_\nu e^{2\Phi/c^2} 4\pi r^2 e^\Lambda
\]

\[
\frac{d}{dr} \left( L e^{2\Phi/c^2} \right) = -c_v \frac{dT}{dt} e^{\Phi/c^2} 4\pi r^2 e^\Lambda, \quad \text{with} \quad \Lambda = \exp(1 - 2GM/c^2 r)^{-1/2}.
\]

\((P, \epsilon)\) : (Pressure, energy density) \quad \(M\) : Enclosed mass

\(\kappa\) : Opacity of matter \quad \(\Phi\) : Gravitational potential

\(L_d\) : Luminosity (thermal conductivity & radiation)

\((L_\nu, \epsilon_\nu)\) : Neutrino (luminosity, emissivity)

\(L = L_d + L_\nu\); Net luminosity

\(c_v\) : Specific heat/volume, \quad \text{Time} \ t \ \text{measured at} \ r = \infty.
Boundary Conditions

Inner boundary conditions:

\[ M(0) = L(0) = L_\nu(0) = 0 \]

Outer boundary conditions:

\[ P_s = \frac{2}{3} g_s / \kappa_s , \quad L_s = L_d(R) = 4\pi R^2 \sigma T_s^4 \]

\[ e^{\Phi/c^2} = \left( 1 - \frac{2GM}{c^2r} \right)^{-1/2} = e^{-\Lambda} \]

\[ g_s = (GM/R^2)e^{\Lambda_s} : \text{Surface gravity} \]

\[ (\kappa_s, T_S) : \text{Opacity and temperature at the surface} \]

Physics ingredients:

- Equation of state \( P = P(\epsilon) \)
- Opacity \( \kappa \) and specific heat \( c_v \)
- Photon and neutrino emissivites
Equation of State

Moderate variation with nucleons-only matter.

Specific Heat

Distribution of $C_v$ in the core among constituents

$$C_V = N(0) \frac{\hbar^2}{3} k_B T \quad N(0) = \frac{m^* P_F}{\pi^2 \hbar^3}$$

At $T=10^9$ K

$$C_V^{\text{paired}} = C_V^{\text{normal}} \times M(T/T_c)$$

$$\approx C_V^{\text{normal}} \times e^{-\Delta(T)/kT}$$

Neutrino Emission Processes

- The direct Urca process:

\[ n \rightarrow p + e^- + \bar{\nu}_e \quad \& \quad p + e^- \rightarrow n + \nu_e \]

cannot occur if the proton abundance is small as energy and momentum are not simultaneously conserved.

- For \( T << T_F \sim 10^{12} \text{ K} \), momenta \( \sim p_{F_i} \) for \( i = n, p \& e \). Neutrino and antineutrino momenta are \( \sim kT/c \ll p_{F_i} \).

- Chemical equilibrium requires \( \mu_n = \mu_p + \mu_e \).

Energy can be conserved for some states close to \( E_{F_i} \).

For momentum conservation, the three triangle inequalities

\[ p_{F_i} + p_{F_j} \geq p_{F_k} , \quad \text{where } i, j \& k \text{ are } p, e \& n , \]

must be satisfied failing which the modified Urca processes, featuring a bystander particle that enables momentum conservation, occur.
Threshold proton fraction

- Number densities: \( n_i = k_{F,i}^3 / (3\pi^2) \) for \( i \) (\( n, p \) or \( e \)).
- Proton fraction: \( x = n_p / (n_p + n_n) \).

At threshold, momentum conservation implies \( k_{Fn} = k_{Fp} + k_{Fe} \).

\[
x_c = \frac{k_{Fp}^3}{k_{Fp}^3 + (k_{Fp} + k_{Fe})^3} = \frac{1}{1 + (1 + k_{Fe}/k_{Fp})^3}.
\]

In charge neutral \( n, p \& e \) matter, \( n_p = n_e \), or \( k_{Fp} = k_{Fe} \).

- Hence, the proton fraction at threshold is \( x_c = 1/9 \).

In charge neutral \( n, p, e \& \mu \) matter, \( n_e + n_\mu = n_p \) and \( \mu_e = \mu_\mu \).

\[
k_{Fe}^3 + (k_{Fe}^2 - m_\mu^2)^{3/2} = k_{Fp}^3.
\]

For \( \mu_e = k_{Fe} \gg m_\mu \), one has \( k_{Fe} = k_{F\mu} = (1/2)^{1/3}k_{Fp} \), which gives

\[
x_c = \frac{1}{1 + (1 + 1/2^{1/3})^3} \approx 0.148.
\]
DURca Threshold Density-I

- Energy per baryon:

\[ E(n, x) = E(n, 1/2) + S_v(n)(1 - 2x)^2 + \cdots , \]

where \( S_v \) is the density dependent bulk symmetry energy; at \( n_s \simeq 0.16 \, \text{fm}^{-3} \), \( S_v(n_s) \equiv S_o \approx 27 - 36 \, \text{MeV} \).

- Beta equilibrium:

\[ \mu_e = \mu_n - \mu_p = -\frac{\partial E}{\partial x}. \]

- Equilibrium proton fraction:

\[ \hbar c (3\pi^2nx)^{1/3} = 4S_v(n)(1 - 2x), \]

The density, \( n_c \), at which \( x = x_c = 1/9 \), from

\[ S_v(n_c) = 51.2 \left( \frac{S_o}{30 \, \text{MeV}} \right) \left( \frac{n_c}{n_s} \right)^{1/3} \, \text{MeV}. \]
DUrca Threshold Density-II

Urca threshold densities

<table>
<thead>
<tr>
<th>$q$</th>
<th>$1/3$</th>
<th>$2/3$</th>
<th>$1$</th>
<th>$4/3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_c/n_s$</td>
<td>$25(9.7)$</td>
<td>$5.0(3.1)$</td>
<td>$2.2(1.8)$</td>
<td>$1.71(1.46)$</td>
</tr>
</tbody>
</table>

The quantity $n_c/n_s$ was calculated for power law symmetry energies $S_v \propto n^q$ using $S_o = 30(35)$ MeV.

- The critical density is sensitive to interactions and the magnitude of the symmetry energy.
- The case $q = 2/3$ and $S_0 = (1/3)(\hbar^2 k_F^2 / 2m) \approx 12.28$ MeV corresponds to free non–relativistic nucleons for which $n_c/n_s \approx 73$!
Models of Dense matter

- (a) Nuclear symmetry energy vs. density for different EOS's.

- (b) Equilibrium proton fractions including muons.

- Solid circles (squares): critical density for DURca for electrons (muons).

- Arrows(crosses): central density of $1.4M_\odot$ (maximum–mass) neutron stars.
Neutrino Emissivities-I

The $\bar{\nu}$ energy emission rate from neutron beta decay:

$$\epsilon_\beta = \frac{2\pi}{\hbar} 2 \sum_i G_F^2 (1 + 3g_A)^2 \ n_1 (1 - n_2)(1 - n_3)$$

$$E_4 \ \delta^{(4)}(p_1 - p_2 - p_3 - p_4),$$

- Including neutrinos from electron capture, $\epsilon_{Urca} = 2\epsilon_\beta$.

$$\epsilon_{Urca} = \frac{457\pi \ G_F^2 (1 + 3g_A^2)}{10080 \ \hbar^{10} c^5} \ m_n m_p \mu_e (kT)^6 \Theta_t$$

$$= 4.00 \times 10^{27} (Y_e n / n_s)^{1/3} T_9^6 \Theta_t \text{ erg cm}^{-3} \text{ s}^{-1},$$

$T_9$ : temperature in units of $10^9 \text{K},$
$n_s$ : 0.16 fm$^{-3},$
$Y_e = n_e / n$ : electron fraction, and
$\Theta_t = \theta (p_{Fe} + p_{Fp} - p_{Fn})$ :
- If the muon Urca process can occur, we gain another factor of 2.
Neutrino Emissivities-II

Effects of strong & weak interactions

- $n$ & $p$ density of states at $p_{F_{n,p}}$ renormalized: $m_{n,p} \to m_{n,p}^*$.  

- In-medium quenching of $g_A$: $|g_A| \to 1$

- Final state interaction modifications of weak interaction ME’s: small, since $n - p$ interactions small at momentum transfers $\sim p_F$.

- At best a reduction of $\sim 5 - 10$.

- Similar corrections for other $\nu -$ emissivities involving nucleons.

The time for a star’s center to cool:

$$\Delta t = - \int \frac{c_v}{\epsilon_{urca}} dT \simeq 30 T_9^{-4} \text{ s},$$

where $T$ is the temperature and $c_v$ is the specific heat per unit volume.

Rule of thumb for surace $T$ is $T_s/10^6 \text{ K} \simeq (T/10^8 \text{ K})^{1/2}$.
# Neutrino Emissivities-III

<table>
<thead>
<tr>
<th>Name</th>
<th>Process</th>
<th>Emissivity</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modiﬁed Urca</td>
<td>( n + n' \rightarrow n + p + e^- + \bar{\nu}_e )</td>
<td>( \sim 10^{20} T_9^8 )</td>
<td>Friman &amp; Maxwell 1979</td>
</tr>
<tr>
<td></td>
<td>( n' + p + e^- \rightarrow n' + n + \nu_e )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaon Condensate</td>
<td>( n + K^- \rightarrow n + e^- + \bar{\nu}_e )</td>
<td>( \sim 10^{24} T_9^6 )</td>
<td>Brown et al., 1988</td>
</tr>
<tr>
<td></td>
<td>( n + e^- \rightarrow n + K^- + \nu_e )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pion Condensate</td>
<td>( n + \pi^- \rightarrow n + e^- + \bar{\nu}_e )</td>
<td>( \sim 10^{26} T_9^6 )</td>
<td>Maxwell et al., 1977</td>
</tr>
<tr>
<td></td>
<td>( n + e^- \rightarrow n + \pi^- + \nu_e )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct Urca</td>
<td>( n \rightarrow p + e^- + \bar{\nu}_e )</td>
<td>( \sim 10^{27} T_9^6 )</td>
<td>Lattimer et al., 1991</td>
</tr>
<tr>
<td></td>
<td>( p + e^- \rightarrow n + \nu_e )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hyperon Urca</td>
<td>( B_1 \rightarrow B_2 + l + \bar{\nu}_l )</td>
<td>( \sim 10^{26} T_9^6 )</td>
<td>Prakash et al., 1992</td>
</tr>
<tr>
<td></td>
<td>( B_2 + l \rightarrow B_1 + \nu_l )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quark Urca</td>
<td>( d \rightarrow u + e^- + \bar{\nu}_e )</td>
<td>( \sim 10^{26} \alpha_c T_9^6 )</td>
<td>Iwamoto 1980</td>
</tr>
<tr>
<td></td>
<td>( u + e^- \rightarrow d + \nu_e )</td>
<td></td>
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</tbody>
</table>

\( T_9 \): Temperature in units of \( 10^9 \) K.
The star becomes isothermal in tens of years.

Direct versus Modified Urca

- Unlike MUrca, Durca exhibits threshold effects.
- Superfluidity abates DUrca cooling.
- Cooper pair breaking & reformation affects both DUrca & MUrca.
Inferred Surface Temperatures

New Cold Objects

Several cases fall below the “Minimal Cooling” paradigm & point to enhanced cooling, if these objects correspond to neutron stars.

Ongoing Work

Preparing to interpret the detection of really cold objects.

- A NS is yet to be seen!
- If rapid cooling occurs, when can thermal emission begin?
- W/O Cooper pair breaking & formation (CBF):
  \[ t_w \propto \left( \frac{R_{sh}}{1 \text{ km}} \right)^2 \times (1 - r_G)^{-3/2} \text{ yr} \simeq 10' \text{s of yr} \]
  (Lattimer et al., ApJ 425 802 (1994)).
- Time scales with CBF?
