QCD Phenomenology and Nucleon Structure

Stan Brodsky, SLAC

Lecture III

National Nuclear Physics Summer School
Impact of AdS/CFT on QCD

in collaboration with Guy de Teramond

5-Dimensional Anti-de Sitter Spacetime

4-Dimensional Flat Spacetime (hologram)

$z_0 = 1/\Lambda_{\text{QCD}}$
AdS/CFT and QCD

Mapping of Poincare’ and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

- Representation of Semi-Classical QCD
- Confinement at Long Distances and Conformal Behavior at short distances
- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)
- Hadron Spectra, Regge Trajectories, Light-Front Wavefunctions
- Goal: A first approximant to physical QCD
Predictions of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum

Fig: Predictions for the light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV. The 56 trajectory corresponds to $L$ even $P = +$ states, and the 70 to $L$ odd $P = -$ states.

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SJB

QCD Phenomenology

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- $SU(6)$ multiplet structure for $N$ and $\Delta$ orbital states, including internal spin $S$ and $L$.

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<th>$L$</th>
<th>Baryon State</th>
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AdS/CFT Holographic Model

\[ \psi(\sigma, b_\perp) \]

\[ |b_\perp|(\text{GeV}^{-1}) \]

3-dimensional photograph: meson LFWF at fixed LF Time

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Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.
QCD: $N_C = 3$  Quarks: $3C$  Gluons: $8C$.

$\alpha_s = \frac{g^2}{4\pi}$ is dimensionless

Classical Lagrangian is scale invariant for massless quarks

If $\beta = \frac{d\alpha_s(Q^2)}{d \log Q^2} = 0$ then QCD is invariant under conformal transformations:

Parisi
• **Polchinski & Strassler**: AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation

• **Goal**: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

• **Holographic Model**: Initial “semi-classical” approximation to QCD: Remarkable agreement with light hadron spectroscopy

• Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H^{LF}_{QCD}$; variational methods
AdS/QCD

- Semi-Classical approximation to massless QCD
- No particle creation, absorption
- Coupling is constant, $\beta = 0$
- Conformal symmetry broken by confinement
Strongly Coupled Conformal QCD and Holography

Conformal Theories are invariant under the Poincaré and conformal transformations with $M^\mu{}^\nu$, $P^\mu$, $D$, $K^\mu$, the generators of $SO(4, 2)$.

QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops. For $\beta = d\alpha_s(Q^2)/dQ^2$, QCD is a conformal theory: Parisi, Phys. Lett. B 39, 643 (1972).

Growing theoretical and empirical evidence that $\alpha_s(Q^2)$ has an IR fixed point:

Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Constituent Counting Rules

- Point-like quark and gluon constituents plus scale-invariant interactions
- Fall-off of Amplitude measures degree of compositeness (twist)
- Reflects near-Conformal Invariance of QCD
- PQCD: Logarithmic Modification by running coupling and Evolution Equations
- Angular and Spin Dependence -- Fundamental Wavefunctions: Hadron Distribution Amplitudes

\[
\frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{cm})}{s[n_{tot}^{-2}]}
\]

\[
s = E_{cm}^2
\]

\[
F_H(Q^2) \sim \left[\frac{1}{Q^2}\right]^{n_H-1}
\]

\[
-t = Q^2
\]

\[
\phi_H(x_i, Q)
\]

Farrar, sjb; Matveev et al

Lepage, sjb; Efremov, Radyushkin

QCD Phenomenology

NNPSS

July 2006
Proton Form Factor

Conformal Behavior: $t^2 F_1(t) = \text{const}$

Remarkable scaling behavior -- no signal for QCD running coupling

Non-perturbative model: Diehl, Kroll

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Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Taveklidze

$s^7dσ/dt(γp → π^+n) \sim \text{const}$

given fixed $θ_{CM}$ scaling

PQCD and AdS/CFT:

$s^{n_{tot}}2dσ/dt(A + B → C + D) = F_{A+B→C+D}(θ_{CM})$

$s^7dσ/dt(γp → π^+n) = F(θ_{CM})$

$n_{tot} = 1 + 3 + 2 + 3 = 9$

No sign of running coupling

Conformal invariance at high momentum transfer!

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Conformal Invariance:

\[ \frac{d\sigma}{dt}(\gamma p \rightarrow MB) = \frac{F(\theta_{cm})}{s^7} \]
Quark-Counting: \[ \frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}} \]

\[ n = 4 \times 3 - 2 = 10 \]

Best Fit
\[ n = 9.7 \pm 0.5 \]

Reflects underlying conformal scale-free interactions

P.V. LANDSHOFF and J.C. POLKINGHORNE

QCD Phenomenology

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Measured distribution for $\gamma\gamma \rightarrow \pi^+\pi^-$ (left) and $\gamma\gamma \rightarrow K^+K^-$ (right) as a function of $W_{\gamma\gamma}$. Also shown are results from TPC/Two-Gamma, the result of a fit to the ALEPH data and a leading twist QCD calculation with two alternative normalizations as described in the text.


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Why do dimensional counting rules work so well?

- PQCD predicts log corrections from powers of $\alpha_s$, logs, pinch contributions

- QCD coupling evaluated in intermediate regime.

- IR Fixed point! DSE: Alkofer, von Smekal et al.

- QED, EW -- define coupling from observable, predict other observable

- Underlying Conformal Symmetry of QCD Lagrangian
Define QCD Coupling from Observable

\[ R_{e^+e^- \rightarrow X(s)} \equiv 3 \sum_q e_q^2 \left[ 1 + \frac{\alpha R(s)}{\pi} \right] \]

\[ \Gamma(\tau \rightarrow Xe\nu)(m_{\tau}^2) \equiv \Gamma_0(\tau \rightarrow u\bar{d}e\nu) \times \left[ 1 + \frac{\alpha_\tau(m_{\tau}^2)}{\pi} \right] \]

Relate observable to observable at commensurate scales

H.Lu, sjb
QCD Effective Coupling from hadronic $\tau$ decay

Menke, Merino, Rathsman, SJB

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Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \]

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

\[ x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z. \]

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
5-Dimensional Anti-de Sitter Spacetime

4-Dimensional Flat Spacetime (hologram)

Black Hole

$z_0 = 1/\Lambda_{QCD}$

Caltech High Energy Seminar, Feb 6, 2006
AdS/CFT

• Use mapping of conformal group SO(4,2) to AdS5

• Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension
  \[ x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z \]

• Holographic model: Confinement at large distances and conformal symmetry in interior \(0 < z < z_0\)

• Match solutions at small \(z\) to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$

• Truncated space simulates “bag” boundary conditions
  \[ \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}} \]
Identify hadron by its interpolating operator at $z \to 0$

$\Delta = 3 + L$: Twist dimension of baryon

$z_0 = \frac{1}{\Lambda_{QCD}}$

Confinement in the 5th dimension

$\Phi(z)$

$z$
Predictons of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum

Fig: Predictions for the light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV. The 56 trajectory corresponds to $L$ even $P = +$ states, and the 70 to $L$ odd $P = -$ states.

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Action for scalar field in $\text{AdS}_5$

$$S[\Phi] = \kappa' \int d^4x \, dz \sqrt{g} \left[ g^{\ell m} \partial_\ell \Phi^* \partial_m \Phi - \mu^2 \Phi^* \Phi \right]$$

where $[\kappa'] = L^{-2}$. \[ g^{\ell m} = \frac{z^2}{R^2} \eta^{\ell m} \quad \sqrt{g} = R^5 / z^5 \]

Action is invariant under scale transformations

$$x^\mu \rightarrow \lambda x^\mu, \quad z \rightarrow \lambda z.$$

$$\Phi(x^\ell) = \Phi(\lambda x^\ell)$$

Variation wrt $\Phi$

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0$$
Solutions of form:  \[ \Phi(x, z) = e^{-iP \cdot x} f(z) \quad P_\mu P^\mu = M^2 \]

\[ S = -\kappa R^3 \int \frac{dz}{z^3} \left[ (\partial_z f)^2 - M^2 f^2 + \frac{(\mu R)^2}{z^2} f^2 \right] \]

Variation of \( S \) wrt \( f \):

\[ z^5 \partial_z \left( \frac{1}{z^3} \partial_z f \right) + z^2 M^2 f - (\mu R)^2 f = 0. \]

\[ [z^2 \partial_z^2 - 3z \partial_z + z^2 M^2 - (\mu R)^2] f = 0, \]

**Introduce confinement, break conformal invariance**

**P-S Boundary Condition**  \( f(z = \frac{1}{\Lambda_{QCD}}) = 0 \)

**Normalization in truncated space**  \[ R^3 \int_0^{\Lambda_{QCD}^{-1}} \frac{dz}{z^3} f^2(z) = 1 \]
Classical solution

\[ f(z) = \frac{\sqrt{2} \Lambda_{QCD}}{R^3/2 J_{\alpha+1}(\beta_{\alpha,k})} z^2 J_{\alpha}(z/\beta_{\alpha,k} \Lambda_{QCD}), \]

where \( \alpha = \sqrt{4 + (\mu R)^2}. \)

\[
S = -\kappa R^3 \int_0^{\Lambda_{QCD}^{-1}} \frac{dz}{z^5} f \left[ -z^5 \partial_z \left( \frac{1}{z^3} \partial_z \right) - z^2 M^2 + (\mu R)^2 \right] f \\
+ \kappa \bar{R}^3 \lim_{z \to 0} \frac{1}{z^3} f \partial_z f
\]

First term vanishes leaving

\[ S_{class} = \kappa R^3 \lim_{z \to 0} \frac{1}{z^3} f \partial_z f. \]

Breitenlohner - Freedman bound \( \alpha \geq 0 \)
\( \alpha = L \) Orbital Angular Momentum, \( (\mu R)^2 = -4 + L^2 \)

- Wave equation in AdS for bound state of two scalar partons with conformal dimension \( \Delta = 2 + L \)

\[
\left[ \frac{z^2 \partial_z^2}{2} - 3z \partial_z + z^2 \mathcal{M}^2 - L^2 + 4 \right] \Phi(z) = 0,
\]

with solution

\[
\Phi(z) = C e^{-i P \cdot x} z^2 J_L(z \mathcal{M}).
\]

- For spin-carrying constituents: \( \Delta \rightarrow \tau = \Delta - \sigma \), \( \sigma = \sum_{i=1}^{n} \sigma_i \).

- The twist \( \tau \) is equal to the number of partons \( \tau = n \).

- Two-quark vector meson described by wave equation

\[
\left[ \frac{z^2 \partial_z^2}{2} - 3z \partial_z + z^2 \mathcal{M}^2 - L^2 + 4 \right] \Phi_\mu(z) = 0,
\]

with solution

\[
\Phi_\mu(x, z) = C e^{-i P \cdot x} z^2 J_L(z \mathcal{M}) \epsilon_\mu.
\]
Match fall-off at small $z$ to Conformal Dimension of State at short distances

- Pseudoscalar mesons: $O_{3+L} = \bar{\psi} \gamma_5 D_{\ell_1} \ldots D_{\ell_m} \psi$ ($\Phi_\mu = 0$ gauge).

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $M_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.
Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

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AdS solution:

\[ \Phi(z) = Ce^{-iP \cdot x} z^2 J_\alpha(zM) \]

At large argument of the Bessel function

\[ \Phi(x, z) = C e^{-iP \cdot x} z^{d/2} \sqrt{\frac{2}{\pi zM}} \cos \left( zM - \frac{\pi}{4} \sqrt{d^2 + 4l(l + 4)} - \frac{\pi}{4} \right). \]

Dirichlet boundary condition:

\[ \Phi(x, z = z_0 = \frac{1}{\Lambda_{QCD}}) = 0 \]

\[ M(n, l) = \frac{\pi}{2} \left[ \frac{1}{2} \left( 1 + \sqrt{d^2 + 4l(l + 4)} \right) + (2n + 1) \right] \Lambda_{QCD} \]

**Quadratic Regge Relation**

In the large \( \ell \) limit:

\[ M^2 = \frac{\pi^2}{4} \ell^2 \Lambda_{QCD}^2 \]

Independent of \( n, d \)

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Baryon Spectrum

- Baryon: twist-three, dimension $\Delta = \frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D\{\ell_1 \ldots D\ell_q \psi D\ell_{q+1} \ldots D\ell_m\}\psi,$$  
$$L = \sum_{i=1}^{m} \ell_i.$$  

Wave Equation:  

$$\left[ z^2 \frac{\partial^2}{\partial z^2} - 3z \frac{\partial}{\partial z} + z^2 \mathcal{M}^2 - \mathcal{L}^2_{\pm} + 4 \right] f_{\pm}(z) = 0$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP \cdot x} z^2 \left[ J_{1+L}(z\mathcal{M}) u_+(P) + J_{2+L}(z\mathcal{M}) u_-(P) \right].$$

- 4-d mass spectrum $\Psi(x, z_o)^{\pm} = 0 \implies$ parallel Regge trajectories for baryons!

$$\mathcal{M}^+_{\alpha, k} = \beta_{\alpha, k} \Lambda_{QCD}, \quad \mathcal{M}^-_{\alpha, k} = \beta_{\alpha+1, k} \Lambda_{QCD}.$$  

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions!
• \( \mu \) determined asymptotically by spectral comparison with orbital excitations in the boundary: 

\[ \mu = L/R \text{ and } \lambda \text{ are the eigenvalues of the Dirac equation on } S^{d+1}: \]

\[ \lambda_\kappa R = \pm \left( \kappa + \frac{d}{2} + \frac{1}{2} \right), \quad \kappa = 0, 1, 2... \]

• Baryon: twist-three, dimension \( \Delta = \frac{9}{2} + L \)

\[ \mathcal{O}_{\frac{9}{2}+L} = \psi D\{\ell_1 \ldots D\ell_q \psi D\ell_{q+1} \ldots D\ell_m\} \psi, \quad L = \sum_{i=1}^{m} \ell_i. \]

• Normalizable AdS fermion mode (lowest KK-mode \( \kappa = 0 \)):

\[ \Psi_{\alpha,k}(x, z) = C_{\alpha,k} e^{-iP \cdot x} z^{\frac{5}{2}} \left[ J_\alpha(z\beta_{\alpha,k}\Lambda_{QCD}) \mu_+(P) + J_{\alpha+1}(z\beta_{\alpha,k}\Lambda_{QCD}) \mu_-(P) \right]. \]

where \( \mu^- = \frac{\gamma P^\mu}{P^\mu} \mu^+ \), \( \alpha = 2 + L \) and \( \Delta = \frac{9}{2} + L \).

• 4-d mass spectrum \( \Psi(x, z_0) = 0 \) \( \implies \) parallel Regge trajectories for baryons:

\[ \mathcal{M}^+_{\nu,n} = \alpha_{\nu,n}\Lambda_{QCD}, \quad \mathcal{M}^-_{\nu,n} = \alpha_{\nu+1,n}\Lambda_{QCD} \]

• Spin-\( \frac{3}{2} \) Rarita-Schwinger eq. in AdS similar to spin-\( \frac{1}{2} \) in the \( \Psi_z = 0 \) gauge for polarization along Minkowski coordinates, \( \Psi_\mu \). See: Volovich, hep-th/9809009.
Fig: Predictions for the light baryon orbital spectrum for \( \Lambda_{QCD} = 0.25 \) GeV. The 56 trajectory corresponds to \( L \) even \( P = + \) states, and the 70 to \( L \) odd \( P = - \) states.

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QCD Phenomenology

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Glueball Spectrum

• AdS wave function with effective mass $\mu$:

$$\left[ z^2 \partial_z^2 - (d - 1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] f(z) = 0,$$

where $\Phi(x, z) = e^{-iP\cdot x} f(z)$ and $P_\mu P^\mu = \mathcal{M}^2$.

• Glueball interpolating operator with twist-dimension minus spin-two, and conformal dimension $\Delta = 4 + L$

$$\mathcal{O}_{4+L} = FD\{\ell_1 \ldots D\ell_m\} F,$$

where $L = \sum_{i=1}^{m} \ell_i$ is the total internal space-time orbital momentum.

• Normalizable scalar AdS mode ($d = 4$):

$$\Phi_{\alpha,k}(x, z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha(z \beta_{\alpha,a} \Lambda_{QCD})$$

with $\alpha = 2 + L$ and scaling dimension $\Delta = 4 + L$. 
Glueball Regge trajectories from gauge/string duality and the Pomeron

Henrique Boschi-Filho,∗ Nelson R. F. Braga,† and Hector L. Carrion‡

*Istituto de Física, Universidade Federal do Rio de Janeiro,

Neumann Boundary Conditions

Dirichlet Boundary Conditions

QCD Phenomenology

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Substitute $f(z) = \left( \frac{z}{R} \right)^{3/2} \phi(z)$

\[
\left[ - \frac{d^2}{dz^2} + V(z) \right] \phi(z) = M^2 \phi(z)
\]

**Conformal Kernel**

\[
V(z) = -\frac{1 - 4\alpha^2}{4z^2}
\]

de Teramond, sjb

**HO Kernel**

\[
V(z) = -\frac{1 - 4\alpha^2}{4z^2} + \kappa^4 z^2
\]

Karch, et al.

**Solutions:**

\[
\phi_\alpha(z) = \kappa^{\alpha+1} \sqrt{\frac{2n!}{(n+\alpha)!}} \, z^{1/2+\alpha} \, e^{-\kappa^2 z^2/2} \, L_n^{\alpha}(\kappa^2 z^2)
\]
Why is $\alpha$ quantized?

\[ S = \lambda \int_0^\infty d\zeta \left[ (\partial_\zeta \phi)^2 - M^2 \phi^2 - \frac{1 - 4\alpha^2}{4\zeta^2} \phi^2 + \kappa^4 \phi^2 \right] \]

\[ S[\phi] = S_{\text{class}}[\phi] + S_{\text{fluct}}[\phi] \]

\[ S_{\text{fluct}} = \lambda \alpha^2 \int_0^\infty \frac{d\zeta}{\zeta^2} \phi^2 = \lambda \kappa^2 \alpha \]

Semi-classical quantization:

Fluctuations should leave $Z$ unchanged

\[ Z[\phi] \sim e^{iS[\phi]} = e^{iS_{\text{class}}[\phi]} . \]

\[ S_{\text{fluct}} = 2\pi \alpha = 2\pi L \]

Thus $\alpha = L$ is integer \quad $\lambda = 2\pi / \kappa^2$

(Matches integral twist-dimension of state)
Dirac’s Amazing Idea: The “Front Form”

Evolve in light-front time!

Figure 1. Dirac’s three forms of Hamiltonian dynamics.

2.4. Forms of Hamiltonian dynamics

Obviously, one has many possibilities to parametrize space—time by introducing some generalized coordinates $x^\mu(x)$. But one should exclude all those which are accessible by a Lorentz transformation. Those are included anyway in a covariant formalism. This limits considerably the freedom and excludes, for example, almost all rotation angles. Following Dirac [123] there are no more than three basically different parametrizations. They are illustrated in Fig. 1, and cannot be mapped on each other by a Lorentz transform. They differ by the hypersphere on which the fields are initialized, and correspondingly one has different "times". Each of these space—time parametrizations has thus its own Hamiltonian, and correspondingly Dirac [123] speaks of the three forms of Hamiltonian dynamics: The instant form is the familiar one, with its hypersphere given by $t=0$. In the front form the hypersphere is a tangent plane to the light cone. In the point form the time-like coordinate is identified with the eigentime of a physical system and the hypersphere has a shape of a hyperboloid.

Which of the three forms should be preferred? The question is difficult to answer, in fact it is ill-posed. In principle, all three forms should yield the same physical results, since physics should not depend on how one parametrizes the space (and the time). If it depends on it, one has made a mistake. But usually one adjusts parametrization to the nature of the physical problem to simplify the amount of practical work. Since one knows so little on the typical solutions of a field theory, it might well be worth the effort to admit also other than the conventional "instant" form. The bulk of research on field theory implicitly uses the instant form, which we do not even attempt to summarize. Although it is the conventional choice for quantizing field theory, it has

\[ \sigma = ct - z \]

\[ \tau = t + z/c \]
\[ P^+ = P^0 + P^z \]

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

**Invariant under boosts! Independent of** \( P^\mu \)

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_\perp i = \vec{0}_\perp \]
Mapping between LF(3+1) and AdS$_5$

\[ \psi(x, \vec{b}_\perp) = \sqrt{x(1-x)} \phi(\zeta) \]

\[ \zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \]

\[ \phi(z) \]

\[ x \quad (1-x) \]

\[ \vec{b}_\perp \]
Relativistic radial equation:  \[ -\frac{d^2}{d\zeta^2} + V(\zeta) \phi(\zeta) = M^2 \phi(\zeta) \]

\[ \zeta^2 = x(1 - x)b_\perp^2. \]

Effective conformal potential:

\[ V(\zeta) = -\frac{1 - 4L^2}{4\zeta^2}. \]
The Form Factor in AdS Space

- Non-conformal metric dual to a confining gauge theory

\[ ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right), \]

where \( A(z) \to 0 \) as \( z \to 0 \) (Polchinski and Strassler, hep-th/0109174).

- Hadronic matrix element for EM coupling with string mode \( \Phi(x, z), \ x^\ell = (x^\mu, z) \)

\[ ig_5 \int d^4x\, dz \, \sqrt{g} \ A^\ell(x, z) \Phi^*(x, z) \overleftarrow{\partial} \ell \Phi_P(x, z). \]

- Electromagnetic probe polarized along Minkowski coordinates,

\[ A_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q, z), \quad A_z = 0, \]

with

\[ J(Q, z) = zQ K_1(zQ), \quad J(Q = 0, z) = J(Q, z = 0) = 1 \]

- Hadronic modes are plane waves along the Poincaré coordinates with four-momentum \( P^\mu \) and invariant mass \( P_\mu P^\mu = M^2 \)

\[ \Phi(x, z) = e^{-iP\cdot x} f(z), \quad f(z) \to z^\Delta, \quad z \to 0. \]
• Propagation of external perturbation suppressed inside AdS.

• At large enough $Q \sim r/R^2$, the interaction occurs in the large-$r$ conformal region. Important contribution to the FF integral from the boundary near $z \sim 1/Q$.

$$J(Q, z), \quad \Phi(z)$$

- Consider a specific AdS mode $\Phi^{(n)}$ dual to an $n$ partonic Fock state $|n\rangle$. At small $z$, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[ \frac{1}{Q^2} \right]^{\tau - 1},$$

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$. 

**General result from AdS/CFT**
Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

Contributions from Feynman large-x and high transverse momenta regimes
Holographic Model for QCD Light-Front Wavefunctions

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_P^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( b = |\vec{b}_\perp| \):

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, \vec{b}_\perp)|^2 \]

\[ = 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \, |\tilde{\psi}(x, b)|^2, \]

Soper
Two parton case

- Change the integration variable $\zeta = |\vec{b}_\perp| \sqrt{x(1-x)}$

$$ F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{\text{max}}=\Lambda_{\text{QCD}}^{-1}} \zeta \, d\zeta \, J_0 \left( \frac{\zeta Q x}{\sqrt{x(1-x)}} \right) |\tilde{\psi}(x, \zeta)|^2, $$

- Compare with AdS form factor for arbitrary $Q$. Find:

$$ J(Q, \zeta) = \int_0^1 dx J_0 \left( \frac{\zeta Q x}{\sqrt{x(1-x)}} \right) = \zeta Q K_1(\zeta Q), $$

the solution for the electromagnetic potential in AdS space, and

$$ \tilde{\psi}(x, \vec{b}_\perp) = \frac{\Lambda_{\text{QCD}}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0 \left( \frac{\sqrt{x(1-x)} |\vec{b}_\perp| \beta_{0,1} \Lambda_{\text{QCD}}}{x(1-x)} \right) \theta \left( \frac{\vec{b}_\perp^2}{x(1-x)} \leq \frac{\Lambda_{\text{QCD}}^{-2}}{x(1-x)} \right) $$

the holographic LFWF for the valence Fock state of the pion $\psi_{qq}/\pi$.

- The variable $\zeta$, $0 \leq \zeta \leq \Lambda_{\text{QCD}}^{-1}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!
Mapping between LF(3+1) and AdS$_5$

\[ \psi(x, \vec{b}_\perp) = \psi(\zeta) = \sqrt{x(1-x)\zeta^2} \]

\[ \zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \]

\[ \phi(z) \]

\[ \left( 1 - x \right) \]
Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right] \phi(\zeta) = M^2 \phi(\zeta)$$

$$\zeta^2 = x(1 - x)b_{\perp}^2.$$  

Effective conformal potential:

$$V(\zeta) = -\frac{1 - 4L^2}{4\zeta^2}.$$  

General solution:

$$\tilde{\psi}_{L,k}(x, \vec{b}_{\perp}) = B_{L,k} \sqrt{x(1 - x)}$$

$$J_L \left( \sqrt{x(1 - x)} |\vec{b}_{\perp}| \beta_{L,k} \Lambda_{QCD} \right) \theta \left( \vec{b}_{\perp}^2 \leq \frac{\Lambda_{QCD}^{-2}}{x(1 - x)} \right),$$
AdS/CFT Prediction for Meson LFWF

Two-parton holographic LFWF in impact space \( \tilde{\psi}(x, \zeta) \) for \( \Lambda_{QCD} = 0.32 \text{ GeV} \): (a) ground state \( L = 0, \ k = 1 \); (b) first orbital exited state \( L = 1, \ k = 1 \); (c) first radial exited state \( L = 0, \ k = 2 \).

The variable \( \zeta \) is the holographic variable \( z = \zeta = |b_\perp|\sqrt{x(1-x)} \).

\[
\tilde{\psi}(x, \zeta) = \frac{\Lambda_{QCD}}{\sqrt{\pi}J_1(\beta_{0,1})} \sqrt{x(1-x)}J_0 (\zeta \beta_{0,1}\Lambda_{QCD}) \theta \left( z \leq \Lambda_{QCD}^{-1} \right)
\]
AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$

\[ b_\perp (\text{GeV})^{-1} \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$b_\perp (\text{GeV})^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1</td>
</tr>
</tbody>
</table>

$\Lambda_{\text{QCD}} = 0.32$ GeV

$\kappa = 0.76$ GeV

Truncated Space

Harmonic Oscillator

QCD Phenomenology

Stan Brodsky, SLAC
\[ \psi(\sigma, b_\perp) \]

\[ |b_\perp|(\text{GeV}^{-1}) \]

3-dimensional photograph: meson LFWF at fixed LF Time
General n-parton case

- Form factor in AdS is the overlap of normalizable modes dual to the incoming and outgoing hadrons $\Phi_P$ and $\Phi_{P'}$ with the non-normalizable mode $J(Q,z)$ dual to the external source

$$F(Q^2) = R^3 \int_0^{\infty} \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) J(Q, z) \Phi_P(z).$$

Polchinski and Strassler, hep-th/0209211

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta).$$

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary $Q$ using identity:

$$\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta)$!
• Define effective single particle transverse density by (Soper, Phys. Rev. D 15, 1141 (1977))

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{\eta}_\perp e^{i \vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp) \]

• From DYW expression for the FF in transverse position space:

\[ \tilde{\rho}(x, \vec{\eta}_\perp) = \sum_{n} \prod_{j=1}^{n-1} \int dx_j \, d^2 \vec{b}_\perp j \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_\perp j - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_\perp j)|^2 \]

• Compare with the form factor in AdS space for arbitrary \( Q \):

\[ F(Q^2) = R^3 \int_0^{\infty} \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z) \]

• Holographic variable \( z \) is expressed in terms of the average transverse separation distance of the spectator constituents \( \vec{\eta} = \sum_{j=1}^{n-1} x_j \vec{b}_\perp j \)

\[
z = \sqrt{\frac{x}{1-x} \left| \sum_{j=1}^{n-1} x_j \vec{b}_\perp j \right|}
\]
Hadronic QCD transverse density $\tilde{\rho}$ is identified with the string mode density $|\Phi|^2$ in AdS space!

$$\tilde{\rho}(x, \zeta) = \frac{R^3}{2\pi} \frac{x}{1-x} e^{3A(\zeta)} \frac{|\Phi(\zeta)|^2}{\zeta^4}$$

The variable $\zeta$ represents the invariant separation between point-like constituents and it is also the holographic variable: $\zeta = z$.

For two-partons

$$\tilde{\rho}(x, \zeta) = \frac{1}{(1-x)^2} \left|\tilde{\psi}(x, \zeta)\right|^2.$$  

Two-parton bound state LFWF

$$\left|\tilde{\psi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) e^{3A(\zeta)} \frac{|\Phi(\zeta)|^2}{\zeta^4}.$$  


Short distance behavior of LFWF: $\tilde{\psi}(x, b_\perp) \sim (b_\perp^2)^{\Delta - 2}$. 

• Our final result: hadronic QCD transverse density \( \tilde{\rho} \) is determined by the modes \( \Phi \) in AdS space!

\[
\tilde{\rho}(x, \zeta) = \frac{R^3}{2\pi} \frac{x}{1 - x} e^{3A(\zeta)} \frac{|\Phi(\zeta)|^2}{\zeta^4}
\]

• The variable \( \zeta, 0 \leq \zeta \leq \Lambda_{QCD}^{-1} \), is related to the average transverse separation between spectator constituents, and it is also the holographic variable \( z, \zeta = z \).

• For the two-particle case

\[
\tilde{\rho}(x, \zeta) = \frac{1}{(1 - x)^2} |\psi(x, \zeta)|^2,
\]

and we recover our previous results

\[
|\psi(x, \zeta)|^2 \simeq \frac{R^3}{2\pi} x(1 - x) \frac{|\Phi(\zeta)|^2}{\zeta^4} \theta \left( \zeta^2 \leq \Lambda_{QCD}^{-2} \right).
\]
Hadron Distribution Amplitudes

\[ \phi(x_i, Q) \equiv \prod_{i=1}^{n-1} \int_{Q} d^2 k_{\perp} \psi_n(x_i, k_{\perp i}) \]

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

**AdS/CFT:**

\[ \phi(x, Q_0) \propto \sqrt{x(1-x)} \]
Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x, z)$

$$ig_5 \int d^4x \, dz \, \sqrt{g} \, A_\mu(x, z) \, \overline{\Psi}(x, z) \gamma^\mu \Psi(x, z),$$

where

$$\Psi(x, z) = e^{-iP \cdot x} \left[ \psi_+(z) u_+(P) + \psi_-(z) u_-(P) \right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \quad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_{\pm} = \frac{1 \pm \gamma_5}{2} u(P).$$

$$\psi_+(z) \equiv \psi^\dagger(z), \quad \psi_-(z) \equiv \psi^\dagger(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant $C$ determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{QCD}}{R^{3/2} \left[ - J_0(\beta_{1,1}) J_2(\beta_{1,1}) \right]^{1/2}}.$$
Consider the spin non-flip form factors in the infinite wall approximation

\[
F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q, z) |\psi_+(z)|^2,
\]
\[
F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q, z) |\psi_-(z)|^2,
\]

where the effective charges \(g_+\) and \(g_-\) are determined from the spin-flavor structure of the theory.

Choose the struck quark to have \(s^z = +1/2\). The two AdS solutions \(\psi_+(z)\) and \(\psi_-(z)\) correspond to nucleons with \(J^z = +1/2\) and \(-1/2\).

For \(SU(6)\) spin-flavor symmetry (proton up)

\[
N_{u\uparrow} = \frac{5}{3}, \quad N_{u\downarrow} = \frac{1}{3}, \quad N_{d\uparrow} = \frac{1}{3}, \quad N_{d\downarrow} = \frac{2}{3}.
\]

Final result

\[
F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q, z) |\psi_+(z)|^2,
\]
\[
F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q, z) \left[ |\psi_+(z)|^2 - |\psi_-(z)|^2 \right],
\]

where \(F_1^p(0) = 1, \ F_1^n(0) = 0\).
**Dirac Proton Form Factor**  
(Valence Approximation)

\[ Q^4 F_1^P(Q^2) \ [\text{GeV}^4] \]

Prediction for \( Q^4 F_1^P(Q^2) \) for \( \Lambda_{\text{QCD}} = 0.21 \ \text{GeV} \) in the hard wall approximation. Analysis of the data is from Diehl (2005). Red points are from Sill (1993). Superimposed Green points are from Kirk (1973).
Dirac Neutron Form Factor
(Valence Approximation)

\[ Q^4 F^m_1(Q^2) \ [\text{GeV}^4] \]

Prediction for \( Q^4 F^m_1(Q^2) \) for \( \Lambda_{QCD} = 0.21 \) GeV in the hard wall approximation. Data analysis from Diehl (2005).
AdS/CFT and QCD

Implications for Exclusive Processes

• Meson distribution amplitude \(\phi(x, Q_0) \propto \sqrt{x(1 - x)}\)

• Dominance of constituent interchange mechanism

• Power-law behavior from small impact separation \(b_\perp\)
  high transverse momentum \(k_\perp\) as well as \(x\) near 1

• High transverse momentum behavior matches PQCD
  LFWF with orbital: Belitsky, Ji, Yuan

• Perfect match of LF and AdS/CFT formulae for form factors
Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm

- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum sum rules

- Exclusive weak decay amplitudes

- Single spin asymmetries

- Factorization theorems, DGLAP, BFKL, ERBL Evolution

- Quark interchange amplitude

- Relation of spin, momentum, and other distributions to physics of the hadron itself.
Advantages of Light-Front Formalism

- *Hidden Color* of Nuclear Wavefunction

- *Color Transparency, Opaqueness*

- Simple proof of Factorization theorems for hard processes (Lepage, sjb)

- *Direct mapping to AdS/CFT* (de Teramond, sjb)

- New Effective LF Equations (de Teramond, sjb)

- Light-Front Amplitude Generator
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

Invariant under boosts. Independent of $P^\mu$

$$H^QCD_{LF} |\psi > = M^2 |\psi >$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Single-spin asymmetries

\[ \vec{S}_p \cdot \vec{q} \times \vec{p}_q \]

Sivers Effect

\[ \vec{S}_p \cdot \vec{q} \times \vec{p}_q \]

QCD S- and P- Coulomb Phases

Light-Front Wavefunction
S and P- Waves

Hwang, Schmidt.

QCD Phenomenology
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Stan Brodsky, SLAC
Final State Interactions Produce T-Odd (Sivers Effect)

- Bjorken Scaling!

- Arises from Interference of Final-State Coulomb Phases in S and P waves

- Relate to the quark contribution to the target proton anomalous magnetic moment

- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero gravitoanomalous magnetic moment)

\[ \vec{S} \cdot \vec{p}_{jet} \times \vec{q} \]
• Quarks Reinteract in Final State
• Analogous to Coulomb phases, but not unitary
• Observable effects: DDIS, SSI, shadowing, antishadowing
• Structure functions cannot be computed from LFWFs computed in isolation
• Wilson line not 1 even in lcg
Prediction for Single-Spin Asymmetry

Hwang, Schmidt. sjb

QCD Phenomenology

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In the context of the quark-parton model, the virtual-photon asymmetry $A_{UT}$ can be represented in terms of parton distribution functions. The figure was taken from Ref. [9].

Sivers asymmetry from HERMES

Gamberg: Hermes data compatible with BHS model
Single Spin Asymmetry In the Drell Yan Process
\[ \vec{S}_p \cdot \vec{p} \times \vec{q'}^* \]
Quarks Interact in the Initial State
Interference of Coulomb Phases for \( S \) and \( P \) states
Produce Single Spin Asymmetry [Siver’s Effect] Proportional to the Proton Anomalous Moment and \( \alpha_s \).
Opposite Sign to DIS! No Factorization

Collins; Hwang, Schmidt.
Key QCD Experiment at GSI

Measure single-spin asymmetry $A_N$ in Drell-Yan reactions

Leading-twist Bjorken-scaling $A_N$ from $S, P$-wave initial-state gluonic interactions

Predict: $A_N(DY) = -A_N(DIS)$ Opposite in sign!

$$Q^2 = x_1x_2s$$

$$Q^2 = 4 \text{ GeV}^2, s = 80 \text{ GeV}^2$$

$$x_1x_2 = .05, x_F = x_1 - x_2$$

Trento
July 5, 2006
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances
MIT Bag Model predicts dominance of quark interchange:

C. de Tar

Blankenbecler, Gunion, sjb
Why is quark-interchange dominant over gluon exchange?

Example: \( M(K^+p \rightarrow K^+p) \propto \frac{1}{ut^2} \)

Exchange of common \( u \) quark

\[
M_{QIM} = \int d^2k_\perp dx \ \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B
\]

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS\(_5\)

Quarks travel freely within cavity as long as separation \( z < z_0 = \frac{1}{\Lambda_{QCD}} \)

LFWFs obey conformal symmetry producing quark counting rules.
AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions.

Non-linear Regge behavior:

$$\alpha_R(t) \rightarrow -1$$
Key QCD Experiment at GSI

\[ \bar{p}p \rightarrow K^+K^- \]

\[ s \leftrightarrow t \leftrightarrow u \text{ crossing of } K^+p \rightarrow K^+p \]

\[ M(\bar{p}p \rightarrow K^+K^-) \propto \frac{1}{ts^2} \]

\[ \frac{d\sigma}{dt} \propto \frac{1}{s^6t^2} \]

at large \( t, u \)
Test of Quark Interchange Mechanism in QCD

\[ \frac{\frac{d\sigma}{dt}}{\frac{d\sigma}{dt}}_{90^\circ} = R(z) \]

\[ \frac{1}{(1-z^2)^{5.2}} \]

\[ P_{LAB} \]
- 12.1
- 14.25
- 16.9
- 19.3
- 21.3
- 11.1
- 10.1
- 9.2
- 7.1

\[ z = \cos \theta_{c.m.} \]
Comparison of Exclusive Reactions at Large $t$

B. R. Baller, (a) G. C. Blazey, (b) H. Courant, K. J. Heller, S. Heppelmann, (c) M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl (d)  

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(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^\pm p \rightarrow p\pi^\pm, p\rho^\pm, \pi^\pm + \Delta^\pm, K^+\Sigma^\pm, (\Lambda^0/\Sigma^0)K^0, K^\pm p \rightarrow pK^\pm; p^\pm p \rightarrow pp^\pm$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

\[
\begin{align*}
\pi^\pm p &\rightarrow p\pi^\pm, \\
K^\pm p &\rightarrow pK^\pm, \\
\pi^\pm p &\rightarrow \rho^\pm, \\
\pi^\pm p &\rightarrow \pi^\pm + \Delta^\pm, \\
\pi^\pm p &\rightarrow K^+\Sigma^\pm, \\
\pi^- p &\rightarrow \Lambda^0K^0, \Sigma^0K^0, \\
p^\pm p &\rightarrow pp^\pm.
\end{align*}
\]
The cross section and upper limits (90% confidence level) measured by this experiment are indicated by the filled circles and arrowheads. Values from this experiment and from previous measurements represent an average over the angular region of $-0.05 < \cos \theta_{\text{e.m.}} < 0.10$. The other measurements were obtained from the following references: \( \pi^+p \) and \( K^+p \) elastic, Ref. 5; \( \pi^-p \rightarrow p\pi^- \), Ref. 6; \( pp \rightarrow pp \), Ref. 7: Allaby, open circle; Akerlof, cross. Values for the cross sections [(Reaction), cross section in nb/(GeV/c)^2] are as follows: (1), 4.6 ± 0.3; (2), 1.7 ± 0.2; (3), 3.4 ± 1.4; (4), 0.9 ± 0.3; (5), 3.4 ± 0.7; (6), 1.3 ± 0.6; (7), 2.0 ± 0.6; (8), < 0.12; (9), < 0.1; (10), < 0.06; (11), < 0.05; (12), < 0.15; (13), 48 ± 5; (14), < 2.1.

Quark Interchange: Dominant Dynamics at large \( t, u \)

Relative Rates Correct
Formula for quark interchange using LFWFs

\[
M_{F,I} = \langle \psi_F | E - K | \psi_I \rangle \\
\equiv \langle \psi_F | \Delta | \psi_I \rangle \\
= \frac{1}{2(2\pi)^3} \int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2} \Delta \psi_C(\vec{k}_\perp - x\vec{r}_\perp, x) \psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x) \psi_B(\vec{k}_\perp, x),
\]

where

\[
\Delta = s - M_A^2 - M_B^2 - K_a - K_b - K_c - K_d \\
= M_A^2 + M_B^2 - S_A(\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp, x) - S_B(\vec{k}_\perp, x) \\
= M_C^2 + M_D^2 - S_C(\vec{k}_\perp - x\vec{r}_\perp, x) - S_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x).
\]
The biggest failure of the interchange mechanism is in the spin correlation. For all angles we predict from Table I

\[ A_{nn} = \frac{1}{3} \frac{1 - (\frac{3}{31})^2 \chi^2}{1 + \frac{1}{3} (\frac{3}{31})^2 \chi^2}, \]  

(3.11)

where

\[ \chi = \frac{f(\theta) - f(\pi - \theta)}{f(\theta) + f(\pi - \theta)}. \]

Thus \( A_{nn} \) is predicted to be within 2\% of \( \frac{1}{3} \) even when \( \chi = 1 \) [\( \chi = 0 \) for the form in Eq. (3.6)]. The data clearly indicate that \( A_{nn} \) is not a constant near \( \frac{1}{3} \).

Our expectation, then, is that there is an additional amplitude which strongly interferes with the quark-interchange contributions at Argonne energies; most plausibly, the quark-interchange contribution is dominant at asymptotic \( t \) and \( u \), and the interfering amplitude is most important at low \( t \) and \( u \). As we shall discuss below, the behavior of \( A_{11} \) and \( A_{ss} \) in the interference region can play an important role in sorting out the possible sub-asymptotic contributions.

These results for the quark-interchange model have also been obtained by Farrar, Gottlieb, Sivers, and Thomas,\(^{12}\) who also consider the possibility that nonperturbative effects (quark-quark scattering via instantons) can explain the data.

\[ \frac{d\sigma}{dt}(pp \rightarrow pp) = C \frac{F_p^2(t) F_p^2(u)}{s^2} \]

\[ \frac{d\sigma}{dt} = \frac{1}{s^{10}} f(\theta_{\text{c.m.}}), \quad f(\theta_{\text{c.m.}}) \sim \left( \frac{1}{1 - \cos^2 \theta} \right)^4. \]
New Perspectives on QCD Phenomena from AdS/CFT

- **AdS/CFT**: Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions
- AdS/CFT provides novel insights into the quark structure of hadrons
Outlook

- Only one scale $\Lambda_{QCD}$ determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension $3, \frac{9}{2}$, and 4 states $\bar{q}q, qqq$, and $gg$ appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.
**Features of Holographic Model**

- Use of holographic light-front wave functions to compute hadronic matrix elements and other observables.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model, modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.
- Precise mapping of string modes to partonic states. String modes inside AdS represent the probability amplitude for the distribution of quarks at a given scale.
- Exact holographic mapping for \( n \)-parton state determines effective QCD transverse charge density in terms of modes in AdS space.
- Holographic mapping allows deconstruction: express the eigenvalue problem in terms of 3+1 QCD degrees of freedom.
Use the AdS/CFT orthonormal LFWFs as a basis to diagonalize the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis
- DLCQ discretization
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Vary, Harinandrath, sjb
Fig. 6. A few selected matrix elements of the QCD front form Hamiltonian $H_{LC}^{QCD}$.

For the given case, in the $p^6$ convention.

In terms of the hadron four-momentum $P = (P^0, P^3, \mathbf{P}^\perp)$ with $P^\pm = P^0 \pm P^3$, the light-front frame independent Hamiltonian $H_{LC}^{QCD}$ for the mass spectrum of the color-singlet states in QCD is given by:

$$H_{LC}^{QCD} |\psi_h\rangle = M^2_h |\psi_h\rangle$$

where $M^2_h$ is the eigenmass squared corresponding to the mass spectrum of the color-singlet states in QCD.

Use AdS/QCD basis functions

NNPSS QCD Phenomenology
July 2006

Pauli, Pinsky, sjb
Stan Brodsky, SLAC
Use AdS/QCD Basis functions to diagonalize the LF Hamiltonian

“…I will sum up by saying that light-front QCD is not for the faint of heart, but for a few good candidates it is a chance to be a leader in a much smaller community of researchers than one faces in the other areas of high-energy physics, with, I believe, unusual promise for interesting and unexpected results.”

Conformal symmetry: Template for QCD

• Initial approximation to PQCD; then correct for non-zero beta function and quark masses

• Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

• Arguments for Infrared fixed-point for $\alpha_s$ Alhofer, et al.

• Effective Charges: analytic at quark mass thresholds, finite at small momenta

• Eigensolutions of Evolution Equation of distribution amplitudes
New Perspectives on QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- QCD is Nearly Conformal
- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra
- Quark-interchange dominates scattering amplitudes
AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:
  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small $x$:
  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:
  Brodsky and de Téramond, hep-th/0310227. E. van Beveren et al.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:
Gluonium spectrum (top-bottom):
Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

D3/D7 branes (top-bottom):

Other aspects of high energy scattering in warped spaces:
Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

Strongly coupled quark-gluon plasma \((\eta/s = 1/4\pi)\):
Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 \ldots
A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists’ best hope for unifying gravity and quantum theory -- into a single coherent theory.

I thought I had discovered the Theory of Everything
But everything canceled out!