The World of Quarks and Gluons:

- Quarks and Gluons: Fundamental constituents of hadrons and nuclei
- Remarkable and novel properties of Quantum Chromodynamics (QCD)
- New Insights from higher space-time dimensions: Holography
QCD Lagrangian

Generalization of QED

\[
L_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{n_f} m_f \bar{\psi}_f \psi_f
\]

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement
• Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.

• Novel QCD Phenomena: hidden color, color transparency, intrinsic charm, anomalous heavy quark phenomena, anomalous spin-spin effects, odderon, anomalous Regge behavior ...

• Remarkable Predictions of AdS/CFT

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities.

—Mark Twain
Quarks in the Proton

\[ p = (u \ u \ d) \]

1 \text{ fm} \quad 10^{-15} m = 10^{-13} \text{ cm}

Bjorken Scaling: Pointlike Quarks

Feynman: “Parton” model

Ne’eman: \( SU(3)_F \)

Zweig: “Aces, Duces, Treys”

Gell Mann: “Three Quarks for Mr. Mark”

QCD Phenomenology

Stan Brodsky, SLAC

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The Quark Structure of the Nucleus

\[ p = (uud) \]
\[ n = (ddu) \]

\[ e_u = +\frac{2}{3} \quad e_d = -\frac{1}{3} \]

\[ 2e_u + e_d = e_p \]
\[ 2 \times (+\frac{2}{3}) + 1 \times (-\frac{1}{3}) = 1 \]

\[ 2e_d + e_u = e_n \]
\[ 2 \times (-\frac{1}{3}) + 1 \times (+\frac{2}{3}) = 0 \]
1967 SLAC Experiment:
Scatter Electrons on protons
in a Hydrogen Target
Discovery of the Quark Structure of Matter

\[ \omega = 2M_p \nu \]
\[ Q^2 = \frac{Q^2}{2M_p \nu} \]

Measure rate as a function of energy loss \( \nu \) and momentum transfer \( Q \)

Scaling at fixed \( x_{\text{Bjorken}} = \frac{Q^2}{2M_p \nu} = 1 \)

\[ \omega_{ep \rightarrow e'X} \]

Discovery of quarks!

Friedman, Kendall, Taylor: Nobel Prize

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QCD Phenomenology

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SLAC Two-Mile Linear Accelerator

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QCD Phenomenology

Stan Brodsky, SLAC
First Evidence for Quark Structure of Matter

Deep Inelastic Electron-Proton Scattering

Gluonic Bremsstrahlung

DGLAP Evolution

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QCD Phenomenology

9

Stan Brodsky, SLAC
$ep \rightarrow e' X$

$E' = E - \nu, \bar{q}$

$p$

$Q^2 = q^2 - \nu^2$

No intrinsic length scale!

Measure rate as a function of energy loss $\nu$ and momentum transfer $Q$

Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p \nu} = \frac{1}{\omega}$

Discovery of Bjorken Scaling

Electron scatters on point-like quarks!
Generations of matter

Quarks

\( u \ c \ t \ d \ s \ b \)

Leptons

\( \nu_e \ \nu_\mu \ \nu_\tau \ e \ \mu \ \tau \)

Quark Masses (GeV)

- Up
- Down
- Charm
- Strange
- Top
- Bottom

Mass (GeV)

0.001 0.01 0.1 1 10 100 1000

Quark

QCD Phenomenology

Stan Brodsky, SLAC
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<td>$B_s^*$</td>
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**Constructing mesons**

$$M = (q \bar{q})$$

$$\pi^+ = (u \bar{d})$$

Pseudoscalar ($J^P = 0^-$) (upper lines) and vector ($J^P = 0^-$) (lower lines) mesons with different flavour content.
The Hadron Spectrum

\[ SU(3)_{\text{flavor}} \]

\[ \Omega^- = (ss) \]

Prediction and Measurement of \( \Omega^- = (ss) \)

Ne’eman, Gell Mann, Zweig
Y. Eisenberg
Samios

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QCD Phenomenology

Stan Brodsky, SLAC
Why are there three colors of quarks?

Pauli Exclusion Principle!

spin-half quarks cannot be in same quantum state!

\[ \Delta^{++} \]

\[ J^z = +\frac{3}{2} \]

\[ u \quad S^z = +\frac{1}{2} \]

\[ u \quad S^z = +\frac{1}{2} \]

\[ u \quad S^z = +\frac{1}{2} \]

Three Colors (Parastatistics) Solves Paradox

Three Colors Combine: **WHITE**

Greenberg: Parastatistics

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QCD Phenomenology

Stan Brodsky, SLAC
QCD Lagrangian

Generalization of QED

\[ L_{QCD} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{n_f} m_f \bar{\psi}_f \psi_f \]

Yang Mills Gauge Principle:
Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement
\[ \mathcal{L} = -\frac{1}{4} F^\alpha_{\mu\nu} F^\mu\nu - \sum_n \bar{\psi}_n \gamma^\mu [\partial_\mu - ig A^\alpha_\mu t_\alpha] \psi_n - \sum_n m_n \bar{\psi}_n \psi_n \]

\[
[t_\beta, t_\gamma] = i C^\alpha_{\beta\gamma} t_\alpha
\]

where \( C^\alpha_{\beta\gamma} \) are the SU(3) algebra structure constants

The gluon field tensors \( F^\alpha_{\mu\nu} \) are defined as

\[
F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + C^\alpha_{\beta\gamma} A^\beta_\mu A^\gamma_\nu.
\]

Quarks couple to gluons through the color currents

\[
J^\mu_\alpha = -ig \sum_n \bar{\psi}_n \gamma^\mu A^\alpha_\mu t_\alpha \psi_n.
\]
QCD

**Fundamental Couplings**

Only quarks and gluons involve basic vertices: Quark-gluon vertex

\[ q \quad g \quad q \]

More exactly

\[ q(r) \quad g(b, \bar{r}) \quad q(b) \]

Gluon vertices

Similar to QED

QCD Phenomenology colored particles couple to gluons

Stan Brodsky, SLAC
In QCD and the Standard Model the beta function is indeed negative!

\[ \beta(g) = \frac{-g^3}{16\pi^2} \left( \frac{11}{3} N_c - \frac{4}{3} N_F \right) \]

\[ \beta = \frac{d\alpha_s(Q^2)}{d\ln Q^2} < 0 \]

logarithmic derivative of the QCD coupling is negative
Coupling becomes weaker at short distances or high momentum transfer

Illustration: Typoform

Stan Brodsky, SLAC

QCD Phenomenology

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Verification of Asymptotic Freedom

\[ \alpha_s(Q) \propto \frac{1}{\ln Q} \]

\[
\frac{\sigma(e^+e^- \to \text{three jets})}{\sigma(e^+e^- \to \text{two jets})}
\]

proportional to \( \alpha_s(Q) \)

Ratio of rate for \( e^+e^- \to q\bar{q}g \) to \( e^+e^- \to q\bar{q} \) at \( Q = E_{CM} = E_{e^-} + E_{e^+} \)

Gross, Wilzcek, Politzer
Khriplovich, 't Hooft

\[ \Lambda^{(5)}_{MS} \quad \alpha_s(M_Z) \]

\[ \begin{align*}
245 \text{ MeV} & \quad 0.1210 \\
211 \text{ MeV} & \quad 0.1183 \\
181 \text{ MeV} & \quad 0.1156
\end{align*} \]
QCD Lagrangian

\[
L_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr}(G^{\mu \nu} G_{\mu \nu}) + \sum_{f=1}^{n_f} \bar{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{n_f} m_f \bar{\psi}_f \psi_f
\]

\[\text{gluon dynamics} \quad \text{quark kinetic energy + quark-gluon dynamics} \quad \text{mass term}\]

\[\text{QCD color charge} \quad \text{field strength tensor} \quad \text{covariant derivative} \quad \text{quark field}\]

\[
\lim_{N_C \to 0} \text{ at fixed } \alpha = C_F \alpha_s, \quad n_\ell = n_F / C_F \quad [C_F = \frac{N_C^2 - 1}{2N_C}]
\]

Analytic limit of QCD: Abelian Gauge Theory

P. Huet, sjb

NNPSS July 2006

QCD Phenomenology

Stan Brodsky, SLAC
In QED (N_C=0) the beta function is positive.

\[
\beta(g) = \frac{-g^3}{16\pi^2} \left( \begin{array}{c} 0 \\ -\frac{4N_F}{3} \end{array} \right)
\]

\[
\beta = \frac{d\alpha_s(Q^2)}{d \ln Q^2} > 0
\]

Logarithmic derivative of the QED coupling is negative.
Coupling becomes stronger at short distances or high momentum transfer.
Asymptotic unification of strong, electromagnetic, and weak forces
Given the elementary gauge theory interactions, all fundamental processes described in principle!

**Example from QED:**

Electron gyromagnetic moment - ratio of spin precession frequency to Larmor frequency in a magnetic field

\[
\frac{1}{2} g_e = 1.001\,159\,652\,201(30) \quad \text{QED prediction (Kinoshita, et al.)}
\]

\[
\frac{1}{2} g_e = 1.001\,159\,652\,193(10) \quad \text{Measurement (Dehmelt, et al.)}
\]

\[g_e \text{ accurate to 11 figures!}\]

**Dirac:** \[ g_e \equiv 2 \]
Radiative Corrections of Eighth- and Tenth-Orders to Lepton g-2

Toichiro Kinoshita

Laboratory for Elementary-Particle Physics
Cornell University
Ithaca, NY 14853, USA
E-mail: tk@hepth.cornell.edu

PHOTON-PHOTON SCATTERING CONTRIBUTION
TO THE SIXTH-ORDER MAGNETIC MOMENT OF THE MUON*

Janis Aldins† and Toichiro Kinoshita
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

and

Stanley J. Brodsky and Andrew J. Dufner
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 July 1969)

We report a calculation of the three-photon-exchange (electron-loop) contribution to
the sixth-order anomalous magnetic moment of the muon. Our result, which contains a
logarithmic dependence on the muon-to-electron mass ratio, brings the theoretical pre-
diction into agreement with the CERN measurements, within the ±1-standard-deviation
experimental accuracy.

\[ \Delta a_{ph-ph} = [(6.4 \pm 0.1) \ln(m_\mu/m_e) + \text{const}] \times (\alpha/\pi)^3. \]
High-Precision Atomic Physics Tests of QED

All Accurate to ppm

- Lamb Shift in Hydrogen
- Hyperfine splitting of muonium and hydrogen
- Muonic Atom spectroscopy
- Positronium Lifetime

Crucial tool of atomic physics: Wavefunctions
Electron-Positron Annihilation

\[ e^+ e^- \rightarrow \gamma^* \rightarrow \mu^+ \mu^- \]
**Electron-Positron Annihilation**

\[ e^+ e^- \rightarrow \gamma^* \rightarrow q\bar{q} \]

Rate proportional to quark charge squared and number of colors

\[ R_{e^+ e^-}(E_{cm}) = N_{\text{colors}} \times \sum_q e_q^2 \]
How to Count Quarks

\[ R = \frac{\sigma(\text{hadrons})}{\sigma(\mu^+\mu^-)} \]

\[ \Delta R = N_C \times e_c^2 = 3 \times \left(\frac{2}{3}\right)^2 = \frac{4}{3} \]

\[ 3 \times \left(-\frac{1}{3}\right)^2 = \frac{1}{3} \]

\[ N_C = 3 \]

\[ R_{e^+e^-}(E_{cm}) = N_{\text{colors}} \times \sum_q e_q^2 \]

J/ψ = (c\bar{c})_{1S} \quad \Upsilon = (b\bar{b})_{1S}
Hadron Dynamics at the Amplitude Level

• DIS studies have primarily focussed on probability distributions: integrated and unintegrated.

• Impact of ISI and FSI: Single Spin Asymmetries, Diffractive Deep Inelastic Scattering, Shadowing, Anti-shadowing

• Test QCD at the amplitude level: Phases, multiparton correlations, spin, angular momentum, exclusive processes

• Wavefunctions: Fundamental QCD Dynamics
Wavefunctions: Fundamental description of composite systems

- Basic quantum mechanical quantities in atomic and nuclear physics
- Physics at the amplitude level
- **Schrödinger** wavefunction in nonrelativistic theory
- Relativistic formulation: Bethe Salpeter amplitudes evaluated at fixed time $t$
- Problem: “Instant” form: Frame-dependent
Dirac’s Amazing Idea: The “Front Form”

Evolve in light-cone time!

$\sigma = ct - z$

$\tau = t + z/c$

Instant Form

Front Form

$z_0 = 1$

$\Lambda_{\text{QCD}}$

$z\Delta$

$\Delta = 3 + L$: conformal dimension of meson

$P^+ = P_0 + P_z$

Fixed $\tau = t + z/c$

$\sigma = ct - z$

$\tau = t + z/c$
\[ P^+ = P^0 + P^z \]

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp i \]

\[ \Psi_n (x_i, \vec{k}_\perp i, \lambda_i) \]

Invariant under boosts! Independent of \( P^\mu \)

\[ \sum_i x_i = 1 \]

\[ \sum_i \vec{k}_\perp i = \vec{0}_\perp \]
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD} |\psi >= M^2 |\psi >$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
In terms of the hadron four-momentum $P = (P^+, P^-, \vec{P}_\perp)$ with $P^\pm = P^0 \pm P^3$, the light-front frame independent mass spectrum of the color-singlet states in QCD can be described by the Hamiltonian $H_{\text{LC}}^{\text{QCD}}$.

The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting $n$-particle states $|n\rangle$ with an infinite number of components

$$|\Psi_h(P^+, \vec{P}_\perp)\rangle = \sum_{n, \lambda} \int \left[dx_i \ d^2\vec{k}_\perp \right] \psi_{n/h}(x_i, \vec{k}_\perp, \lambda) \times |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp, \lambda\rangle$$

$$\sum_n \int [dx_i \ d^2\vec{k}_\perp] \ |\psi_{n/h}(x_i, \vec{k}_\perp, \lambda_i)|^2 = 1$$

Compute LFWFS from first principles

**QCD Phenomenology**

Stan Brodsky, SLAC

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\[ \sum_{i=1}^{n} k_{i}^+ = \sum_{i=1}^{n} x_i \tilde{P}^+ = \tilde{P}^+ \]

\[ \sum_{i=1}^{n} (x_i \tilde{P}_\perp + k_{i\perp}) = \tilde{P}_\perp \]

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ \vec{\ell}_j \equiv (\vec{k}_\perp \times \vec{b}_\perp)_j = (\vec{k}_\perp \times \frac{i \partial}{\partial \vec{k}_\perp})_j \]

\[ \text{n-1 Intrinsic Orbital Angular Momenta} \]

\[ \text{Frame Independent} \quad j = 1, 2, \cdots (n - 1) \]
Angular Momentum on the Light-Front

\[ J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z. \]

Conserved LF Fock state by Fock State

\[ l_j^z = -i \left( k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right) \]

n-1 orbital angular momenta
LFWFs of Electron (n=2)

\[
\psi_{\frac{1}{2}+1}^\uparrow (x, \vec{k}_\perp) = -\sqrt{2} \frac{|-k_1^1+|k_2^2|}{x|1-x|} \varphi ,
\]
\[
\psi_{\frac{1}{2}-1}^\uparrow (x, \vec{k}_\perp) = -\sqrt{2} \frac{|+k_1^1+|k_2^2|}{1-x} \varphi ,
\]
\[
\psi_{-\frac{1}{2}+1}^\uparrow (x, \vec{k}_\perp) = -\sqrt{2} (M - \frac{m}{x}) \varphi ,
\]
\[
\psi_{-\frac{1}{2}-1}^\uparrow (x, \vec{k}_\perp) = 0 ,
\]

where

\[
\varphi = \varphi (x, \vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}_\perp^2 + m^2)/x - (\vec{k}_\perp^2 + \lambda^2)/(1-x)} .
\]

\[M \rightarrow m + \lambda^l\]

Spin-1  mass \(\lambda^l\)

Spin-1/2 mass \(m\)

Drell, sjb
Hwang, Schmidt, sjb

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QCD Phenomenology
38

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Quantum Mechanics: Uncertainty in p, r, spin

Relativistic Quantum Field Theory: Uncertainty in particle number n

- Positronium n=2
  \( e^+ e^- \)

- Lamb Shift n=3
  \( e^+ e^- \gamma \)

- Hyperfine splitting n=3
  \( e^+ e^- \gamma \)

- Vacuum Polarization n=4
  \( e^+ e^- e^+ e^- \)

\( g_\text{e} \equiv \frac{1}{2} \left( 1.001159652201(30) \right) \)

\( g_\text{e} \text{ accurate to 11 figures!} \)
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp) \quad x_i = \frac{k_i^+}{P^+}$$

$$H_{QLF}^{QCD} |\psi> = M^2 |\psi>$$

Invariant under boosts. Independent of $P^\mu$
Central Property of Quantum Field Theory

Quantum Fluctuations

Fluctuations in

* Particle number $n = 2, 3, \ldots$

* Off-shellness $E \neq \sum E_i$

* Size, momenta, speed coordinates

* Orbital angular momentum

\[ J_z = \sum_{i=1}^n S^z_i + \sum_{i=1}^n L^z_i \]

Dirac:

Light Front Quantization

Fixed $\tau = t + x/c$

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^3}{p_0 + p_3} \]

\[ M_n^2 = \frac{m_i^2}{x_i^2} \left( \frac{k_i^2 + m_i^2}{x_i} \right) \]

\[ |\Psi_p\rangle = \frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} |n\rangle \langle n | \Psi_p\rangle \]

\[ \Psi_{n/p}(x_1, k_1^+, x) \]
Hadrons Fluctuate in Particle Number

- Proton Fock States
  \[ |uud >, |uudg >, |uuds\bar{s} >, |uudc\bar{c} >, |uudb\bar{b} > \cdots \]
- Strange and Anti-Strange Quarks not Symmetric
  \[ s(x) \neq \bar{s}(x) \]
- “Intrinsic Charm”: High momentum heavy quarks
- “Hidden Color”: Deuteron not always p + n

- Orbital Angular Momentum Fluctuations - Anomalous Magnetic Moment
Properties of Intrinsic Sea

\[ Q(x) \neq \overline{Q}(x) \]

QED analogy

Coulomb interactions break \( \Upsilon \pm \) symmetry

\[ \langle x_s^- \rangle < \langle x_s \rangle \]

\( x_s \sim -x_F \)
\[ S^- (x) = x [s(x) - \bar{s}(x)] \]
Test of $S(x) \neq \bar{S}(x)$ in $\bar{p}p$ reactions

Asymmetric distribution! $D_{c\bar{s}}$ harder than $D_{s\bar{c}}$ reflects $\bar{S}(x)$ harder than $S(x)$ in $\bar{p}$!

Conventional wisdom: $D_{c\bar{s}}$ and $D_{s\bar{c}}$ identical
Light-cone wavefunctions encode all helicity, transverse distributions.

\[ q^2 \lambda_p = \int_{\lambda_p} \]

\[ q(x, \Lambda) = \sum_{n, \xi} \left| \psi_n^{(\Lambda)} (x, k_x, \xi) \right|^2 \delta(x-x_0) \delta(\xi-\xi_0) \theta(\Delta^2 - m^2) \]
Exact Representation of Form Factors using LFWFs

Hadron form factors can be expressed as a sum of overlap integrals of light-front wave functions:

$$F(q^2) = \sum_n \int [dx_i] \left[ d^2\vec{k}_{\perp i} \right] \sum_j e_j \psi^*_n(x_i, \vec{k}'_{\perp i}, \lambda_i) \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the variables of the light-cone Fock components in the final-state are given by

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i) \vec{q}_{\perp},$$

for a struck constituent quark and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_{\perp},$$

for each spectator. The momentum transfer is $q^2 = -\vec{q}_{\perp}^2 = -2P \cdot q = -Q^2$. The measure of the phase-space integration is

$$[dx_i] = \prod_{i=1}^n \frac{dx_i}{\sqrt{x_i}} \delta \left(1 - \sum_{j=1}^n x_j\right),$$

$$[d^2\vec{k}_{\perp i}] = \left(16\pi^3\right) \prod_{i=1}^n \frac{d^2\vec{k}_{\perp i}}{16\pi^3} \delta^{(2)} \left(\sum_{\ell=1}^n \vec{k}_{\perp \ell}\right).$$
Light-Front Wavefunctions

Space-like Form factors computed
from diagonal $n = n'$ overlap

$q^+ = 0, \quad q^2 = -q_{\perp}^2 = -Q^2$

$$\langle p_1 + q, \lambda' \mid J^+(0) \mid p_2 \rangle$$

$$F_{2n'}(q^2) = \sum_n c_n E_n \int d^2k_2 \int d\alpha d\lambda$$

$$\Psi_n(x, k_2^2, \lambda) \Psi_n(x, k_2^2, \lambda')$$

$$k_2' = \left\{ \begin{array}{c}
\frac{k_2^2}{1 - x} - x Q^2 \\
\frac{k_2^2}{1 - x}
\end{array} \right. \quad \text{Struck parton}$$

$$\text{no ghosts (i.e.8.)}$$

$$\text{no vacuum graphs}$$

$$\text{no infinite sum of irreducible kernels}$$

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\[
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times \\
\left[ -\frac{1}{q_L} \psi_a^\dagger(x_i, k'_\perp, \lambda_i) \psi_a(x_i, k_\perp, \lambda_i) + \frac{1}{q_R} \psi_a^\dagger(x_i, k'_\perp, \lambda_i) \psi_a(x_i, k_\perp, \lambda_i) \right] \\
k'_\perp = k_\perp - x_i q_\perp \\
k'_j = k_\perp + (1 - x_j) q_\perp
\]

Must have \( \Delta \ell_z = \pm 1 \) to have nonzero \( F_2(q^2) \)
Anomalous Magnetic Moment $B(0)$

$F_2(0) = \frac{Q^2}{2\pi}$

$B(0) = \frac{Q^2}{8\pi} - \frac{Q^2}{8\pi} = 0$

$B(q^2) \sim \frac{q^2}{4\pi \sqrt{s'}}$

Equivalence Principle: $B(0) = 0$

Okun + Kobzarev (63)

X. Ji, Teryaev

LF: $\lim_{q^2 \to 0} \sum_{i=1}^n P_i \to$ Fock state

Result of Lorentz prop of LF wavefunction.

Huang, Ma, Schindl, 74

key Question for QCD Th

$B_p(0) = 0$?

Important indicator of lattice errors.
Light-Cone Wavefunction Representations of Anomalous Magnetic Moment and Electric Dipole Moment

In the case of a spin-\( \frac{1}{2} \) composite system, the Dirac and Pauli form factors \( F_1(q^2) \) and \( F_2(q^2) \), electric dipole moment form factor \( F_3(q^2) \) are defined by

\[
\langle P'|J^\mu(0)|P \rangle = U(P') \left[ F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5q_\alpha \right] U(P), \quad (47)
\]

**Compute matrix elements of good current \( J^+ \)**

\[
F_1(q^2) = \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle = \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle, \quad (48)
\]

\[
F_2(q^2) = \frac{1}{2M} \left[ + \frac{1}{-q^1 + iq^2} \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle + \frac{1}{q^1 + iq^2} \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle \right], \quad (49)
\]

\[
F_3(q^2) = \frac{i}{2M} \left[ + \frac{1}{-q^1 + iq^2} \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle - \frac{1}{q^1 + iq^2} \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle \right]. \quad (50)
\]
\[ \frac{F_2(q^2)}{2M} = \sum_a \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \sum_j e_j \frac{1}{2} \times \]
\[ \left[ + \frac{1}{-q^1 + iq^2} \psi^\dagger_a(x_i, \vec{k}'_\perp i, \lambda_i) \psi_a(x_i, \vec{k}_\perp i, \lambda_i) + \frac{1}{q^1 + iq^2} \psi^\dagger_a(x_i, \vec{k}'_\perp i, \lambda_i) \psi_a(x_i, \vec{k}_\perp i, \lambda_i) \right] \]

Drell, sbj,

\[ \frac{F_3(q^2)}{2M} = \sum_a \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \sum_j e_j \frac{i}{2} \times \]
\[ \left[ + \frac{1}{-q^1 + iq^2} \psi^\dagger_a(x_i, \vec{k}'_\perp i, \lambda_i) \psi_a(x_i, \vec{k}_\perp i, \lambda_i) - \frac{1}{q^1 + iq^2} \psi^\dagger_a(x_i, \vec{k}'_\perp i, \lambda_i) \psi_a(x_i, \vec{k}_\perp i, \lambda_i) \right] , \]

Gardner, Hwang, sbj,

\[ \vec{k}'_\perp i = \vec{k}_\perp i + (1 - x_i) \vec{q}_\perp \]  
struck quark  
[ \vec{k}'_\perp i = \vec{k}_\perp i - x_i \vec{q}_\perp \]  
spectator

QCD Phenomenology

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$F_3(q^2) = F_2(q^2) \times \tan \phi$

Fock state by Fock state

Gardner, Hwang, sjb,

New relation between $d_n$ and $d_p$
Nuclear Chromodynamics:
Novel Effects of QCD in Nuclear Systems

• QCD Color Transparency and Opaqueness
• Hidden Color
• Exclusive Nuclear Reactions, $x > 1$
• Nuclear shadowing and antishadowing
• Diffractive Phenomena
What if we ask for a specific final state?

Probability decreases with number of constituents!

$e^+ e^- \rightarrow p \bar{p}$

$s = (E_{e^+} + E_{e^-})^2$

$R(e^+e^- \rightarrow H \bar{H}) \propto |F(s)|^2$

$|F(s)| \propto \left(\frac{1}{s}\right)^{n_q - 1}$
Dispersive description of the ratio $GE / GM$

The "inverse problem"

Two photon contribution to $e^+e^- \rightarrow \ldots$

\[ \Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M_N} \sigma^{\mu\nu} q_\nu F_2(q^2) \]

Electric and Magnetic Form Factors

$G_E(q^2) = F_1(q^2) + \tau F_2(q^2)$
$G_M(q^2) = F_1(q^2) + F_2(q^2)$
$\tau = \frac{q^2}{4M_N^2}$

Elastic scattering
$e^- p \rightarrow e^- p$
\[ \frac{d\sigma}{d\Omega} = \alpha^2 E'_e \cos^2 \frac{\theta}{2} \left[ G_E^2 + \tau \left( 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right) G_M^2 \right] \frac{1}{1 + \tau} \]

Annihilation
$e^- e^- \rightarrow p\bar{p}$
\[ \frac{d\sigma}{d\Omega} = \alpha^2 \sqrt{1 - 1/\tau} \left[ (1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right] \]
Form Factors \( \ell p \rightarrow \ell' p' \left< p' \lambda' | J^+ (0) | p \lambda \right> \)

\[
F_{\lambda \lambda'} (Q^2) = \sum_n x, \bar{k}_\perp \rightarrow x, \bar{k}_\perp + q_\perp
\]

\[
p, \lambda \rightarrow \psi_n \rightarrow \psi_n \rightarrow p+q, \lambda'
\]

\[
\nabla \text{ Large } Q^2
\]

\[
\phi \rightarrow \psi_n \rightarrow \phi
\]

\[
T_H = \sum \rightarrow \gamma^* \rightarrow y_1 + \ldots
\]

\[
x_1 \rightarrow \gamma \rightarrow y_1
\]

\[
x_2 \rightarrow y_2
\]

\[
x_3 \rightarrow y_3
\]

\[
\frac{\alpha_s^2}{Q^4} \rightarrow f (x_i, y_i)
\]

Scaling from PQCD or AdS/CFT

QCD Phenomenology

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Hadron Distribution Amplitudes

\[ \phi(x_i, Q) \equiv \prod_{i=1}^{n-1} \int_Q d^2\vec{k}_\perp \, \psi_n(x_i, \vec{k}_\perp) \]

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Lepage; SJB
Efremov, Radyuskin
Proton Form Factor

Conformal Behavior: $t^2 F_1(t) = \text{const}$

Non-perturbative model: Diehl, Kroll

Remarkable scaling behavior

Non-perturbative model:

QCD Phenomenology

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Primary Test of QCD Factorization, Scaling

$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

Conformal Behavior: $$t^2 F_1(t) = \text{const}$$
Quark Counting Rules for Exclusive Processes

- Power-law fall-off of the scattering rate reflects degree of compositeness
- The more composite -- the faster the fall-off
- Power-law counts the number of quarks and gluon constituents
- Form factors: probability amplitude to stay intact
  \[ F_H(Q) \propto \frac{1}{(Q^2)^{n-1}} \quad n = \# \text{ elementary constituents} \]
Quark counting rules predict: \[ [Q^2]^{n_H-1} F_H(Q^2) \rightarrow \text{constant} \]
Timelike proton form factor in PQCD

\[ G_M(Q^2) \rightarrow \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} \left( \log \frac{Q^2}{\Lambda^2} \right)^\gamma_n^B + \gamma_n^\bar{B} \]

\[ \times \left[ 1 + \mathcal{O} \left( \alpha_s(Q^2), \frac{m^2}{Q^2} \right) \right] \]

Lepage and Sjb
PQCD and Exclusive Processes

\[ M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q) \]

- Iterate kernel of LFWFs when at high virtuality; distribution amplitude contains all physics below factorization scale
- Rigorous Factorization Formulae: Leading twist
- Underly Exclusive B-decay analyses
- Distribution amplitude: gauge invariant, OPE, evolution equations, conformal expansions
- BLM scale setting: sum nonconformal contributions in scale of running coupling
- Derive Dimensional Counting Rules/Conformal Scaling
In the large $\ell$ limit:

$$M^2 = \frac{\pi^2}{4} \frac{\ell^2}{\Lambda^2_{QCD}}$$

Timelike Proton Form Factor

Quark counting for 3 quarks in proton:

$$n_q - 1 = 3 - 1 = 2$$
Test of quark counting rule: timelike form factors

Proton timelike form factor.

\[ Q^2 |F_K(13.48 \text{ GeV}^2)| = 0.85 \pm 0.05(\text{stat}) \pm 0.02(\text{syst}) \text{ GeV}^2 \]

\[ Q^4 |G^p_M(13.48 \text{ GeV}^2)|/\mu_p = 0.91 \pm 0.13(\text{stat}) \pm 0.06(\text{syst}) \text{ GeV}^4 \]

The proton magnetic form factor result agrees with that measured in the reverse reaction \( p\bar{p} \rightarrow e^+e^- \) at Fermilab. **The kaon form factor measurement is the first ever direct measurement at \( |Q^2| > 4 \text{ GeV}^2 \).**

The pion form factor is being measured.
Conformal Behavior of LFWFs Predicted by AdS/CFT Leads to PQCD Scaling Laws

- Bjorken Scaling of DIS
- Counting Rules of Structure Functions at large $x$
- Dimensional Counting Rules for Exclusive Processes and Form Factors