QCD Phenomenology and Nucleon Structure

Stan Brodsky, SLAC

Lecture IV

National Nuclear Physics Summer School
Hadron Dynamics at the Amplitude Level

- DIS studies have primarily focussed on probability distributions: integrated and unintegrated.

- Test QCD at the amplitude level: Phases, multi-parton correlations, spin, angular momentum, exclusive amplitudes

- Impact of ISI and FSI: Single Spin Asymmetries, Diffractive Deep Inelastic Scattering, Shadowing, Antishadowing

- Wavefunctions on the light front: fundamental QCD dynamics of hadrons, nuclei

- Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Dirac’s Amazing Idea:  
The “Front Form”

Evolve in light-cone time!

\[
\begin{align*}
\sigma &= ct - z \\
\tau &= t + z/c
\end{align*}
\]

\[
\begin{align*}
\sigma &= ct - z \\
\tau &= t + z/c
\end{align*}
\]

Instant Form

Front Form
$P^+ = P^0 + P^z$

Fixed $\tau = t + z/c$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp i$

\[ \Psi_n (x_i, \vec{k}_\perp i, \lambda_i) \]

Invariant under boosts! Independent of $P^\mu$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_\perp i = \vec{0}_\perp$
‘Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
‘tis an immortal thing

Montgomery
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$\psi(x, k_{\perp})$

Invariant under boosts. Independent of $P^\mu$

$x_i = \frac{k_i^+}{P^+}$

$H_{LF}^{QCD}|\psi \rangle \geq M^2|\psi \rangle$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Mapping between LF(3+1) and AdS$_5$

\[ \psi(x, \vec{b}_\perp) = \sqrt{x(1-x)} \phi(\zeta) \]

\[ \zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \]

\[ \kappa = 0.77 \text{GeV} \]
Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

\[
\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta) \]

\[\zeta^2 = x(1-x)b^2_\perp.\]

Effective conformal potential:

\[V(\zeta) = -\frac{1 - 4L^2}{4\zeta^2}.\]

General solution:

\[\tilde{\psi}_{L,k}(x, \vec{b}_\perp) = B_{L,k} \sqrt{x(1-x)}\]

\[J_L \left( \sqrt{x(1-x)} |\vec{b}_\perp| \beta_{L,k} \Lambda_{QCD} \right) \theta \left( \vec{b}_\perp^2 \leq \frac{\Lambda_{QCD}^{-2}}{x(1-x)} \right),\]
AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$

$\Lambda_{QCD} = 0.32 \text{ GeV}$

$\kappa = 0.76 \text{ GeV}$

Truncated Space

Harmonic Oscillator
Deep Inelastic Lepton Proton Scattering

\[ q(x, Q^2) = \sum_n \int_{k_{\perp}^2 \leq Q_{\perp}^2} d^2k_{\perp} |\Psi_n(x, k_{\perp})|^2 \]

\( x = x_q \)

Imaginary Part of Forward Virtual Compton Amplitude

All spin, flavor distributions

Light-Front Wave Functions \( \psi_n(x_i, k_{\perp i}, \lambda_i) \)

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(a) Light Cone Fock Expansion

\[ |p\rangle = \psi_{uud} |uud\rangle + \psi_{uudg} |uudg\rangle + \ldots \]

\[ \langle p | qqq \rangle : \psi_{uud} \]

\[ \psi_n (x_i, \vec{k}_{i\perp}, \lambda_i) : \sum_{i=1}^{n} x_i = 1, \sum_{i=1}^{n} \vec{k}_{i\perp} = 0 \]

(b) Distribution Amplitude

\[ \phi^{(x,Q)}_{M} = \int d^{2}k_{\perp} \psi_2 \]

\[ m_n^2 < Q^2 \]

(d) Form Factors

\[ \langle l^+ (0) | p \lambda \rangle \]

\[ F_{\lambda \lambda'} (Q^2) = \sum_{n} \]

Large \( Q^2 \)

\[ \psi_n \]

\[ \psi_n \]

\[ p, \lambda \]

\[ p+q, \lambda' = \lambda \]

\[ T_H = \sum \]

\[ x_1 \]

\[ y_1 \]

\[ x_2 \]

\[ y_2 + \ldots \]

\[ x_3 \]

\[ y_3 \]
(f) Virtual Compton $\gamma^* p \rightarrow \gamma' p'$
$\langle p' \lambda' | J^\mu(z) J^\nu(0) | p \lambda \rangle$

Large $-q^2 = Q^2$

(g) Vector Meson Leptoproduction $\gamma^* p \rightarrow V^0 p'$

Large $-q^2 = Q^2$

(h) Weak Exclusive Decay $\langle D | J^+(0) | B \rangle$

$B^0 \rightarrow D^+$

$B^0 \rightarrow D^+$

$B^0 \rightarrow D^+$

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QCD Phenomenology

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Use Diffraction to Resolve Hadron Substructure

- Measure Light-Front Wavefunctions
- Test AdS/CFT predictions
- Novel Aspects of Hadron Wavefunctions: Intrinsic Charm, Hidden Color, Color Transparency/Opaqueness
- Diffractive Di-Jet Production
- Nuclear Shadowing and Antishadowing
- New Mechanism for Higgs Production
Fluctuation of a Pion to a Compact Color Dipole State

Color-Transparent Fock State For High Transverse Momentum Di-Jets

Same Fock State Determines Weak Decay

QCD Phenomenology

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Evaluation of QCD Matrix Elements: Example $f_\pi$

- Pion decay constant defined by the matrix element of EW current $J^{+}_W$:

$$\langle 0 | \bar{\psi}_u \gamma^+(1-\gamma_5)\psi_d | \pi^- \rangle = i\sqrt{2}P^+f_\pi,$$

with

$$|\pi^-\rangle = |du\rangle = \frac{1}{\sqrt{N_C}}\frac{1}{\sqrt{2}}\sum_{c=1}^{N_C}\left( b^\dagger_c d^\dagger_c u^\uparrow - b^\dagger_c d^\dagger_c u^\downarrow \right) |0\rangle.$$

- Use light-cone expression:

$$f_\pi = 2\sqrt{N_C}\int_{0}^{1}dx \int \frac{d^2k_\perp}{16\pi^3} \psi_{\bar{q}q/\pi}(x, k_\perp).$$


- Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{QCD}}{8J_1(\beta_{0,1})} = 83.4 \text{ Mev},$$

for $\Lambda_{QCD} = 0.2 \text{ GeV}$. Experiment: $f_\pi = 92.4 \text{ Mev}$. 
Predictions from AdS/CFT

\[ \psi(x, k_\perp) \]

\[ \kappa = 0.77 \text{GeV} \]

\[ F_2^A(q^2_\perp) \sim e^{-\frac{1}{3} \frac{R^2_A}{q^2_\perp}} \]
The \( x \) distribution of diffractive dijets from the platinum target for \( 1.25 \leq k_t \leq 1.5 \text{ GeV/c} \) (left) and for \( 1.5 \leq k_t \leq 2.5 \text{ GeV/c} \) (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.
Solving the LF Heisenberg Eqn.

- Discretized Light-Cone Quantization (DLCQ)  
  Minkowski space!  
  Pauli, sjb

- Many 1+1 model field theories completely solved using 
  DLCQ  Hornbostel, Pauli, sjb; Klebanov

- UV Regularization: 3+1 Pauli Villars  
  Hiller, McCartor, sjb

- Transverse Lattice  
  Bardeen, Peterson, Rabinovici, Burkardt, Dalley

- Bethe-Salpeter/Dyson-Schwinger at fixed LF time

- Angular Structure of Solutions known  
  Karmanov, Hwang, sjb

- Use AdS/CFT model solutions and AdS/LF 
  Equations as starting point!  Vary, de Teramond sjb
\[ H^{QCD}_{LC} |\psi_h\rangle = M^2_h |\psi_h\rangle \]

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Light-Front QCD Heisenberg Equation

DLCQ

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QCD Phenomenology

Pauli, Pinsky, sjb

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Structure function of boson constituent in 3+1 Yukawa theory

Three-particle Fock state truncation

Pauli-Villars Regularization

Hiller, McCartor, sjb

QCD Phenomenology

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Use AdS/CFT basis (complete and orthonormal) to diagonalize LF QCD Hamiltonian
GPDs & Deeply Virtual Exclusive Processes

“handbag” mechanism

Deeply Virtual Compton Scattering (DVCS)

\[ H(x, \xi, t), E(x, \xi, t), \ldots \]

\[ \xi = \frac{x_B}{2-x_B} \]

\[ x - \text{longitudinal quark momentum fraction} \]

\[ 2\xi - \text{longitudinal momentum transfer} \]

\[ \sqrt{t} - \text{Fourier conjugate to transverse impact parameter} \]
**Time-like Deeply Virtual Compton Scattering**

**Time-like Generalized Parton Distributions**

Interference of timelike DVCS amplitude with timelike form factor produces charge asymmetry

\[ e^+ e^- \rightarrow H^+ H^- \gamma \]

**QCD Phenomenology**

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**Time-like Deeply Virtual Compton Scattering**

**J=0 Fixed Pole**

**Signal for fundamental pointlike structure**

$e^+e^- \rightarrow H^+H^-\gamma$

Local “seagull” interaction of two photons at same point produces isotropic real amplitude, independent of photon virtuality at fixed pair mass

$$T_{seagull} = F(M^2_{HH})$$
Single-spin asymmetries

\[ \vec{S}_p \cdot \vec{q} \times \vec{p}_q \]

Sivers Effect

Hwang, Schmidt.

Light-Front Wavefunction
S and P- Waves

QCD S- and P-Coulomb Phases
Final State Interactions Produce T-Odd (Sivers Effect)

- Bjorken Scaling!

- Arises from Interference of Final-State Coulomb Phases in S and P waves

- Relate to the quark contribution to the target proton anomalous magnetic moment

- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero gravitoanomalous magnetic moment)

\[ \vec{S} \cdot \vec{p}_{jet} \times \vec{q} \]
In the context of the quark-parton model, the virtual-photon asymmetry $A_{UT}$ can be represented in terms of parton distributions.

The figure was taken from Ref.[9].

W.-D. Nowak / Nuclear Physics A 755 (2005) 325c–328c 327c

Sivers asymmetry from HERMES

Gamberg: Hermes data compatible with BHS model

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Key QCD Experiment at GSI

Measure single-spin asymmetry $A_N$ in Drell-Yan reactions

Leading-twist Bjorken-scaling $A_N$ from $S,P$-wave initial-state gluonic interactions

Predict: $A_N(DY) = -A_N(DIS)$ Opposite in sign!

$Q^2 = x_1 x_2 s$

$Q^2 = 4 \text{ GeV}^2, s = 80 \text{ GeV}^2$

$x_1 x_2 = .05, x_F = x_1 - x_2$
Measure Time-like T-odd SSA

Test both Sivers and Collins Effect in Quark Fragmentation

$e^+e^- \rightarrow \vec{V} \ jet \ X$

$\gamma^*$

$q$

$\rho^0 \rightarrow \pi^+\pi^-$

$X$

$\vec{\epsilon}_\rho \cdot \vec{q} \times \vec{p}_\rho$

$\epsilon_{\mu\nu\sigma\tau} \epsilon^\mu_V \ p^\nu_V \ p^\sigma_{\ jet} \ q^\tau$

Measure spin projection of detected hadron normal to production plane; use asymmetric B-factory
• Quarks Reinteract in Final State

• Analogous to Coulomb phases, but not unitary

• Observable effects: DDIS, SSI, shadowing, antishadowing

• Structure functions cannot be computed from LFWFs computed in isolation

• Wilson line not 1 even in lcg
First Evidence for Quark Structure of Matter

Deep Inelastic Electron-Proton Scattering

Gluonic Bremsstrahlung
DGLAP Evolution

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Production of new types of quarks from quantum fluctuations

strange, charm, bottom, top quarks and antiquarks

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Diffractive Deep Inelastic Scattering

Proton Remains Intact in Final State

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Remarkable observation at HERA

Fraction $r$ of events with a large rapidity gap, $\eta_{\text{max}} < 1.5$, as a function of $Q'^2_{\text{DA}}$ for two ranges of $x_{\text{DA}}$. No acceptance corrections have been applied.

In a large fraction (~10–15%) of DIS events, the proton escapes intact, keeping a large fraction of its initial momentum.

This leaves a large *rapidity gap* between the proton and the produced particles.

The $t$-channel exchange must be *color singlet $\rightarrow$ a pomeron??*
Final State Interaction Produces Diffractive DIS

Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHMPS)
Enberg, Hoyer, Ingelman, SJB
Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

QCD Phenomenology

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Integration over on-shell domain produces phase $i$

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target
Final State Interactions in QCD

Feynman Gauge

Light-Cone Gauge

Result is Gauge Independent
Conventional Model: Pomeron acts as constituent of proton

Problem: Wrong Phase
Real; must be imaginary

Need Final State Interactions!
QCD Mechanism for Rapidity Gaps

Wilson Line: \( \overline{\psi(y)} \int_0^y dx \, e^{iA(x) \cdot dx} \, \psi(0) \)

\[ q^+ = 0 \]

\[ \gamma^* \]

\[ \beta X_g \]

\[ (1-\beta) X_g \]

\[ X_g \]

\[ \delta x \approx \frac{1}{v} \]

\[ \text{color singlet} \]

\[ \text{Rap Gap} \]

\[ 1-2005 \]

\[ 8711A24 \]
Consequences for DDIS

- Underlying hard scattering sub-process is the same in diffractive and non-diffractive events
- Same $Q^2$ dependence of diffractive and inclusive PDFs (remember: hard radiation not resolved)
- and same energy ($W$ or $x_B$) dependence

$$\frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \text{ independent of } x_B \text{ and } Q^2 \text{ (as in data)}$$

- Also describes: vector meson leptoproduction

Note:
- In pomeron models the ratio depends on $x_B^{1-\alpha_P}$ which is ruled out
- In a two-gluon model with two hard gluons, the diffractive cross section depends on $[f_{g/p}(x_B, Q^2)]^2$
Rescattering gluons have small momenta
\[ \Rightarrow \beta \text{ dependence of diffractive PDFs arises from underlying (non-perturbative)} \quad g \rightarrow q\bar{q} \text{ and } g \rightarrow gg \]

**Effective IP** distribution and quark structure function:

\[ f_{IP/p}(x_{IP}) \propto g(x_{IP}, Q_0^2) \]
\[ f_{q/IP}(\beta, Q_0^2) \propto \beta^2 + (1 - \beta)^2 \]

Diffractive amplitudes from rescattering are dominantly *imaginary* — as expected for diffraction
(Ingelman–Schlein IP model has real amplitudes)

ZEUS data on cross section ratios

\[ \frac{d\sigma^{DDIS}}{d\beta dQ^2 dx_{IP}} / \frac{d\sigma^{DIS}}{dx dQ^2} = \frac{x_{IP} F_2^D(x_{IP}, x, Q^2)}{F_2(x_{IP}, x, Q^2)} \]

Same W dependence
The Pomereron formalism

$F_2^D$ is fitted to HERA data $\rightarrow$ good description

Lines given by fit with NLO QCD evolution
Sum Eikonal Interactions

Similar to Color Dipole Model

Lab Frame Picture

\[ \gamma^*(q) \rightarrow P_1 P_2 \]

\[ P \rightarrow P' \]

\[ k \]

\[ k_1 \]

\[ k_2 \]

\[ + \ldots \]

QCD Phenomenology

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• Quarks Reinteract in Final State

• Analogous to Coulomb phases, but not unitary

• Observable effects: DDIS, SSI, shadowing, antishadowing

• Structure functions cannot be computed from LFWFs computed in isolation

• Wilson line not 1 even in lcg
\[
Q^4 \frac{d\sigma}{d Q^2 d x_B} = \frac{\alpha_{\text{em}}}{16\pi^2} \frac{1 - y}{y^2} \frac{1}{2 M \nu} \int \frac{dp_2^-}{p_2^-} d^2 \vec{r}_T \, d^2 \vec{R}_T \, |\tilde{M}|^2
\]

where

\[
|\tilde{M}(p_2^-, \vec{r}_T, \vec{R}_T)| = \left| \frac{\sin \left[ g^2 W(\vec{r}_T, \vec{R}_T)/2 \right]}{g^2 W(\vec{r}_T, \vec{R}_T)/2} \tilde{A}(p_2^-, \vec{r}_T, \vec{R}_T) \right|
\]

is the resummed result. The Born amplitude is

\[
\tilde{A}(p_2^-, \vec{r}_T, \vec{R}_T) = 2e g^2 MQ p_2^- V(m_\parallel r_T) W(\vec{r}_T, \vec{R}_T)
\]

where \( m_\parallel^2 = p_2^- M x_B + m^2 \) and

\[
V(m r_T) \equiv \int \frac{d^2 p_T}{(2\pi)^2} \frac{e^{i \vec{r}_T \cdot \vec{p}_T}}{p_T^2 + m^2} = \frac{1}{2\pi} K_0(m r_T).
\]

The rescattering effect of the dipole of the \( q\bar{q} \) is controlled by

\[
W(\vec{r}_T, \vec{R}_T) \equiv \int \frac{d^2 k_T}{(2\pi)^2} \frac{1 - e^{i \vec{r}_T \cdot k_T}}{k_T^2} e^{i \vec{R}_T \cdot k_T} = \frac{1}{2\pi} \log \left( \frac{|\vec{R}_T + \vec{r}_T|}{R_T} \right).
\]

**Precursor of Nuclear Shadowing**

**FSI not Unitary Phase!**

**BHMPS**
Deep Inelastic Lepton Proton Scattering

Imaginary Part of Forward Virtual Compton Amplitude

\[ q(x, Q^2) = \sum_n \int_{k_{\perp}^2 \leq Q^2} d^2 k_{\perp} |\Psi_n(x, k_{\perp})|^2 \]

\[ x = x_q \]

All spin, flavor distributions

Light-Front Wave Functions \[ \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]
Photon Diffractive Structure Function

\[ e^+ + e^- \rightarrow V \gamma^* \]

\[ V^0 X \]

Diffractive deep inelastic scattering on a photon target

\[ k^2 \sim 0 \]

\[ T (\gamma^* \rightarrow H^+ H^- \gamma) \]
The Odderon

- Three-Gluon Exchange, $C=\text{\textminus}$, $J=1$, Nearly Real Phase

- Interference of 2-gluon and 3-gluon exchange leads to matter/antimatter asymmetries

- Asymmetry in jet asymmetry in $\gamma p \rightarrow c\bar{c}p$

- Analogous to lepton energy and angle asymmetry $\gamma Z \rightarrow e^+e^-Z$

- Pion Asymmetry in $\gamma p \rightarrow \pi^+\pi^-p$

The Odderon: Another source of antishadowing

Odderon: A collider test for another source of antishadowing

Merino, Rathsman, sjb
Anti-Shadowing

\[ Q^2 = 5 \text{ GeV}^2 \]


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Nuclear Shadowing in QCD

Shadowing depends on understanding diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF!

Dynamical effect due to virtual photon interacting in nucleus
The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken $x_B$:

$$\frac{1}{M x_B} = \frac{2\nu}{Q^2} \geq L_A.$$ 

If the scattering on nucleon $N_1$ is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\bar{q}$ flux reaching $N_2$.

→ Shadowing of the DIS nuclear structure functions.

HERA DDIS produces observed nuclear shadowing

QCD Phenomenology

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Integration over on-shell domain produces phase $i$

Need Imaginary Phase to Generate

Pomeron

Need Imaginary Phase to Generate

T-Odd Single-Spin Asymmetry

**Physics of FSI not in Wavefunction of Target**

Shadowing depends on understanding diffraction in DIS
The one-step and two-step processes in DIS on a nucleus.

If the scattering on nucleon $N_1$ is via $C = -\text{Reggeon or Odderon exchange}$, the one-step and two-step amplitudes are **constructive in phase**, enhancing the $q$ flux reaching $N_2$

$\rightarrow$ **Antishadowing** of the DIS nuclear structure functions
Phase of two-step amplitude relative to one step:

\[ \frac{1}{\sqrt{2}}(1 - i) \times i = \frac{1}{\sqrt{2}}(i + 1) \]

Constructive Interference

Depends on quark flavors!

Thus antishadowing is not universal

Different for couplings of \( \gamma^*, Z^0, W^\pm \)
Shadowing and Antishadowing in Lepton-Nucleus Scattering

- Shadowing: Destructive Interference of Two-Step and One-Step Processes. *Pomeron Exchange*

- Antishadowing: Constructive Interference of Two-Step and One-Step Processes! *Reggeon and Odderon Exchange*

- Antishadowing is Not Universal! Electromagnetic and weak currents: different nuclear effects! *Potentially significant for NuTeV Anomaly*
Antiquark Interacts with Target Nucleus at Effective Energy $\hat{s} \propto 1/x_{B,j}$

$$\sigma_{\bar{q}N} \sim \hat{s}^{\alpha_R - 1} \rightarrow F_{2N}(x_{b,j}) \sim x^1 - \alpha_R$$ at small $x_{b,j}$

Shadowing of antiquark-nucleus cross section $\sigma_{\bar{q}A} \sim A^\alpha$

produces same $A$ dependence of nuclear structure function
Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior
The nuclear shadowing and antishadowing effects at $\langle Q^2 \rangle = 1$ GeV$^2$.

Shadowing and Antishadowing of DIS Structure Functions

Nuclear Effect not Universal!
Inspired by the above relation, we will examine nuclear effects on sin²θW by the following observable for the process νµ + A → νµ + X:

\[ R_{\nu}^Z (x) = \frac{d\sigma(\nu\mu + A \rightarrow \nu\mu + X)}{dx} / \frac{d\sigma(\nu\mu + N \rightarrow \nu\mu + X)}{dx} \]

Ratios \( F_2^A / F_2^N \) (solid curves) and \( F_3^A / F_3^N \) (dashed curves)
Model predictions

\[ R_Z^\nu(x) = \frac{d\sigma(\nu_\mu + A \rightarrow \nu_\mu + X)/dx}{d\sigma(\nu_\mu + N \rightarrow \nu_\mu + X)/dx} \]

- Bigger antishadowing for $\bar{\nu}$
- Different NC-CC effects only for $\bar{\nu}$

QCD Phenomenology

Stan Brodsky, SLAC
Coherence of multiscattering nuclear processes

Different antishadowing for

\[ \text{Estimate 20\% effect on extraction of } \sin^2 \theta_W \text{ for NuTeV} \]

Need new experimental studies of antishadowing in

- Parity-violating DIS
- Spin Dependent DIS
- Charged and Neutral Current DIS

Different antishadowing for

\{ \text{Shadowing, Antishadowing} \}

\{ \text{Neutral currents, Charged currents, Electromagnetic currents} \}
Nuclear Shadowing and Anti-Shadowing in QCD

- Relation to Diffractive DIS and Final-State Interactions
- Novel Color Effects
- Non-Universality of Antishadowing
- Implications for NuTeV


Hard Diffraction from Rescattering

Unification:
- Diffractive Deep Inelastic Scattering (DDIS)
- Nuclear Shadowing & Antishadowing
- Single Spin Asymmetries (Sivers Effect)
- Diffractive Di-jets, Tri-jets
- Fundamental Features of Gauge Theory, Color
Novel Diffractive Phenomena and New Insights Into QCD from AdS/CFT

- Ashery Diffractive Di-Jet Production:
  - First measurement of hadron wavefunction
  - Verification of QCD Color Transparency
  - Related phenomena: Diffractive deep inelastic scattering and vector meson electroproduction
  - Nuclear shadowing and antishadowing
  - New “Exclusive Diffractive Mechanism” for high $x_F$ Higgs Production
\[ |p, S_z > = \sum_{n=3}^{\infty} \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) |n, \vec{k}_\perp i, \lambda_i > \]

**sum over states with \( n=3, 4, \ldots \) constituents**

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k^+ i}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z} \]

are boost invariant.

\[ \sum_i k^+ i = P^+, \sum_i x_i = 1, \sum_i \vec{k}_\perp i = 0_\perp. \]

**Intrinsic glue, sea quarks, charm, bottom**

\[ Fixed \ LF \ time \]
Hadrons Fluctuate in Particle Number

- Proton Fock States
  \[ |uud\>, |uudg\>, |uuds\bar{s}\>, |uudc\bar{c}\>, |uudb\bar{b}\> \ldots \]

- Strange and Anti-Strange Quarks not Symmetric
  \[ s(x) \neq \bar{s}(x) \]

- “Intrinsic Charm”: High momentum heavy quarks

- “Hidden Color”: Deuteron not always \( p + n \)

- Orbital Angular Momentum Fluctuations - Anomalous Magnetic Moment
|\text{uudc}\bar{c} > \text{Fluctuation in Proton} \\
\text{QCD: Probability } \sim \frac{\Lambda_{QCD}^2}{M_Q^2} \\
|e^+e^-\ell^+\ell^- > \text{Fluctuation in Positron} \\
\text{QED: Probability } \sim \frac{(m_e\alpha)^4}{M_\ell^4} \\

\text{OPE derivation - M.Polyakov et al.} \\
\text{c}\bar{c} \text{ in Color Octet} \\

\text{Distribution peaks at equal rapidity (velocity)} \\
\text{Therefore heavy particles carry the largest momentum fractions} \\

\textbf{High x charm!}
\[ |p, S_z > = \sum_{n=3}^{\infty} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i > \]

**sum over states with n=3, 4, ...constituents**

The Light Front Fock State Wavefunctions
\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]
are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction
\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z} \]
are boost invariant.

\[ \sum_i k_i^+ = P^+, \sum_i x_i = 1, \sum_i \vec{k}_{\perp i} = \vec{0}_\perp. \]

**Intrinsic glue, sea quarks, charm, bottom**

**Fixed LF time**
HADRONS: complex relativistic system

Fluctuations in particle no., size, spin, color

Gluing intrinsic to hadron structure

\[ \bar{u}(x) \neq \bar{d}(x) \Rightarrow \text{correlations} \]

\[ S(x) \neq \bar{S}(x) ? \]

Sea not from gluon splitting alone

\[ q(x) = \bar{q}(x) \]

\[ \bar{u}(x) = \bar{d}(x) \]

"Hidden color" in nuclei,

\[ \psi_d \neq \psi_n \times \psi_p \]
Measure $c(x)$ in Deep Inelastic Lepton-Proton Scattering
QCD Phenomenology

Stan Brodsky, SLAC

July 2006

NNPSS

75
Two components to charm structure function

\[ DGLAP \]

*Extrinsic*

\[ c(x) \sim (1-x)^6 \]

\[ g(x) \]

*Intrinsic (high x)*

\[ c(x) \text{ peaks at } x \approx 0.4 \]

\[ P_{\text{jet}} \sim \frac{\log \frac{Q^2}{m_c^2}}{m_c^2} \]

\[ P_{\text{jet}} \sim \frac{C(x)}{m_c^2} \]

*non-abelson effect!*

\[ c(x) \]
Measurement of Charm Structure Function


Evidence for Intrinsic Charm

DGLAP / Photon-Gluon Fusion Factor of 30 too small
Predictions for Inclusive Charm Production Distributions at the ISR. Assumes active and spectator charm distribution in proton patterned on IC, plus coalescence of valence and charm quarks.

- EMC data: $c(x, Q^2) > 30 \times \text{DGLAP}$
  $Q^2 = 75 \text{ GeV}^2$, $x = 0.42$

- High $x_F \ pp \rightarrow J/\psi X$

- High $x_F \ pp \rightarrow J/\psi J/\psi X$

- High $x_F \ pp \rightarrow \Lambda_c X$

- High $x_F \ pp \rightarrow \Lambda_b X$

- High $x_F \ pp \rightarrow \Xi(ccd) X \ (\text{SELEX})$
Diffractive Dissociation of Intrinsic Charm

Coalescence of Comoving Charm and Valence Quarks Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$
Production of Color-Octet IC Fock State

Scattering on Nucleon via one Gluon

Coalescence of Color-Singlet Pair into Charmonium State

QCD Phenomenology
Shadowing of $pA \rightarrow J/\Psi X$

$J/\Psi$ Production on Front Surface
No Absorption of Propagating $J/\Psi$

$\sigma(p + A \rightarrow J/\Psi + X) \propto A^{2/3}$

Elastic scattering of IC Fock state:

$|uud\rangle_{8C}|c\bar{c}\rangle_{8C} > + N_1 \rightarrow |uud\rangle_{8C}|c\bar{c}\rangle_{8C} > + N_1$

followed by:

$|uud\rangle_{8C}|c\bar{c}\rangle_{8C} > + N_2 \rightarrow J/\Psi + X$

Depleted flux on downstream nucleons

Color-Opaque
Color-Octet Intrinsic Charm!
Remarkably Strong Nuclear Dependence for Fast Charmonium

M. Leitch
Nuclear effects in Quarkonium production

\[ p + A \text{ at } s^{1/2} = 38.8 \text{ GeV} \]

\[ \sigma(p+A) = A^\alpha \sigma(p+N) \]

Strong \( x_F \) - dependence

Nuclear effects scale with \( x_F \), not \( x_2 \) !!!

M. Leitch

QCD Phenomenology

Stan Brodsky, SLAC
Nuclear Dependence of Quarkonium Production

NA3 data for $\frac{d\sigma}{dx_F}(p(\pi)A \rightarrow J/\psi X)$: hard $A^1$ and “diffractive” $A^{2/3}$ components

Diffractive contribution extends to large $x_F$

$A^\alpha(x_F)$ not $A^\alpha(x_2)$: PQCD Factorization Violated!
Hard Component $\frac{d\sigma}{dx_F}(p(\pi)A \rightarrow J/\psi X)$

The fit: $gg$ fusion (dashed)
$q\bar{q}$ fusion (dashed-dot)
total (solid)

$A^1$ Component
• IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \rightarrow J/\psi X$
(Mueller, Gunion, Tang, SJB)

• Color Octet IC Explains $A^{2/3}$ behavior at high $x_F$ (NA3, Fermilab)
(Kopeliovitch, Schmidt, Soffer, SJB)

• IC Explains $J/\psi \rightarrow \rho\pi$ puzzle
(Karliner, SJB)

• IC leads to new effects in $B$ decay
(Gardner, SJB)

Color Opaqueness
Double Charmonium Production

\[ \pi A \rightarrow J/\psi J/\psi X \]

Intrinsic charm contribution to double quarkonium hadroproduction

R. Vogt, S.J. Brodsky

The probability distribution for a general \( n \)-particle intrinsic \( c\bar{c} \) Fock state as a function of \( x \) and \( k_{\perp} \) is written as

\[
\frac{dP_{ic}}{\prod_{i=1}^{n} dx_{i} d^{2}k_{T,i}} = N_{n} \alpha_{s}^{4}(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^{n} k_{T,i}) \delta(1 - \sum_{i=1}^{n} x_{i})}{(m_{h}^{2} - \sum_{i=1}^{n} (m_{T,i}^{2}/x_{i}))^{2}},
\]

The \( \psi \bar{\psi} \) pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of \( J/\psi \)'s from the pions are shown in (b) and (d). Our calculations are compared with the \( \pi^{-}N \) data at 150 and 280 GeV/c [1]. The \( x_{J/\psi} \) distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single \( J/\psi \)'s is twice the number of pairs.

NA3 Data

NNPSS
July 2006

QCD Phenomenology

Stan Brodsky, SLAC
Double Intrinsic Charm

Production of a Double-Charm Baryon
Intrinsic Charm Mechanism for Exclusive Diffraction Production

\[ p \ p \rightarrow J/\psi \ p \ p \]

\[ x_{J/\psi} = x_{c} + x_{\bar{c}} \]

Exclusive Diffractive High-\(X_{F}\) Higgs Production

Kopeliovitch, Schmidt, Soffer, sjb

Intrinsic \(c\bar{c}\) pair formed in color octet 8\(C\) in proton wavefunction

Large Color Dipole

Collision produces color-singlet \(J/\psi\) through color exchange

RHIC Experiment

QCD Phenomenology

Stan Brodsky, SLAC
Intrinsic Charm Mechanism for Exclusive Diffraction Production

Kopeliovitch, Schmidt, Soffer, sjb
Anomalous QCD Effects

- Hidden Color of Nuclear Wavefunction
- Odderon Trajectory: Charm jet asymmetry
- Anomalous Regge Behavior: J=0 Fixed Pole
- Proton-Proton Scattering: Color Transparency Breakdown and $A_{NN}$
- Non-Universality of Antishadowing
- Intrinsic Heavy Quarks at large $x$
- Anomalous scaling of single-particle inclusive at high $p_T$
Conformal symmetry: Template for QCD

- Initial approximation to PQCD; then correct for non-zero beta function and quark masses

- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

- Arguments for Infrared fixed-point for $\alpha_s$

- Effective Charges: analytic at quark mass thresholds, finite at small momenta

- Eigensolutions of Evolution Equation of distribution amplitudes
The Renormalization Scale Problem

\[ \rho = C_0 \alpha_s(Q) \left[ 1 + C_1(Q) \frac{\alpha_s(Q)}{\pi} \right. \\
+ \left. C_2(Q) \frac{\alpha_s^2(Q)}{\pi^2} + \cdots \right] \]

How does one set renormalization scale \( Q \)?

QCD Phenomenology

Stan Brodsky, SLAC
\( e^+ e^- \rightarrow \mu^+ \mu^- \)

\[ \mu^2 \equiv s \]

**Scale of \( \alpha_{QED}(\mu^2) \) unique!**

**The QED Effective Charge**

- Complex
- Analytic through mass thresholds
- Distinguishes between timelike and spacelike momenta

**Analyticity essential!**
Electron-Electron Scattering in QED

\[ M_{ee\to ee}(++; ++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u) \]

- No renormalization scale ambiguity!
- Two separate physical scales.
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.
- If one chooses a different scale, one must sum an infinite number of graphs -- but then recover same result!
- Number of active leptons correctly set
- **Analytic:** reproduces correct behavior at lepton mass thresholds
The Renormalization Scale Problem

• No renormalization scale ambiguity in QED

• Gell Mann-Low-Dyson QED Coupling defined from physical observable;

• Sums all Vacuum Polarization Contributions

• Renormalization Scale in QED scheme: Identical to Photon Virtuality

• Analytic: Reproduces lepton-pair thresholds

• Examples: muonic atoms, g-2, Lamb Shift

• Time-like and Space-like QED Coupling related by analyticity

• Uses Dressed Skeleton Expansion

M. Binger, sjb
Lessons from QED: Summary

• Effective couplings are complex analytic functions with the correct threshold structure expected from unitarity

• Multiple “renormalization” scales appear

• The scales are unambiguous since they are physical kinematic invariants

• Optimal improvement of perturbation theory
Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb


• All terms associated with nonzero beta function summed into running coupling

• Resulting series identical to conformal series

• Renormalon n! growth of PQCD coefficients from beta function eliminated!

• In general, BLM scale depends on all invariants
BLM Scale Setting

\[ \rho = C_0 \alpha_{\overline{\text{MS}}}(Q) \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \left( -\frac{3}{2} \beta_0 A_{\text{VP}} + \frac{33}{2} A_{\text{VP}} + B \right) + \cdots \right] \]

by

\[ \rho = C_0 \alpha_{\overline{\text{MS}}}(Q^*) \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q^*)}{\pi} C_1^* + \cdots \right] , \]

where

\[ Q^* = Q \exp(3A_{\text{VP}}) , \]
\[ C_1^* = \frac{33}{2} A_{\text{VP}} + B . \]

The term \( 33A_{\text{VP}}/2 \) in \( C_1^* \) serves to remove that part of the constant \( B \) which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by \( \beta_0 = 11 - \frac{2}{3} n_f \).

Use \( n_f \) dependence at NLO to identify \( A_{\text{VP}} \)

Use skeleton expansion

Gardi, Rathsman, sjb

Conformal Coefficient
\[ R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right]. \]

\[ R_{e^+e^-}(Q^2) = 3 \sum_{q} e_q^2 \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} + \frac{\alpha_{\overline{\text{MS}}}^2}{\pi^2} (1.98 - 0.115n_f) \right. \]

\[ + \cdots \left. \right] \]

\[ \rightarrow 3 \sum_{q} e_q^2 \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q^*)}{\pi} + \frac{\alpha_{\overline{\text{MS}}}^2(Q^*)}{\pi^2} 0.08 \right. \]

\[ + \cdots \left. \right] , \]

\[ Q^* = 0.710Q. \] Notice that \( \alpha_R(Q) \) differs from \( \alpha_{\overline{\text{MS}}}(Q^*) \) by only \( 0.08\alpha_{\overline{\text{MS}}}/\pi \), so that \( \alpha_R(Q) \) and \( \alpha_{\overline{\text{MS}}}(0.71Q) \) are effectively interchangeable (for any value of \( n_f \)).
Deep-inelastic scattering. The moments of the nonsinglet structure function \( F_2(x, Q^2) \) obey the evolution equation

\[
Q^2 \frac{d}{dQ^2} \ln M_n(Q^2) = - \frac{\gamma_n^{(0)}}{8\pi} \alpha_{\text{MS}}(Q) \left[ 1 + \frac{\alpha_{\text{MS}}}{4\pi} \frac{2\beta_0 \beta_n + \gamma_n^{(1)}}{\gamma_n^{(0)}} + \cdots \right]
\]

\[
\rightarrow - \frac{\gamma_n^{(0)}}{8\pi} \alpha_{\text{MS}}(Q^*_n) \left[ 1 - \frac{\alpha_{\text{MS}}(Q^*_n)}{\pi} C_n + \cdots \right],
\]

where, for example,

\[
Q_2^* = 0.48Q,\quad C_2 = 0.27,
\]

\[
Q_{10}^* = 0.21Q,\quad C_{10} = 1.1.
\]

For \( n \) very large, the effective scale here becomes \( Q_n^* \sim Q/\sqrt{n} \)

BLM scales for DIS moments
Three-Jet Rate

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in $e^+e^-$ annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low $y$. In particular, the latter two methods predict increasing values of $\mu$ as the jet invariant mass $M < \sqrt{(ys)}$ decreases.

Other Jet Observables:

Kramer & Lampe

Rathsman
Features of BLM Scale Setting

- All terms associated with nonzero beta function summed into running coupling
- Conformal series preserved
- BLM Scale $Q^*$ sets the number of active flavors
- Correct analytic dependence in the quark mass
- Only $n_f$ dependence required to determine renormalization scale at NLO
- Result is scheme independent: $Q^*$ has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit!
\[ \lim_{N_C \to 0} \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F \]

QCD → Abelian Gauge Theory

Analytic Feature of SU(Nc) Gauge Theory

Huet, sjb
Relate Observables to Each Other

• Eliminate intermediate scheme
• No scale ambiguity
• Transitive!
• Commensurate Scale Relations
• Example: Generalized Crewther Relation
\[
\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\text{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\text{MS}}(Q)}{\pi}\right)^2 \left[ \left(\frac{41}{8} - \frac{11}{3} \zeta_3\right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3} \zeta_3\right) f \right] \\
+ \left(\frac{\alpha_{\text{MS}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{90445}{2592} - \frac{2737}{108} \zeta_3 - \frac{55}{18} \zeta_5 - \frac{121}{432} \pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12} \zeta_3 + \frac{55}{3} \zeta_5\right) C_A C_F - \frac{23}{32} C_F^2 \\
+ \left[ \left(-\frac{970}{81} + \frac{224}{27} \zeta_3 + \frac{5}{9} \zeta_5 + \frac{11}{108} \pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6} \zeta_3 - \frac{10}{3} \zeta_5\right) C_F \right] f \\
+ \left(\frac{151}{162} - \frac{19}{27} \zeta_3 - \frac{1}{108} \pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6} \zeta_3\right) \frac{d^{abc} d^{abc}}{C_F d(R)} \left(\sum_f Q_f\right)^2 \right\}. 
\]

\[
\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\text{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\text{MS}}(Q)}{\pi}\right)^2 \left[ \frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f \right] \\
+ \left(\frac{\alpha_{\text{MS}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{5437}{648} - \frac{55}{18} \zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9} \zeta_3\right) C_A C_F + \frac{1}{32} C_F^2 \\
+ \left[ \left(-\frac{3535}{1296} - \frac{1}{2} \zeta_3 + \frac{5}{9} \zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18} \zeta_3\right) C_F \right] f + \frac{115}{648} f^2 \right\}. 
\]
\[ \int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right] \]

\[ \frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left( \frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left( \frac{\alpha_R(Q^{***})}{\pi} \right)^3 \]

**Geometric Series in Conformal QCD**

**Generalized Crewther Relation**

add Light-by-Light

Lu, Kataev, Gabadadze, Sjb
Generalized Crewther Relation

\[ \left[ 1 + \alpha_R \left( \frac{s^*}{\pi} \right) \right] \left[ 1 - \frac{\alpha g_1 (q^2)}{\pi} \right] = 1 \]

\( \sqrt{s^*} \approx 0.52Q \)

Conformal relation true to all orders in perturbation theory
PMS violates transitivity

Relation between observables $A \leftrightarrow B$

Independent of choice of scheme or theoretical convention!
Commensurate Scale Relation:

\[ \alpha_s(Q_B) = \alpha_s(Q_A) \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{Q_B}{Q_A} \right)^{\lambda B/A} + \ldots \right] \]

\[ \frac{Q_B}{Q_A} = \lambda B/A \]

Peterman
\begin{align*}
\lambda B/A &= \frac{\lambda b/c}{\lambda a/c} & \text{transition} \\
\lambda b/a &= \lambda a/b & \text{symmetry} \\
\lambda a/a &= 1 & \text{idem} \\
\end{align*}

Renormalisation "Group"
Leading Order Commensurate Scales

Translate between schemes at LO
Production of four heavy-quark jets

\( T(\gamma^* \rightarrow Q\bar{Q}Q\bar{Q}) \propto \alpha_{4Q}(k_g^2) \)

Defines analytic QCD effective charge

time-like values not same as space-like
coupling similar to “pinch” scheme
complex for time-like argument
Unification in Physical Schemes

“PHYSICAL RENORMALIZATION SCHEMES AND GRAND UNIFICATION”

\[
\alpha_i(Q) = \frac{\alpha_i(Q_0)}{1 + \hat{\Pi}_i(Q) - \hat{\Pi}_i(Q_0)}
\]
i=1,2,3

\[
\hat{\Pi}_i(Q) = \frac{\alpha_i}{4\pi} \sum_p \beta_i^{(p)} \left( L_{s(p)} \left( \frac{Q^2}{m_p^2} \right) + \cdots \right)
\]

“log-like” function:

\[
L_{s(p)} \approx \log(e^{\eta_p} + \frac{Q^2}{m_p^2})
\]

\[
\eta_p = \frac{8}{3}, \frac{5}{3}, \frac{40}{21}
\]

For spin \( s(p) = 0, \frac{1}{2}, \) and 1

Elegant and natural formalism for all threshold effects
The Pinch Technique

(Cornwall, Papavassiliou)

Gauge-invariant gluon self-energy!

Gauge-dependent

\[ q \cdot V(p, k) = S^{-1}(p) - S^{-1}(k) \]

Gauge-dependent

Natural generalization of QED charge
Asymptotic Unification

\[ \alpha_i^{-1}(Q) \]

\[ Q(\text{GeV}) \]

Binger, sjb

QCD Phenomenology

Stan Brodsky, SLAC
Analyticity and Mass Thresholds

$\overline{MS}$ does not have automatic decoupling of heavy particles

Must define a set of schemes in each desert region and match

$$\alpha_s^{(f)}(M_Q) = \alpha_s^{(f+1)}(M_Q)$$

• The coupling has **discontinuous derivative** at the matching point

• At higher orders the coupling itself becomes **discontinuous**!

• Does not distinguish between spacelike and timelike momenta

"AN ANALYTIC EXTENSION OF THE MS-BAR RENORMALIZATION SCHEME"

Unification in Physical Schemes

• Smooth analytic threshold behavior with automatic decoupling

• More directly reflects the unification of the forces

• Higher “unification” scale than usual
General Structure of the Three-Gluon Vertex

3 index tensor \( \hat{\Gamma}_{\mu_1\mu_2\mu_3} \) built out of \( g_{\mu\nu} \) and \( p_1, p_2, p_3 \)

with \( p_1 + p_2 + p_3 = 0 \)

Full calculation, general masses, spin

14 basis tensors and form factors

"THE FORM-FACTORS OF THE GAUGE-INARIANT THREE-GLUON VERTEX"

Binger, sjb

Cornwall and Papavassiliou performed the PT construction:

The "pinched" parts are added to the "regular" 3 gluon vertex

Later shown to = BFMFG

Integrals were not evaluated…

gauge dependent

gauge invariant
<table>
<thead>
<tr>
<th>Massless</th>
<th>Massive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_G + 4F_Q + (10-d)F_S = 0$</td>
<td>$F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$</td>
</tr>
<tr>
<td>$\Sigma_{QG}(F) \equiv \frac{d-2}{2}F_Q + F_G$ = simple</td>
<td>$\Sigma_{MQG}(F) \equiv \frac{d-1}{2}F_{MQ} + F_{MG}$ = simple</td>
</tr>
</tbody>
</table>
Multi-scale Renormalization of the Three-Gluon Vertex

\[ \tilde{g}(p_1^2, p_2^2, p_3^2) \]

\[ g(p_1^2) \quad g(p_2^2) \quad g(p_3^2) \]

gauge-invariant subset of rad. cor.

coupling at each vertex absorb the rad. cor.

QCD Phenomenology
3 Scale Effective Charge

\[ \tilde{\alpha}(a,b,c) \equiv \frac{g^2(a,b,c)}{4\pi} \]  
(First suggested by H.J. Lu)

\[
\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi} \beta_0 \left( \frac{1}{\varepsilon} L(a,b,c) + \cdots \right)
\]

\[
\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi} \beta_0 \left[ L(a,b,c) - L(a_0,b_0,c_0) \right]
\]

\[ L(a,b,c) = 3\text{-scale “log-like” function} \]

\[ L(a,a,a) = \log(a) \]
3 Scale Effective Scale

\[ L(a, b, c) \equiv \log(Q_{\text{eff}}^2(a, b, c)) + i \text{Im} L(a, b, c) \]

Governed strength of the three-gluon vertex

\[
\frac{1}{\tilde{\alpha}(a, b, c)} = \frac{1}{\tilde{\alpha}(a_0, b_0, c_0)} + \frac{1}{4\pi} \beta_0 \left[ L(a, b, c) - L(a_0, b_0, c_0) \right]
\]

\[
\hat{\Gamma}_{\mu_1 \mu_2 \mu_3} \propto \sqrt{\tilde{\alpha}(a, b, c)}
\]

Generalization of BLM Scale to 3-Gluon Vertex
Properties of the Effective Scale

\[ Q_{\text{eff}}^2 (a, b, c) = Q_{\text{eff}}^2 (-a, -b, -c) \]

\[ Q_{\text{eff}}^2 (\lambda a, \lambda b, \lambda c) = |\lambda| Q_{\text{eff}}^2 (a, b, c) \]

\[ Q_{\text{eff}}^2 (a, a, a) = |a| \]

\[ Q_{\text{eff}}^2 (a, -a, -a) \approx 5.54 |a| \]

\[ Q_{\text{eff}}^2 (a, a, c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c| \]

\[ Q_{\text{eff}}^2 (a, -a, c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c| \]

\[ Q_{\text{eff}}^2 (a, b, c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c| \]

Surprising dependence on Invariants
The Effective Scale

\( Q_{\text{eff}}^2 (10 \text{ GeV}^2, 10 \text{ GeV}^2, p^2) \)

\( Q_{\text{eff}}^2 (-10 \text{ GeV}^2, -10 \text{ GeV}^2, p^2) \)

\( Q_{\text{eff}}^2 (10 \text{ GeV}^2, p^2, p^2) \)

\( Q_{\text{eff}}^2 (-10 \text{ GeV}^2, p^2, p^2) \)
Future Directions

Gauge-invariant four gluon vertex

\[ L_4(p_1, p_2, p_3, p_4) \]

\[ Q_{4\text{eff}}^2(p_1, p_2, p_3, p_4) \]

Hundreds of form factors!
Summary and Future

• **Multi-scale analytic** renormalization based on **physical, gauge-invariant** Green’s functions

• **Optimal** improvement of perturbation theory with **no scale-ambiguity** since physical kinematic invariants are the arguments of the (multi-scale) couplings
Conventional renormalization scale-setting method:

- Guess arbitrary renormalization scale and take arbitrary range. Wrong for QED and Precision Electroweak.
- Prediction depends on choice of renormalization scheme.
- Variation of result with respect to renormalization scale only sensitive to nonconformal terms; no information on genuine (conformal) higher order terms.
- Conventional procedure has no scientific basis.
- FAC and PMS give unphysical results.
- Renormalization scale not arbitrary: Analytic constraint from flavor thresholds.
Use Physical Scheme to Characterize QCD Coupling

- Use Observable to define QCD coupling or Pinch Scheme
- Analytic: Smooth behavior as one crosses new quark threshold
- New perspective on grand unification

Binger, Sjb
Factorization scale

\[ \mu_{\text{factorization}} \neq \mu_{\text{renormalization}} \]

- Arbitrary separation of soft and hard physics

- Dependence on factorization scale not associated with beta function - present even in conformal theory

- Keep factorization scale separate from renormalization scale

\[ \frac{d\mathcal{O}}{d\mu_{\text{factorization}}} = 0 \]

- Residual dependence when one works in fixed order in perturbation theory.
Use BLM!

- Satisfies Transitivity, all aspects of Renormalization Group; scheme independent
- Analytic at Flavor Thresholds
- Preserves Underlying Conformal Template
- Physical Interpretation of Scales; Multiple Scales
- Correct Abelian Limit \((N_c = 0)\)
- Eliminates unnecessary source of imprecision of PQCD predictions
- Commensurate Scale Relations: Fundamental Tests of QCD free of renormalization scale and scheme ambiguities
- BLM used in many applications, QED, LGTH, BFKL, ...
Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level

- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances

- Model for LFWFs, meson and baryon spectra: many applications!

- New basis for diagonalizing Light-Front Hamiltonian

- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.

- Quark Interchange dominant force at short distances
Essential to test QCD

- J-PARC
- GSI antiprotons
- 12 GeV Jlab
- BaBar/Belle: ISR, two-gamma, timelike DVCS
- RHIC/LHC Nuclear Collisions; LHCb
- electron-proton, electron-nucleus collisions
Novel Tests of QCD at GSI

Polarized antiproton Beam  Secondary Beams

- Characteristic momentum scale of QCD: 300 MeV
- Many Tests of AdS/CFT predictions possible
- Exclusive channels: Conformal scaling laws, quark-interchange
- $\bar{p}p$ scattering: fundamental aspects of nuclear force
- Color transparency: Coherent color effects
- Nuclear Effects, Hidden Color, Anti-Shadowing
- Anomalous heavy quark phenomena
- Spin Effects: $A_N$, $A_{NN}$
Hadron Dynamics at the Amplitude Level

- DIS studies have primarily focussed on probability distributions: integrated and unintegrated.
- Test QCD at the amplitude level: Phases, multi-parton correlations, spin, angular momentum, exclusive amplitudes
- Impact of ISI and FSI: Single Spin Asymmetries, Diffractive Deep Inelastic Scattering, Shadowing, Antishadowing
- Hadron wavefunctions: Fundamental QCD Dynamics
- Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space